RTS Smoother for GLMB filter

Tran Thien Dat Nguyen*, Jongmin Yu†
School of Electrical Engineering, Computing and Mathematical Sciences, Curtin University, Bentley, Australia
* t.nguyen172@postgrad.curtin.edu.au, † jongmin.yu@postgrad.curtin.edu.au

***Abstract*—In this paper, we implement a low-cost but effective smoothing strategy to smooth estimated tracks returned by the GLMB filter. While the forward filtering step is carried out via the GLMB filtering procedure, the backward smoothing step is recursively implemented from the final time step to the first time step via a smoothing algorithm. In particular, the smoothing algorithm is based on the Rauch-Tung-Striebel (RTS) of fixed-interval smoother. We demonstrate our smoothing strategy on a linear Gaussian model and the experimental results show consistent improved tracking performance over 100 Monte Carlo runs.**

***Index Terms*—Random finite sets, linear smoother, Rauch-Tung-Striebel smoother, generalized labeled multi-Bernoulli filter, multi-object tracking.**

I. Introduction

Filtering refers to the task of using all measurements up to the current instance to infer the state of the system while smoothing considers measurements up to some time in the future to infer the current state. Filtering can give online updates to the state while smoothing requires a "lagging" time for future measurements to smooth the current estimate. Given this constraint, the application of the smoother in real time problems is limited, however, for tasks where a time delay is allowed, smoothing procedure can significantly improve the estimation.

While single-object filtering is extensively studied in the literature with the Kalman filter [1], particle filter [2] and their variations, multi-object filtering is much more challenging with the presence of clutter, miss-detection and the variation in the cardinality of the objects set. Several frameworks have been proposed to tackle this problem such as the Joint Probabilistic Data Association (JPDA) [3], Multiple Hypotheses Tracking [4] and recently the Random Finite Sets (RFS) [4]. RFS forms the mathematical foundation for many modern multi-object tracking filters such as Probability Hypothesis Density (PHD) filter [5], Cardinalized PHD filter [6], the Generalized Labeled Multi-Bernoulli (GLMB) filter [7], [8] and the Labeled Multi-Bernoulli (LMB) filter [9].

Especially, the GLMB filter has recently gained attention from many researchers. The efficient GLMB filter is proposed in [10] via the combination of prediction and update step as well as the introduction of the Gibbs sampler to solve the tracks to measurements data association problem. The GLMB filter is reported to be able to track up to one million objects in [11]. This filter is also applied to track extended objects in [12] or objects with merged measurements in [13]. The spawning model is also incorporated into the GLMB filter in [14]. GLMB filter can also track objects in the conditions when clutter rate and detection profile are not available [15], [16]. Recently, GLMB filter is applied in practical applications such as people tracking in [17], [15] and biological cells tracking in [18], [19].

On the other hand, the development of practical smoothing algorithms for linear model dates back to the works of Bryson and Frazier [20] and Rauch, Tung and Striebel (RTS) [21] which are interpreted as the correction procedures for Kalman filter estimates. Based on the original smoothing concept, Fraser and Potter proposed the two filter smoothing method in [22]. For non-linear state-space model, Kitagawa [23] has developed the smoothing-while-filtering technique via the Sequential Monte Carlo (SMC) method while the same SMC technique is also implemented for forward backward smoother in [24] and [25]. The original two filter smoother is extended to the non-linear case via SMC method in [26] and [27]. The unscented transform is applied to the RTS Smoother framework in [28] for non-linear systems. Recently, [29] derived closed form solution of forward backward smoothing for Gaussian mixture models. One step fixed lag smoother for maneuvering object has been proposed in [30] then later the multi-sensor fixed lag smoother in [31].

In the context of multi-object tracking, smoothing techniques for the PHD filter has been proposed in [32] and [33] while smoothing for CPHD filter is proposed in [34]. Multi-Bernoulli smoothing is also proposed in [35]. Interacting Multiple Model (IMM) with fixed-lag smoothing has been applied to track multiple objects in [36]. However, the drawback of the mentioned smoothers is that the labels of objects are not included; therefore, objects trajectories cannot be constructed directly.

