Bounding of double-differenced correlated errors of multi-GNSS observations using RTK for AV positioning

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Contents

IM for Autonomous vehicles

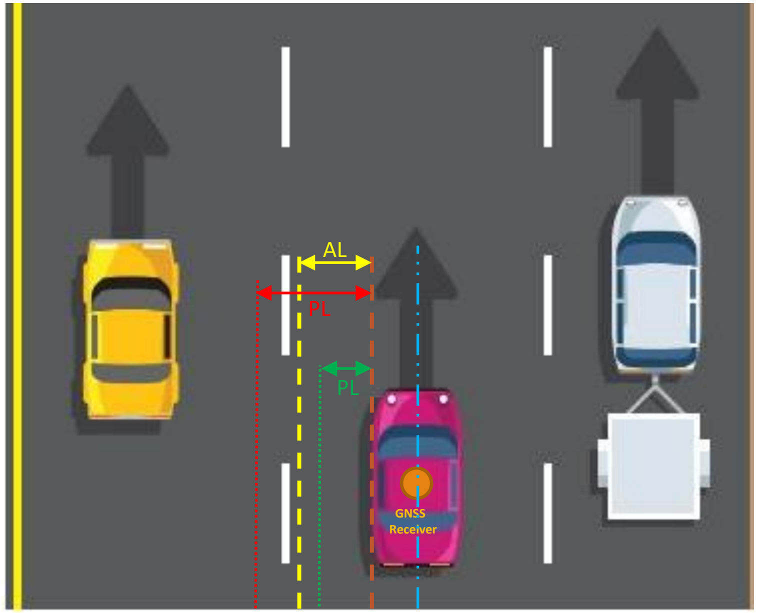
- Real-time positioning is key component in Autonomous Vehicles (AV).
- Safety is paramount for AV.
- We need to guarantee the reliability of positioning, i.e. Positioning integrity monitoring (IM).

Background: Integrity monitoring

Positioning Integrity Monitoring is about how much trust the user can put in the obtained position.

Integrity definitions:

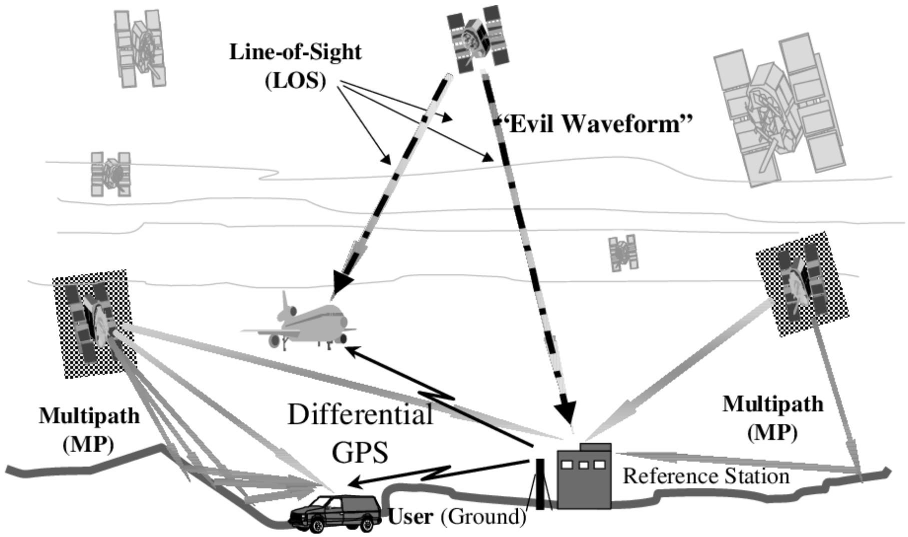
- Alert Limit (AL)
- Protection level (PL)
- Probability of misleading information (P_{HMI})
- Probability of false alert (P_{FA})



ARAIM

- Advanced Receiver Autonomous Integrity Monitoring or ARAIM
- Developed for aviation applications with multi-frequency observations and multi-hypothesis for faults

Airborne applications	Ground applications
Smoothed code observations	Phase observations
Snapshot positioning	Precise positioning (RTK)
Tens of meters accuracy	cm/dm accuracy



Multicorrelator techniques for robust mitigation of threats to GPS signal quality (R. E. Phelts, 2001)

ARAIM baseline steps and equations

Calculating the all-in-view position $\hat{x}^{(0)}$

$$\Delta\hat{x} = \left(G^T W G\right)^{-1} G^T W\, r$$

$\Delta\hat{x}$ is position solution update at each iteration

Define the threat model with (K) fault modes and calculate the following for each fault mode subset:

1) Probability ($P^{(K)}$) 2) Position solution ($\hat{x}^{(K)}$)

Commencing FDE

SST

$$T_{k,q} = K_{fa,q}\, \sigma_{ss,q}^{(K)}$$

$$\tau_{k,q} = \frac{\left|\hat{x}_q^{(K)} - \hat{x}_q^{(0)}\right|}{T_{k,q}} \leq 1$$

Chi-square test

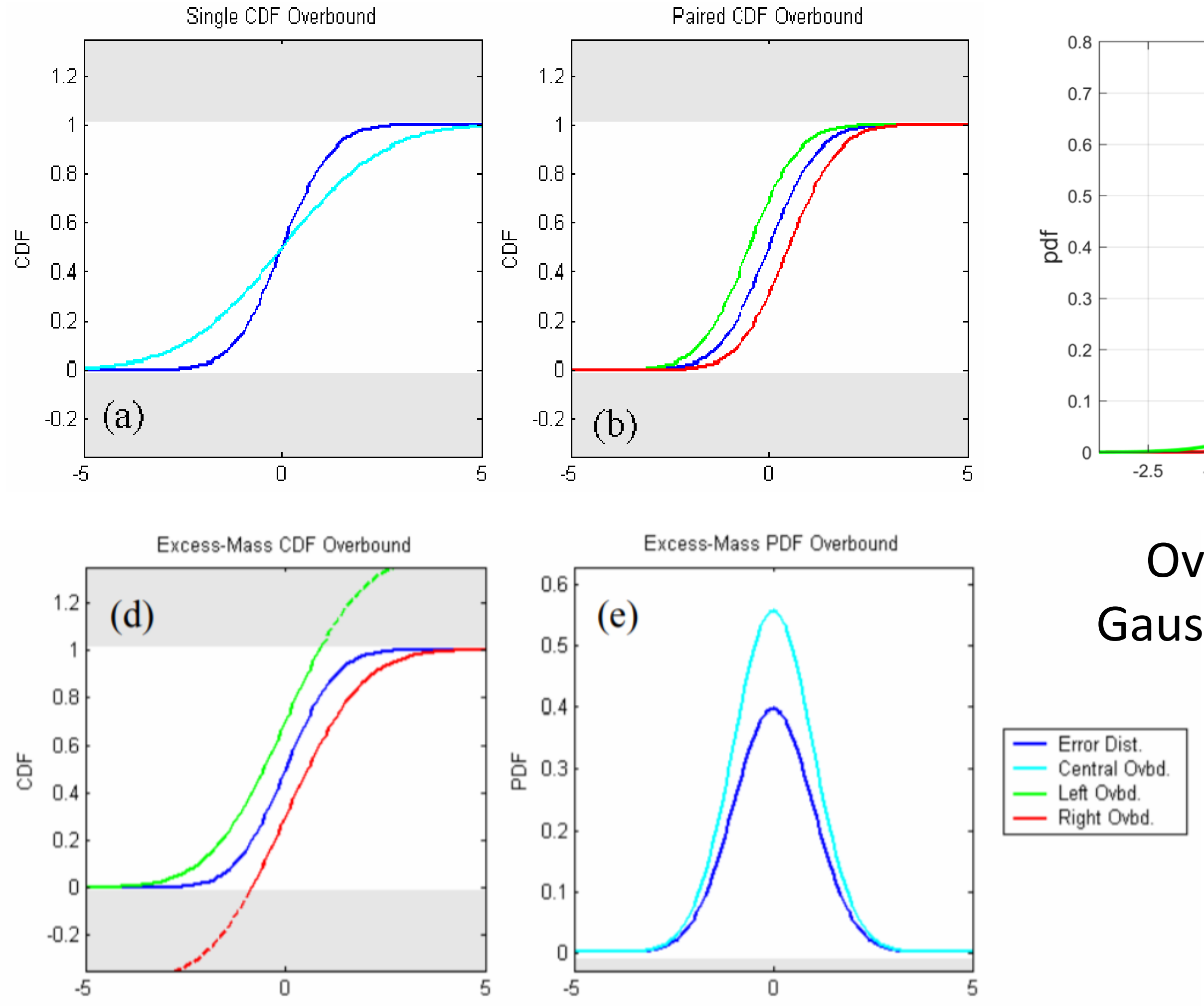
$$\chi^2 = y^T \left(W - WG\left(G^T W G\right)^{-1} G^T W\right) y$$

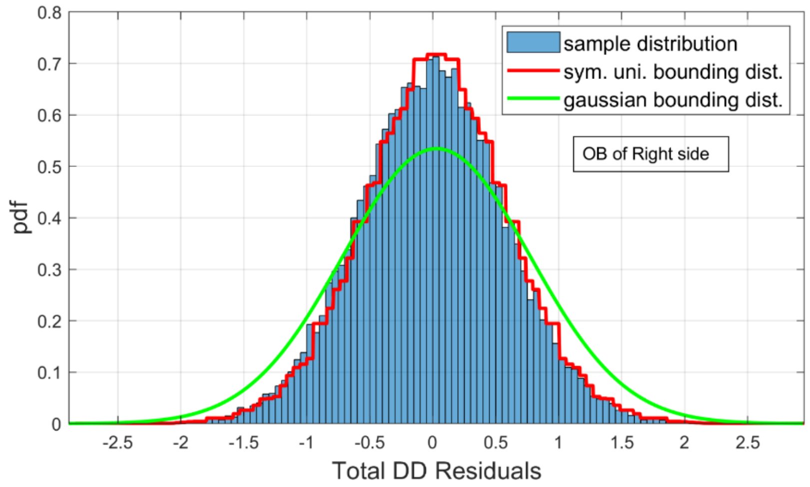
$$T_{\chi^2} = \left(\chi^2_{deg}\right)^{-1} \left(1 - P_{FA-Chi^2}\right)$$

When FDE passes, Commencing PL computation

$$2Q\left[\frac{PL - b_q^{(0)}}{\sigma_q^{(0)}}\right] + \sum_{K=1}^{N_{fault\ modes}} P_{fault,K}\, Q\left[\frac{PL - T_{K,q} - b_q^{(K)}}{\sigma_q^{(K)}}\right] = \frac{1}{2} PHMI_{Horizontal}$$

Overbounding of the CDF of the error distribution





Overbounding using Two Step Gaussian Bounding Method (TSGB)

Integrity Overview
(Todd Walter, Bruce DeCleene 2015)

Research objective

Building a stochastic model that represents the double-differenced multi-GNSS observations errors for precise positioning of autonomous vehicles utilizing an error bounding technique.