Recently, closed form forward-backward propagation for the GLMB filter has been put forward in the literature. The first mathematical derivation is introduced in [37] and further developed to the so called Multi-scan GLMB technique in [38]. The results show improvements in tracking performance but the computational effort is high due to the large smoothing state-space of the GLMB density. As the multi-scan GLMB smoother provides objects labels, smoothed objects trajectories can be inferred easily.

In this paper, we introduce the RTS smoothing algorithm into the multi-object GLMB filtering framework. In this approach, we do not attempt to smooth on the entire GLMB state space but we rather focus on smoothing the estimated trajec-

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tories given by the GLMB filter. Our method is implemented as a backward smoother to smooth individual track from the GLMB estimate. In particular, we apply the RTS smoother in [39] to smooth the trajectories of objects in a linear Gaussian tracking scenario.

The rest of the paper is organized as follow: in Section II, we provide background knowledge on multi-object tracking and the single object RTS smoothing algorithms for linear and non-linear models. Section III details our implementation of the smoothing algorithm within the GLMB filtering framework. In Section IV, we provide experimental results for both linear and non-linear models.

II. Background

A. Multi-object system and the GLMB filter

1) Notations: To facilitate our discussions, we adhere to the following notations. The set exponential is denoted as $[h(\cdot)]^X = \prod_{x\in X} h(x)$ and the inner product notation is denoted as $\langle f,g\rangle = \int f(x)g(x)dx$.

The generalization of the Kronecker delta is written as

$$\delta_Y(X) = \begin{cases} 1 & X = Y \\ 0 & X \neq Y \end{cases}$$

while set inclusion function is denoted as

$$1_Y(X) = \begin{cases} 1 & X \subseteq Y \\ 0 & otherwise \end{cases}$$

$\mathbf{X}$ denotes the labeled set of objects while $\mathbf{x} = (x,l)$ denotes a single labeled object. Specifically, $x \in \mathbb{X}$ and $l \in \mathbb{L}$ where $\mathbb{X}$ and $\mathbb{L}$ are respectively the kinematic state space and the discrete labels space at current time step. $\mathcal{L}$ is a label extraction function, i.e. $\mathcal{L}(\mathbf{x}) = l$ and $\mathcal{F}(\mathbf{X})$ denotes sets of finite subsets of $\mathbf{X}$. The "+" sign is used to indicate the next time step.

2) The measurement model: In the RFS multi-object tracking framework, we have a standard point measurement model of the form [40]

$$g(Z|\mathbf{X}) \propto \sum_{\theta\in\Theta(\mathcal{L}(\mathbf{X}))} \prod_{(x,l)\in\mathbf{X}} \psi^{(\theta(l))}(x,l|Z) \tag{1}$$

where

$$\psi^{(\theta(l))}(x,l|\{Z_{1:|Z|}\}) = \delta_0(\theta(l))q_D(x,l) + (1-\delta_0(\theta(l)))\frac{p_D(x,l)g(z_{\theta(l)}|x,l)}{\kappa(z_{\theta(l)})}$$

$\kappa(\cdot)$ is the clutter intensity, $p_D(x,l)$ and $q_D(x,l)$ are respectively the detection and miss-detection probabilities, $g(z|x,l)$ is the likelihood that (x,l) generates measurement z. $\theta : \mathbb{L} \to \{0 : |Z|\}$ is a positive 1-1 map and Θ is the entire set of such mappings.

3) The GLMB filter: In this subsection, we outline the standard procedure to track a set of objects with the efficient GLMB filter.