$$
\begin{bmatrix}
\sigma_{G1,2}^2 & q_{G1,2,3}\,\sigma_{G1,2}\,\sigma_{G1,3} & q_{G1,2,4}\,\sigma_{G1,2}\,\sigma_{G1,4} & 0 & 0 & 0 & 0 & 0 & 0 \\
q_{G1,2,3}\,\sigma_{G1,2}\,\sigma_{G1,3} & \sigma_{G1,3}^2 & q_{G1,3,4}\,\sigma_{G1,3}\,\sigma_{G1,4} & 0 & 0 & 0 & 0 & 0 & 0 \\
q_{G1,2,4}\,\sigma_{G1,2}\,\sigma_{G1,4} & q_{G1,3,4}\,\sigma_{G1,3}\,\sigma_{G1,4} & \sigma_{G1,4}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{E1,2}^2 & q_{E1,2,3}\,\sigma_{E1,2}\,\sigma_{E1,3} & q_{E1,2,4}\,\sigma_{E1,2}\,\sigma_{E1,4} & 0 & 0 & 0 \\
0 & 0 & 0 & q_{E1,2,3}\,\sigma_{E1,2}\,\sigma_{E1,3} & \sigma_{E1,3}^2 & q_{E1,3,4}\,\sigma_{E1,3}\,\sigma_{E1,4} & 0 & 0 & 0 \\
0 & 0 & 0 & q_{E1,2,4}\,\sigma_{E1,2}\,\sigma_{E1,4} & q_{E1,3,4}\,\sigma_{E1,3}\,\sigma_{E1,4} & \sigma_{E1,4}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{C1,2}^2 & q_{C1,2,3}\,\sigma_{C1,2}\,\sigma_{C1,3} & q_{C1,2,4}\,\sigma_{C1,2}\,\sigma_{C1,4} \\
0 & 0 & 0 & 0 & 0 & 0 & q_{C1,2,3}\,\sigma_{C1,2}\,\sigma_{C1,3} & \sigma_{C1,3}^2 & q_{C1,3,4}\,\sigma_{C1,3}\,\sigma_{C1,4} \\
0 & 0 & 0 & 0 & 0 & 0 & q_{C1,2,4}\,\sigma_{C1,2}\,\sigma_{C1,4} & q_{C1,3,4}\,\sigma_{C1,3}\,\sigma_{C1,4} & \sigma_{C1,4}^2
\end{bmatrix}
$$

A simple example of a variance covariance matrix at one epoch encompassing 12 satellites, i.e. three DD from each constellation.

Methodology

1. Collect one year of RINEX data from two Curtin CORS
2. Compute Double-differenced errors
3. Mass data classification, processing and analyzing
4. bounding using TSGB method

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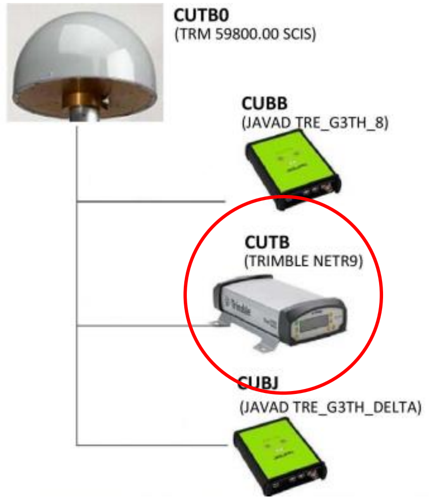
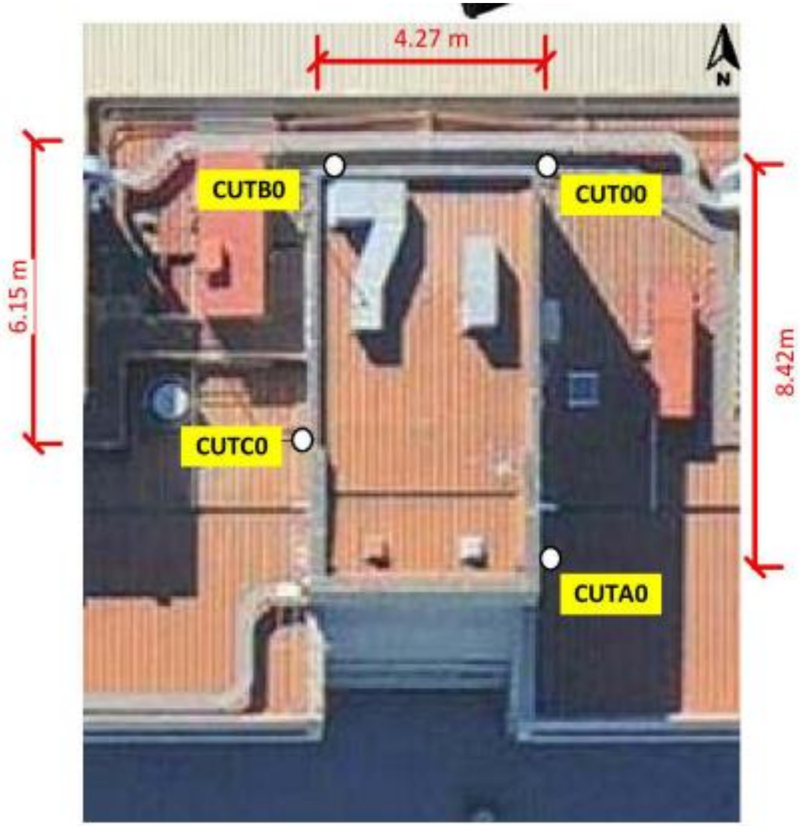
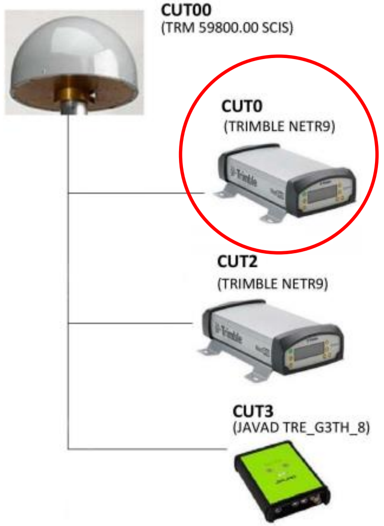
Methodology: Data collection



Receivers configuration with station CUTB



Plane view of CORS on top of building 402 – Curtin University



Receivers configuration with station CUT0

Methodology

Collect one year of RINEX data from two Curtin CORS

Compute Double-differenced errors

Mass data classification, processing and analyzing

bounding using TSGB method

Methodology: Errors extraction

- For phase $\varphi^{jk}_{rb,i} = \rho^{jk}_{rb} + \lambda_i(B^{j}_{rb,i} - B^{k}_{rb,i}) + \varepsilon_\varphi + b_\varphi$
- For code $P^{jk}_{rb,i} = \rho^{jk}_{rb} + \varepsilon_P + b_P$
- $\varphi^{jk}_{rb,i}$ and $P^{jk}_{rb,i}$: The DD value of phase and code measurements, respectively for satellites j and k and stations r and b for signal i.
- ρ^{jk}_{rb} : The DD value of the true range between the stations and the satellites.
- λ_i : The wavelength of the observed signal.
- $(B^{j}_{rb,i} - B^{k}_{rb,i})$: The DD value of ambiguity values.
- ε_φ and ε_p: are DD phase and code noise, respectively.
- b_φ and b_P: are DD phase and code biases, respectively.

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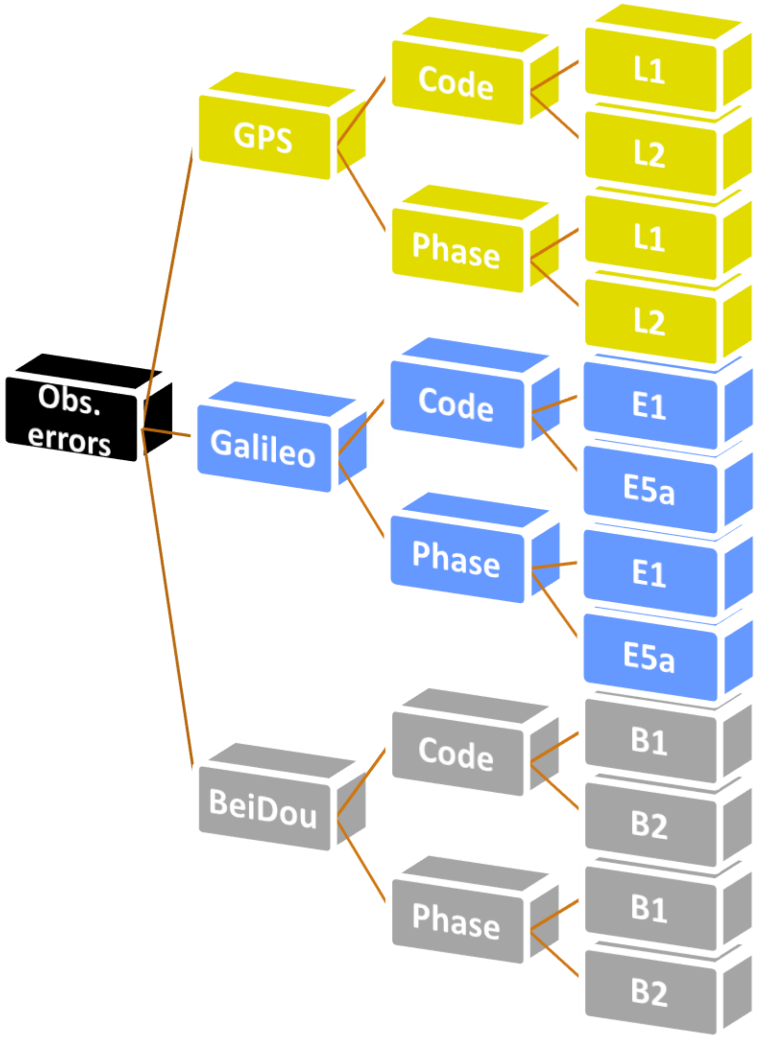
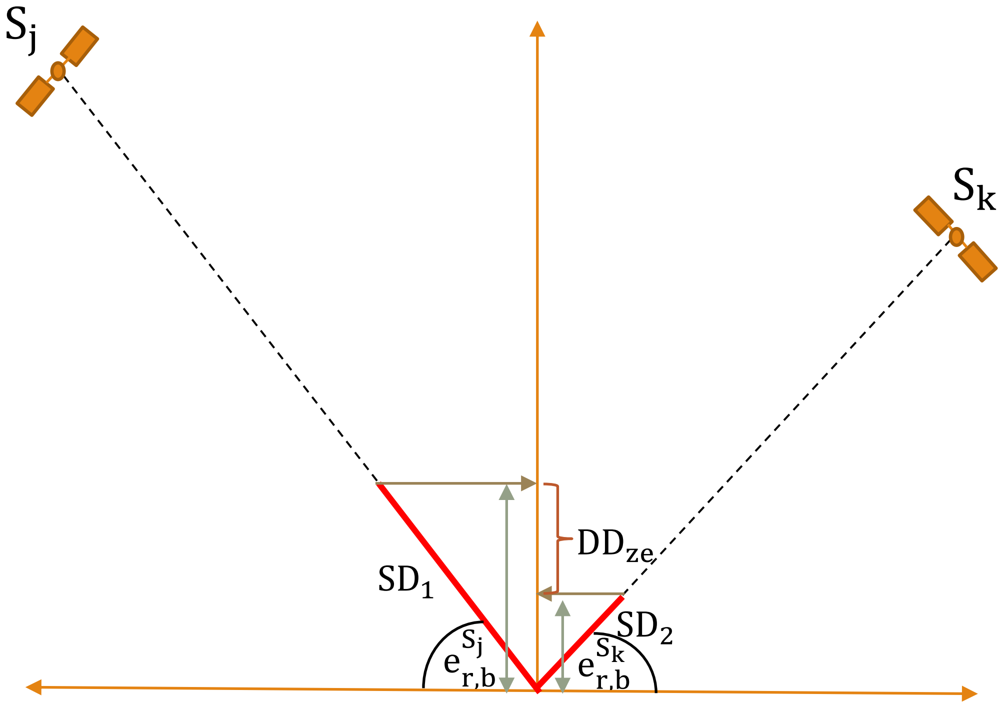
Methodology: Data analysis – 1st empirical method

- Classification:



- Mapping:



$$\varepsilon_{rb}^{jk} = \varepsilon_{rb}^{j} - \varepsilon_{rb}^{k}$$
$$\varepsilon_{rb_{ze}}^{jk} = \varepsilon_{rb_{ze}}^{j} - \varepsilon_{rb_{ze}}^{k}$$
$$\varepsilon_{rb_{ze}}^{jk} = \varepsilon_{rb}^{j} f(e_r^{S_j}) - \varepsilon_{rb}^{k} f(e_b^{S_k})$$