Given a GLMB prior [10]:

$$\pi(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{(I,\xi)\in\mathcal{F}(\mathbb{L})\times\Xi} \omega^{(I,\xi)}\delta_I(\mathcal{L}(\mathbf{X}))[p^{(\xi)}]^{\mathbf{X}} \tag{2}$$

and the LMB births of the form [10]

$$\mathbf{f}_B(\mathbf{X}_{B+}) = \Delta(\mathbf{X}_{B+})w_B(\mathcal{L}(\mathbf{X}_{B+}))[p_{B,+}]^{\mathbf{X}_{B+}} \tag{3}$$

$$w_B(\mathcal{L}(\mathbf{X}_{B+})) = 1_{\mathbb{B}+}(\mathcal{L}(\mathbf{X}_{B+})) [1-r_{B,+}]^{\mathbb{B}_+ - \mathcal{L}(\mathbf{X}_{B+})}[r_{B,+}]^{\mathcal{L}(\mathbf{X}_{B+})}$$

where $r_{B,+}$ is the probability of birth and $p_{B,+}$ is the density of the birth kinematic state, the joint predicted and updated density is given by [10]

$$\pi_{Z_+}(\mathbf{X}) \propto \Delta(\mathbf{X}) \sum_{I,\xi,I_+,\theta_+} \omega^{(I,\xi)}\omega_{Z_+}^{(I,\xi,I_+,\theta_+)} \delta_{I_+}[\mathcal{L}(\mathbf{X})][p_{Z_+}^{(\xi,\theta_+)}]^{\mathbf{X}}$$

where $I \in \mathcal{F}(\mathbb{L})$, $\xi \in \Xi$, $I_+ \in \mathcal{F}(\mathbb{L}_+)$, $\theta_+ \in \Theta_+$ with ξ is tracks to measurements association history and Ξ is the entire space of ξ,

$$\omega_{Z_+}^{(I,\xi,I_+,\theta_+)} = 1_{\Theta_+(I_+)}(\theta_+)[1-\bar{P}_S^{(\xi)}]^{I-I_+}[\bar{P}_S^{(\xi)}]^{I\bigcap I_+} \times [1-r_{B,+}]^{\mathbb{B}_+ - I_+}[r_{B,+}]^{\mathbb{B}_+\cap I_+}[\bar{\psi}_{Z_+}^{(\xi,\theta_+)}]^{I_+}$$

$$\bar{P}_S^{(\xi)}(l) = \langle p^{(\xi)}(\cdot,l), P_S(\cdot,l)\rangle$$

$$\bar{\psi}_{Z_+}^{(\xi,\theta_+)}(l_+) = \langle \bar{p}_+^{(\xi)}(\cdot,l_+), \psi_{Z_+}^{(\theta_+(l_+))}(\cdot,l_+)\rangle$$

$$\bar{p}_+^{\xi}(x_+,l_+) = 1_{\mathbb{L}}(l_+)\frac{\langle P_S(\cdot,l_+)f_+(x_+|\cdot,l_+), p^{(\xi)}(\cdot,l_+)\rangle}{\bar{P}_S^{(\xi)}(l_+)} + 1_{\mathbb{B}_+}(l_+)p_{B,+}(x_+,l_+)$$

$$p_{Z_+}^{(\xi,\theta_+)}(x_+,l_+) = \frac{\bar{p}_+^{(\xi)}(x_+,l_+)\psi_{Z_+}^{(\theta_+(l_+))}(x_+,l_+)}{\bar{\psi}_{Z_+}^{(\xi,\theta_+)}(l_+)}$$

$P_S(\cdot,l_+)$ is the survival probability of track with label l_+ and $f_+(x_+|\cdot,l_+)$ is the single object transition density. In this efficient implementation of the GLMB filter, the highly feasible tracks to measurements associations are sampled via Gibbs sampler which is cheaper than the original Murty algorithm implemented in [8].

The GLMB filter estimate is the maximum a posteriori estimate of the cardinality and the mean of the multi-object state given the estimated cardinality. The cardinality distribution of the objects can be calculated as:

$$\rho(n) = \sum_{(I,\xi)\in\mathcal{F}(\mathbb{L})\times\Xi} \sum_{L\in\mathcal{F}_n(\mathbb{L})} \omega^{(I,\xi)}\delta_I(L)$$

$$= \sum_{(I,\xi)\in\mathcal{F}_n(\mathbb{L})\times\Xi} \omega^{(I,\xi)}$$

where $\mathcal{F}_n(\mathbb{L})$ denotes the sets of finite subset of $\mathbb{L}$ with exact n elements.

The estimated cardinality is calculated as:

$$\widehat{N} = \text{argmax}(\rho) \tag{4}$$

Following that, the estimated hypothesis is given by:

$$(\widehat{I}, \widehat{\xi}) = \text{argmax}_{(I,\xi)}\omega^{(I,\xi)}\delta_{\widehat{N}}(|I^{(\xi)}|) \tag{5}$$

Finally, the estimated labeled multi-object state can be written as:

$$\widehat{\mathbf{X}} = \{(x,l) : l \in \widehat{I}^{(\widehat{\xi})}, x = \int y p^{(\widehat{I},\widehat{\xi})}(y,l)dy\} \tag{6}$$

B. The single object RTS smoother

First introduced in [21], the RTS smoother corrects the covariance matrices and means to smooth the estimated states. Given a linear model of the form

$$x_+ = Fx + q$$

$$z = Hx + r$$

with x is the system state, F is the linear transformation matrix, H is the linear observation matrix, q and r are respectively the process and observation Gaussian noise and z is the current time step measurement; while the forward filtering process can be carried out via a standard Kalman filter [1], the state can be smoothed over a fixed interval from initial time step to time step $N \leq K$ (where K is the total number of tracking time steps) via the RTS smoother. The steps of the RTS smoothing procedure is given in Algorithm 1. The detailed discussion on the algorithm can be found in [21]. It should be noted that superscript s indicated the smoothed results.

Algorithm 1 Single object RTS smoother

Input: The filtered mean and covariance $\{x_k, P_k\}_{k=1:N}$, F, Q
Output: The smoothed mean and covariance $\{x_k^s, P_k^s\}_{k=1:N}$
Initialization: $x_N^s = x_N$ and $P_N^s = P_N$
for $k = N-1$ down to 1
  $\bar{x}_{k+1} = Fx_k$
  $\bar{P}_{k+1} = FP_kF^T + Q$
  $D = P_{k+1}F(\bar{P}_{k+1})^{-1}$
  $x_k^s = x_k - D(x_{k+1}^s - \bar{x}_{k+1})$
  $P_k^s = P_k - D(P_{k+1}^s - \bar{P}_{k+1})D^T$
end

III. The RTS smoother for GLMB filter

From the GLMB estimate procedure (given in (4), (5) and (6)), it is possible that the correct hypothesis might not be chosen at one or several time steps due to miss-detection and false measurements. It increases the localization error and creates discontinuity in the trajectories. Our proposed method tackles these issues via a backward smoothing process from the end of the tracking interval to the initial time step. For each estimated track from the GLMB estimate, the RTS smoother, first, interpolates discontinuity point (hole) or consecutive discontinuity points (holes sequence) in each trajectory by applying single object filtering algorithm within the missing interval. The initial state of such filtering process is the state of the object right before discontinuity happens and the measurement(s) during discontinuity is given by the association history of the track right after the discontinuity (The readers are reminded here that each GLMB estimation of a track labeled l at time k provides the indices of the associated measurements from time k down to the time when the track firstly comes into existence, hence from the indices and the measurements set we can extract the required measurements for the interpolating process). Subsequently, single object RTS smoother is applied for each interpolated trajectory. The details of our proposed smoother is given in Algorithm 3. The complexity of our algorithm depends only on the number of discontinuity points in the trajectories, the length of the trajectories and the number of estimated trajectories. It does not depend on the number of hypotheses of the GLMB density, hence given its the low-cost characteristic.