Methodology: Data analysis – 1st empirical method

The variance matrix for undifferenced observations at stations r and b:

$$Q_{rb} = \begin{bmatrix} \sigma_0^2 f(e_r^{S_1}) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_0^2 f(e_b^{S_1}) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_0^2 f(e_r^{S_2}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_0^2 f(e_b^{S_2}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_0^2 f(e_r^{S_n}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0^2 f(e_b^{S_n}) \end{bmatrix}$$

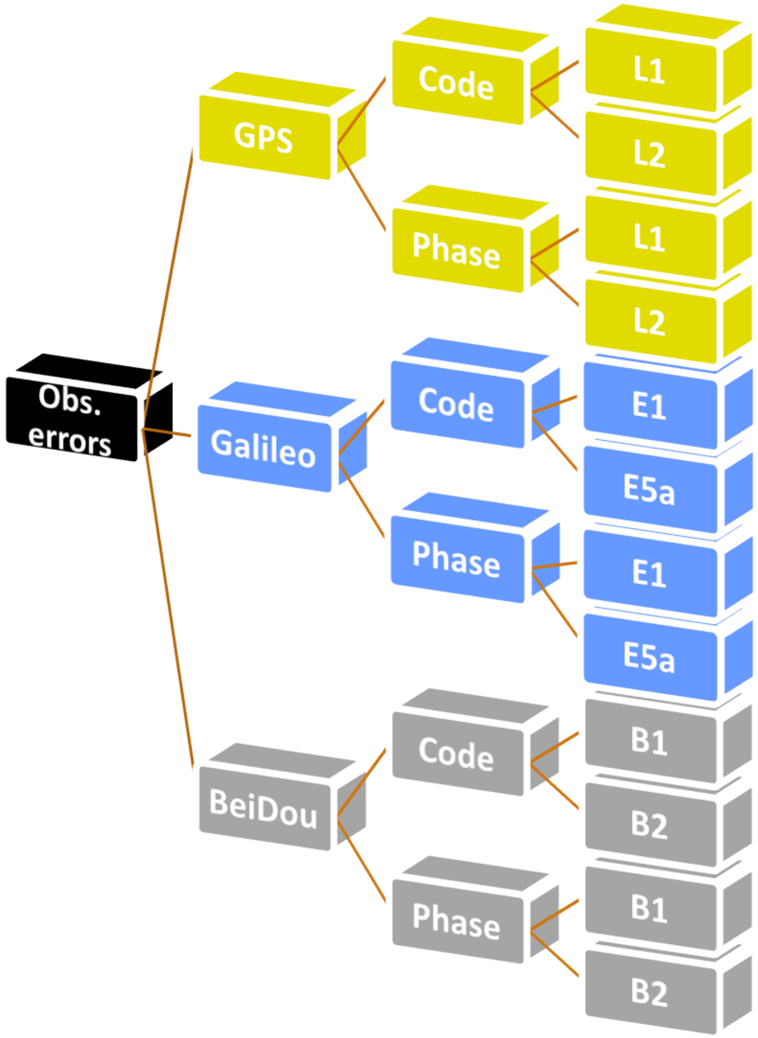
$f(e_r^{S_n})$ and $f(e_b^{S_n})$ are the mapping function dependent on EA $e_r^{S_n}$ or $e_b^{S_n}$, for satellites s1 to sn .

The VC matrix for DD observations: $\mathrm{Q_{DD}} = \mathrm{A(Q_{rb})A^T}$

The correlation coefficient between DD observations:

$$q_{ij} = \frac{f\left(e_r^{S_1}\right) + f\left(e_b^{S_1}\right)}{\sqrt{\left(f\left(e_r^{S_1}\right) + f\left(e_b^{S_1}\right) + f\left(e_r^{S_i}\right) + f\left(e_b^{S_i}\right)\right)} \times \sqrt{\left(f\left(e_r^{S_1}\right) + f\left(e_b^{S_1}\right) + f\left(e_r^{S_j}\right) + f\left(e_b^{S_j}\right)\right)}}$$

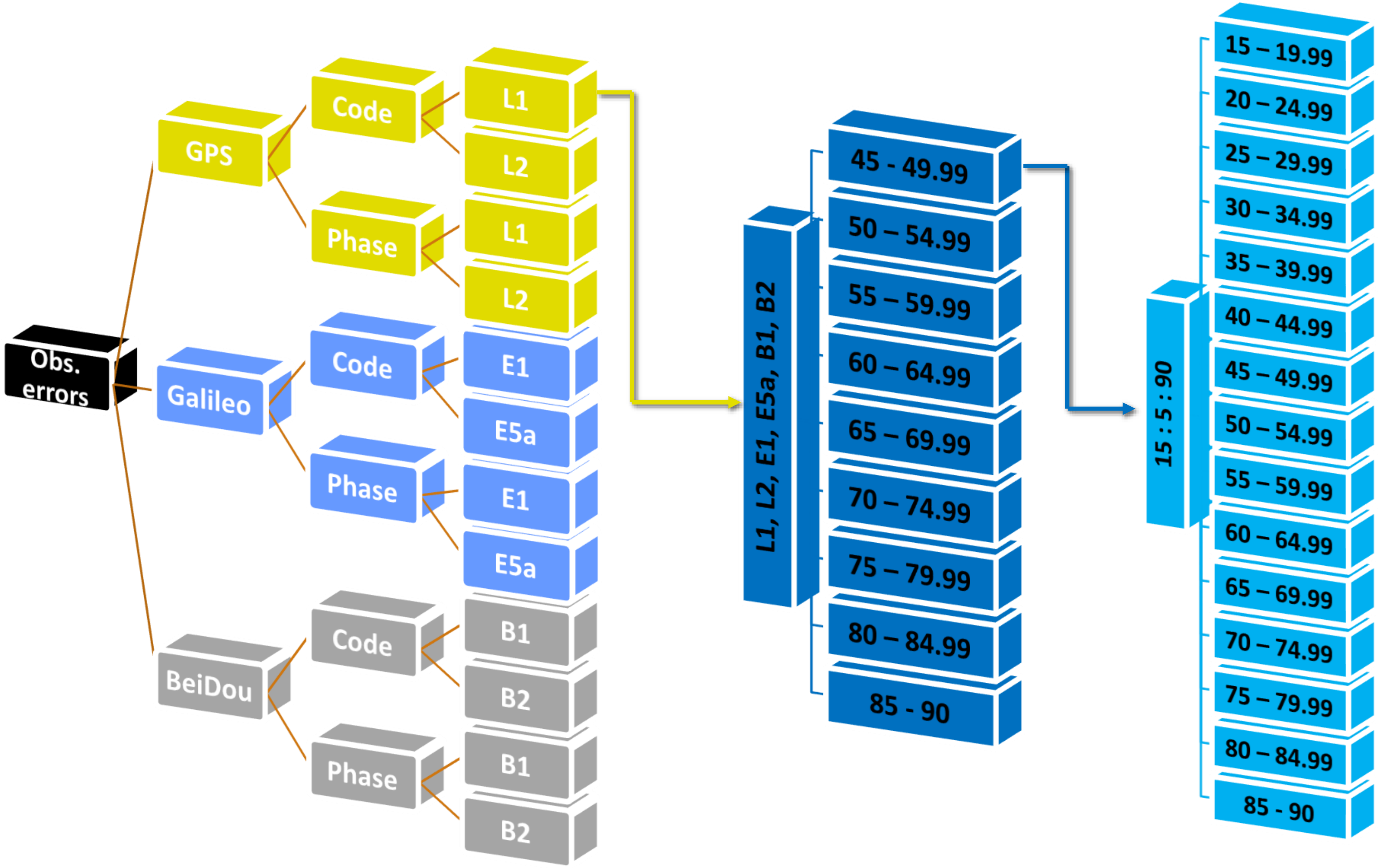
Methodology: Data analysis – 2nd empirical method