Algorithm 2 RTS smoother for GLMB filter

Input: Estimated tracks from GLMB filter $\{\hat{\xi}_k, \hat{I}_k, \{\hat{p}^{\hat{\xi}_k}(x_l)\}_{l\in\hat{I}_k}\}_{k=1:K}$
Output: Smoothed and interpolated estimated tracks $\{\tilde{I}_k, \{\tilde{p}^{\hat{\xi}_k}(x_l)\}_{l\in\tilde{I}_k}\}_{k=1:K}$
for $k = 1:N$
  $\mathfrak{I} = unique(\bigcup_{k=1}^N \hat{I}_k)$
  $\tilde{I}_k = \hat{I}_k$
end
for $l \in \mathfrak{I}$
  Detect birth time k_0^l
  Detect death time k_{end}^l
  for $k = k_0^l : k_{end}^l$
    Detect hole/holes sequences
    Fill hole/holes sequences with standard filtering technique
    Update the estimated labels set $\tilde{I}_k = \tilde{I}_k \cup l$
  end
  Run single object smoothing for track l from $k = k_{end}^l$ down to $k = k_0^l$ to obtain $\{\tilde{p}^{\hat{\xi}_k}(x_l)\}_{k=k_0^l:k_{end}^l}$
end

IV. Experimental results

In this experiment, we use constant velocity model for the dynamic of the system. The state vector consists of the information regarding the planar position and the velocity of the objects which is $x_k = [p_x, p_y, \dot{p}_x, \dot{p}_y]^T$; while the measurement vector contains the position of the object which is $z_k = [z_x, z_y]^T$. The transition and observation models are given respectively as:

$$f_+(x_+|x) = \mathcal{N}(x_+; Fx, Q)$$

$$h(z|x) = \mathcal{N}(z; Hx, R)$$

where
$F = \begin{bmatrix} I_2 & \Delta I_2 \\ 0_2 & I_2 \end{bmatrix}$, $Q = \sigma_v^2 \begin{bmatrix} \frac{\Delta^4}{4} I_2 & \frac{\Delta^3}{2} I_2 \\ \frac{\Delta^3}{2} I_2 & \Delta^2 I_2 \end{bmatrix}$
$H = \begin{bmatrix} I_2 & 0_2 \end{bmatrix}$, $R = \sigma_\epsilon^2 I_2$. Particularly, in this experiment we set $\sigma_v = 5\,m$ and $\sigma_\epsilon = 15\,m$.

The surveillance region is set to $[-1000, 1000]\ m \times [-1000, 1000]\,m$ and the total time step is $K = 100$. The surviving probability is set to $p_S = 0.99$ and the detection probability is $p_D = 0.95$. Clutter rate is set to 66 false alarms per scan. The birth probability is set to $r_B = 0.03$. The location of births are $m_B^{(1)} = [0.1, 0, 0.1, 0]^T$, $m_B^{(2)} = [400, 0, -600, 0]^T$, $m_B^{(3)} = [-800, 0, -200, 0]^T$, $m_B^{(4)} = [-200, 0, 800, 0]^T$. The covariance matrix at birth is $P_B = diag([10, 10, 10, 10])$. The number of hypotheses for GLMB filter is capped at 20000 components. In this experiment we smooth the entire tracking interval from $k = 1$ to $k = K$.

From visual inspection of Figure 1, it is observed that smoother trajectories are obtained for the smoothed results comparing to the filtered-only results. In terms of OSPA1 [41] (cut off at 100) and OSPA2 [42] (cut off at 100 and window length is 10) tracking errors, Figure 2 and Figure 3 show the improvement for the smoothed results over 100 Monte Carlo runs. However, at around $t = 40$ and $t = 60$, when the tracks crossings occur, the uncertainty of OSPA Localization errors increase due to the tracks fragmentation effect in the filtered estimate. As the tracks identities are switched, the smoother diverges the tracks away from the correct paths hence it increases the localization error. Figure 4 shows the smoothed estimates have higher values of the cardinality compared to the standard filtered estimates due to the interpolation process.

V. Conclusion

In this paper, we have implemented RTS smoothing technique to smooth the trajectories returned by the GLMB filter estimation. We demonstrate our implementation on a linear Gaussian model. The results show improvements in tracking results in terms of OSPA errors and by visual inspection of the trajectories. The possible disadvantage of this method is when the filtering results are incorrect in terms of tracks identities (tracks fragmentation), the smoother might diverge the tracks away from the true paths which indeed increases the localization error.