Each group of the frequencies' categories is then sorted into 9 groups based on the elevation angle of the pivot satellite with an interval of five degrees.

Each group of the pivot satellite's categories is then sorted into 15 groups based on the elevation angle of the secondary satellite with an interval of five degrees.

Methodology: Data analysis – 2nd empirical method



Methodology: Data analysis – 2nd empirical method

The correlation coefficient was calculated between each category using Pearson's Correlation Coefficient:

$$\mathrm{q_{ij}} = \frac{n\left(\sum \varepsilon_i \, \varepsilon_j\right) - \left(\sum \varepsilon_i\right)\left(\sum \varepsilon_j\right)}{\sqrt{\left[n \sum {\varepsilon_i}^2 - \left(\sum \varepsilon_i\right)^2\right]\left[n \sum {\varepsilon_j}^2 - \left(\sum \varepsilon_j\right)^2\right]}}$$

where i and j are the correlated categories and *n* is the number of data in a certain category.

Methodology

Collect one year of RINEX data from two Curtin CORS

Compute Double-differenced errors

Mass data classification, processing and analyzing

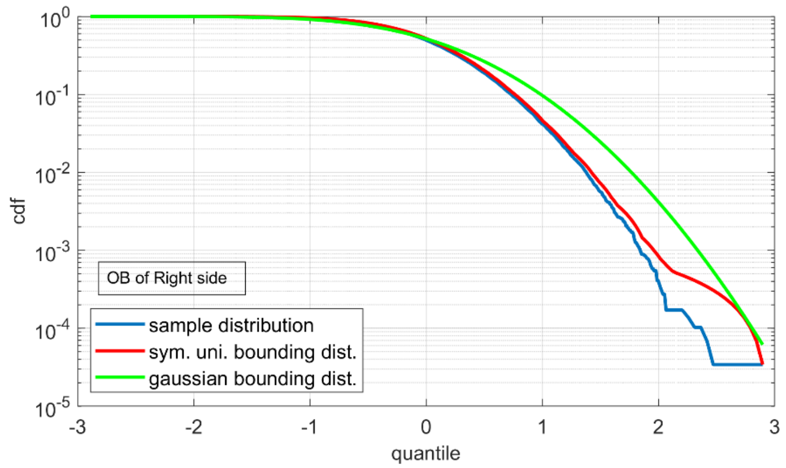
bounding using TSGB method

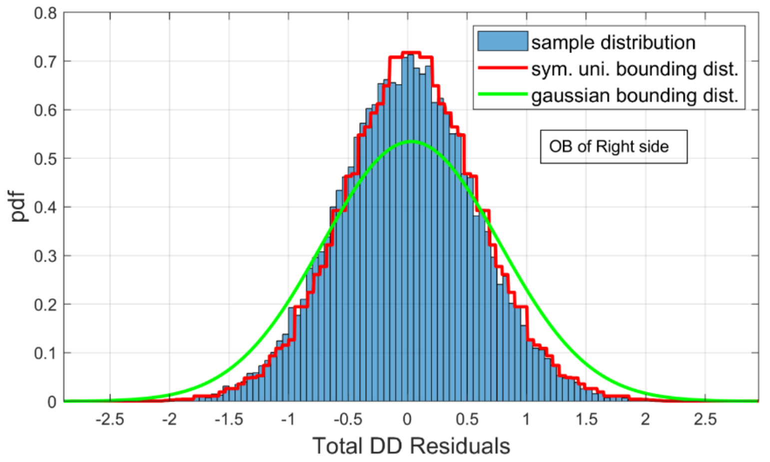
Methodology: TSGB implementation

- Forming a symmetric and unimodal distribution.
- performed from both sides of the data and the mean of each half is computed.
- The second step is to find a standard deviation of a Gaussian distribution for each half that achieves the bounding status.

$$F(x) \leq F_s(x) \quad for\ any\ x$$

$$F_s(x) \leq \frac{1}{\sqrt{2\pi}} \int_{x-m}^{+\infty} e^{-\frac{t^2}{2\sigma^2}}\, dt \quad for\ any\ x \geq m$$



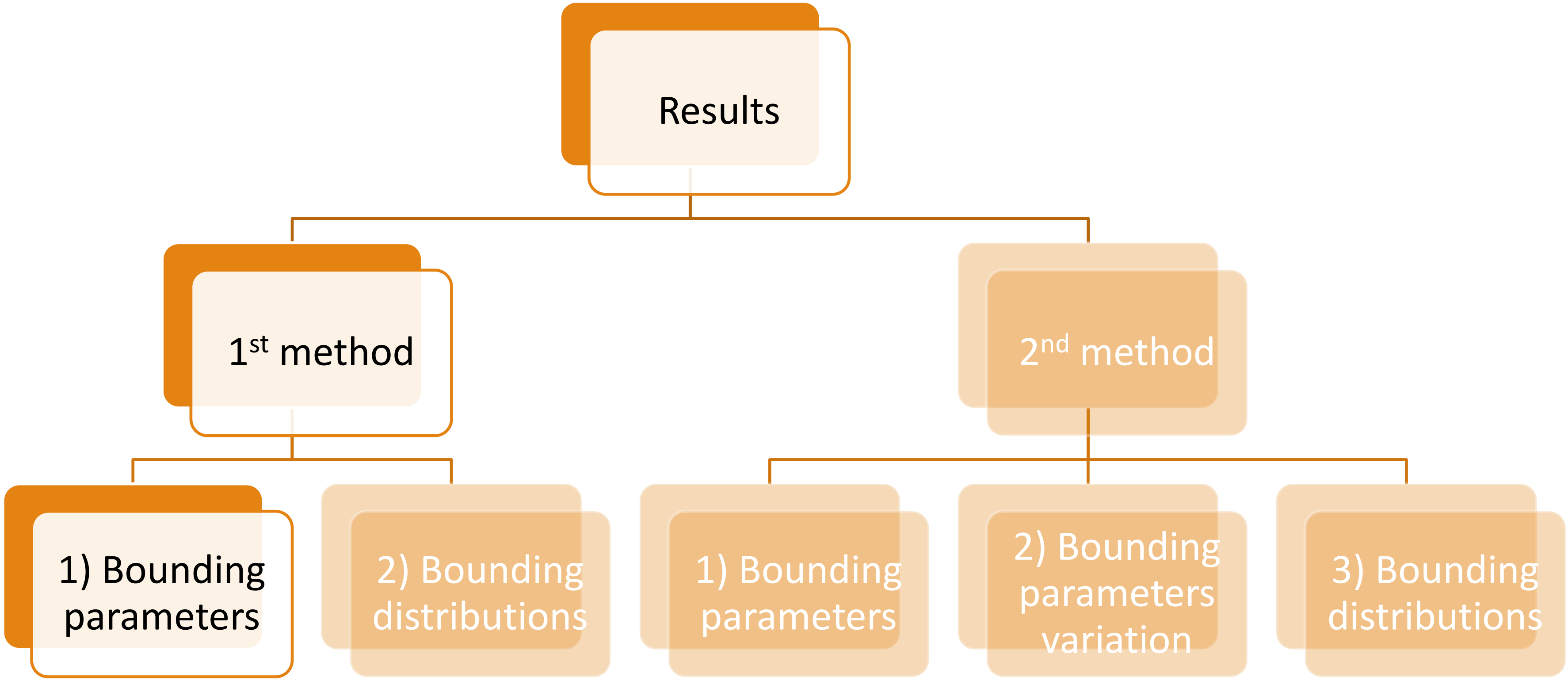


Overbounding using Two Step Gaussian Bounding Method (TSGB)

Results

- Results
  - 1st method
    - 1) Bounding parameters
    - 2) Bounding distributions
  - 2nd method
    - 1) Bounding parameters
    - 2) Bounding parameters variation
    - 3) Bounding distributions