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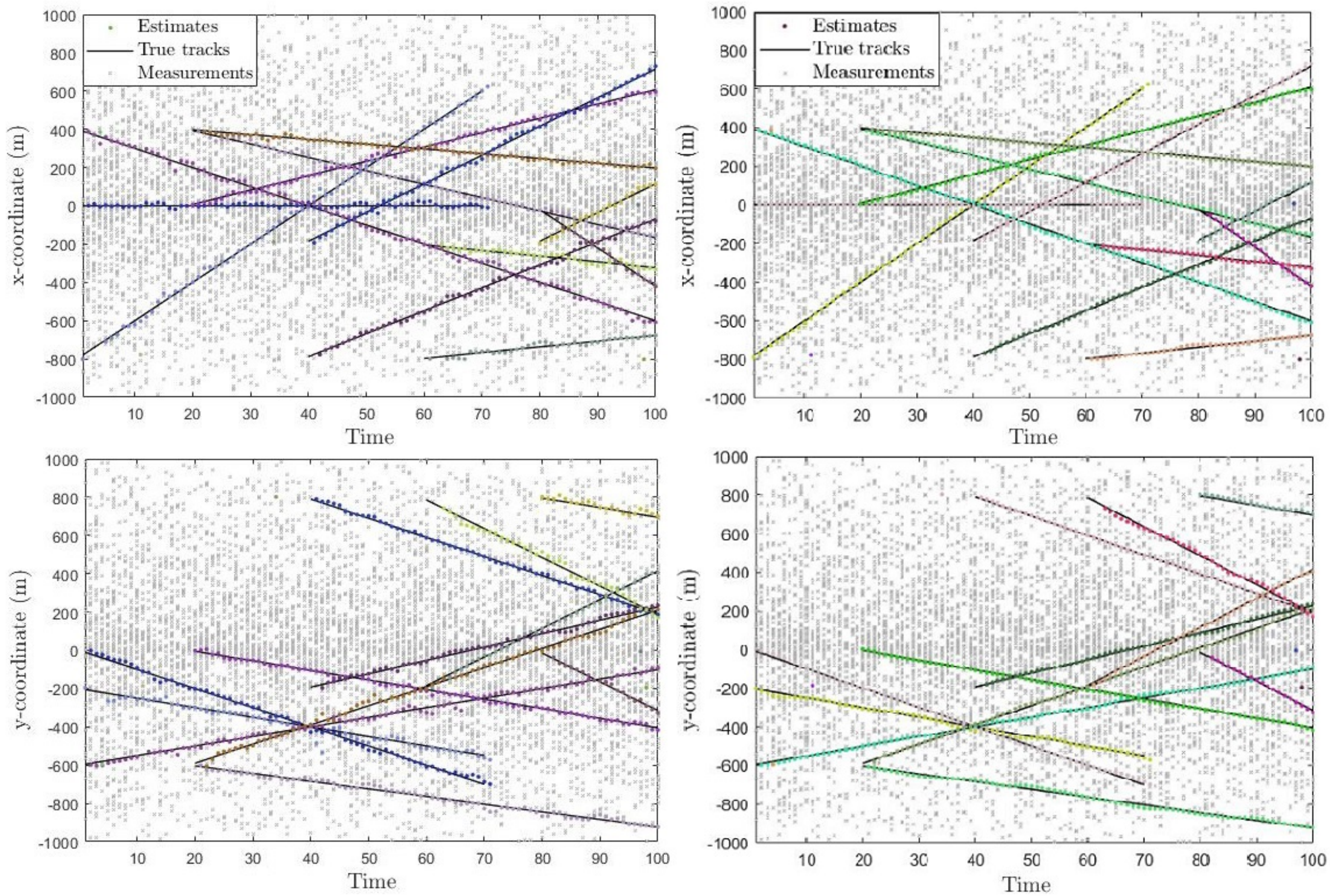

Fig. 1. Filtered-only trajectories (left) and smoothed trajectories (right)

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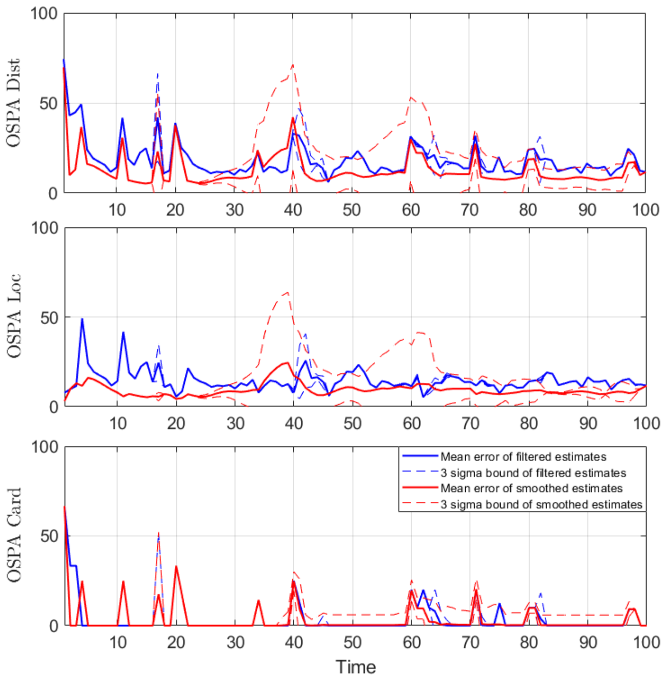

Fig. 2. OSPA1 error over 100 Monte Carlo runs

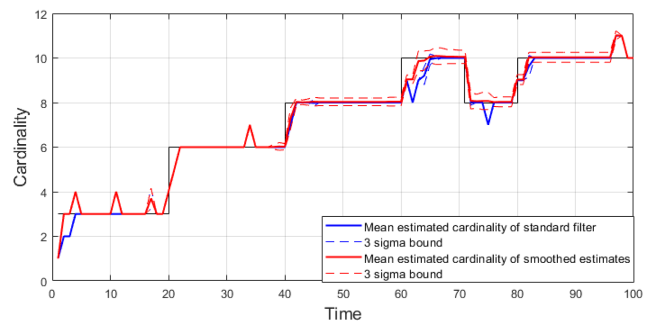

Fig. 4. Cardinality of the objects over 100 Monte Carlo runs

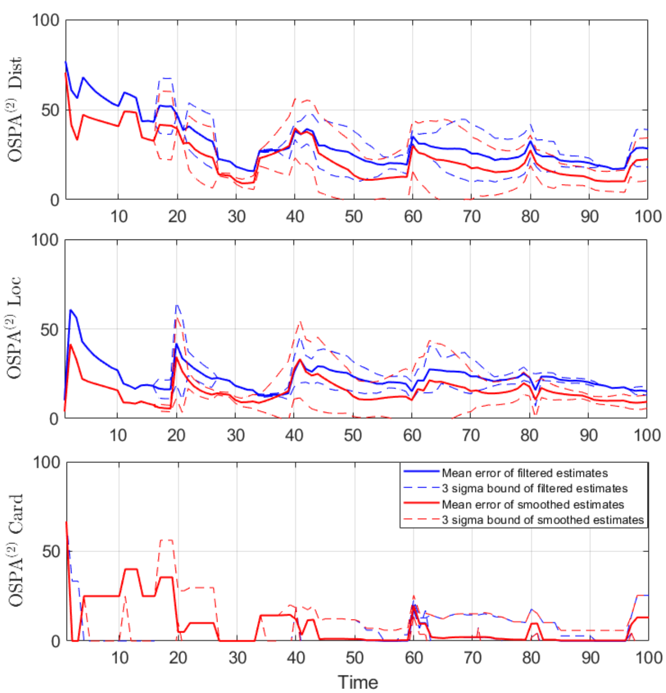

Fig. 3. OSPA2 error over 100 Monte Carlo runs