Results: Bounding parameters of the 1st method

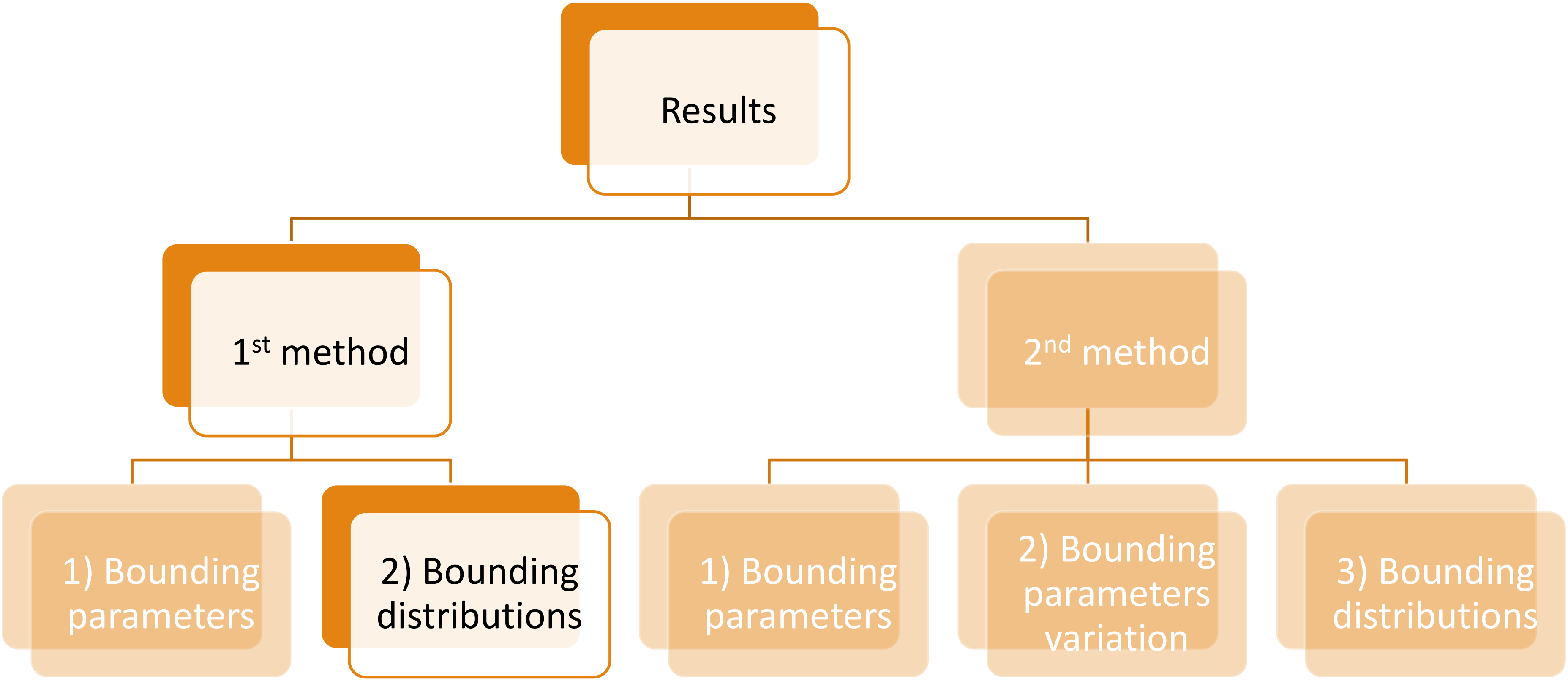


Results: Bounding parameters of the 1st method

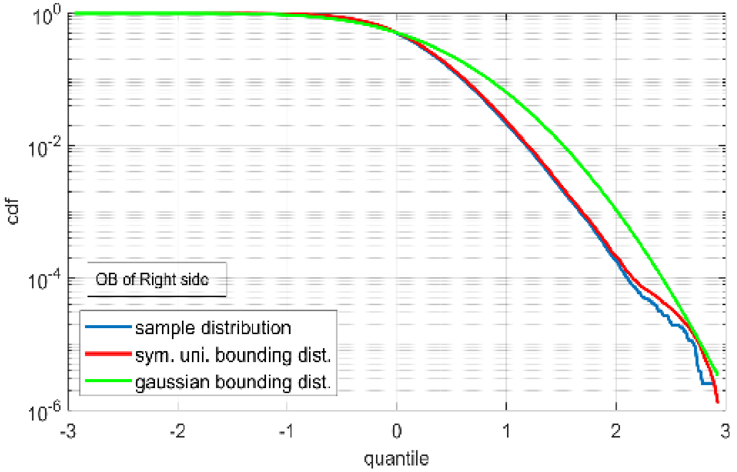
Bounding parameters at zenith of the three investigated systems at different signals and frequencies

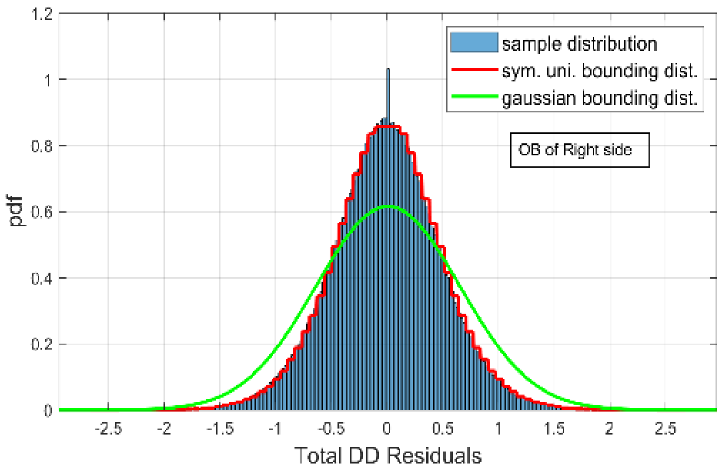
System	Signal	Frequency	Mean (m)	Standard Deviation (m)
GPS	Code	L1	0.03588	0.78321
		L2	0.03484	0.72368
	Phase	L1	0.00032	0.01381
		L2	0.00049	0.01345
Galileo	Code	E1	0.01254	0.45621
		E5a	0.01711	0.34738
	Phase	E1	0.00054	0.01291
		E5a	0.00054	0.01242
BeiDou	Code	B1	0.02508	0.69540
		B2	0.13323	0.42987
	Phase	B1	0.00171	0.01419
		B2	0.00257	0.01406

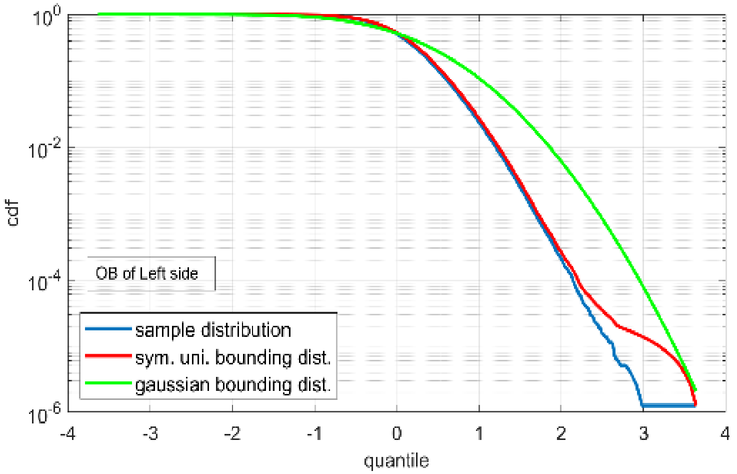
Results: Bounding distributions of the 1st method

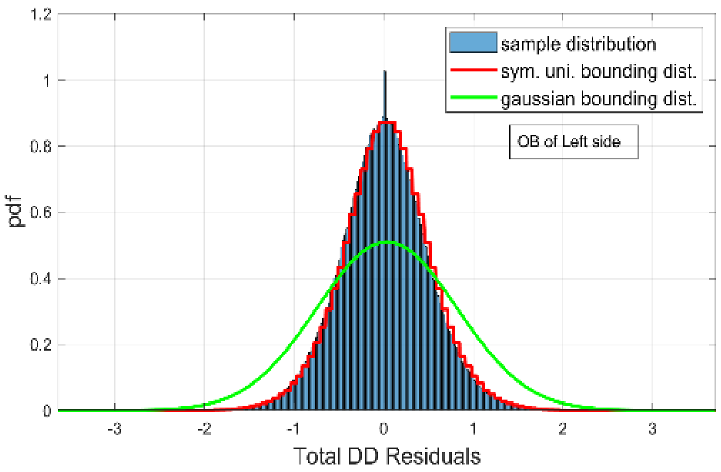


Results: Bounding distributions of the 1st method



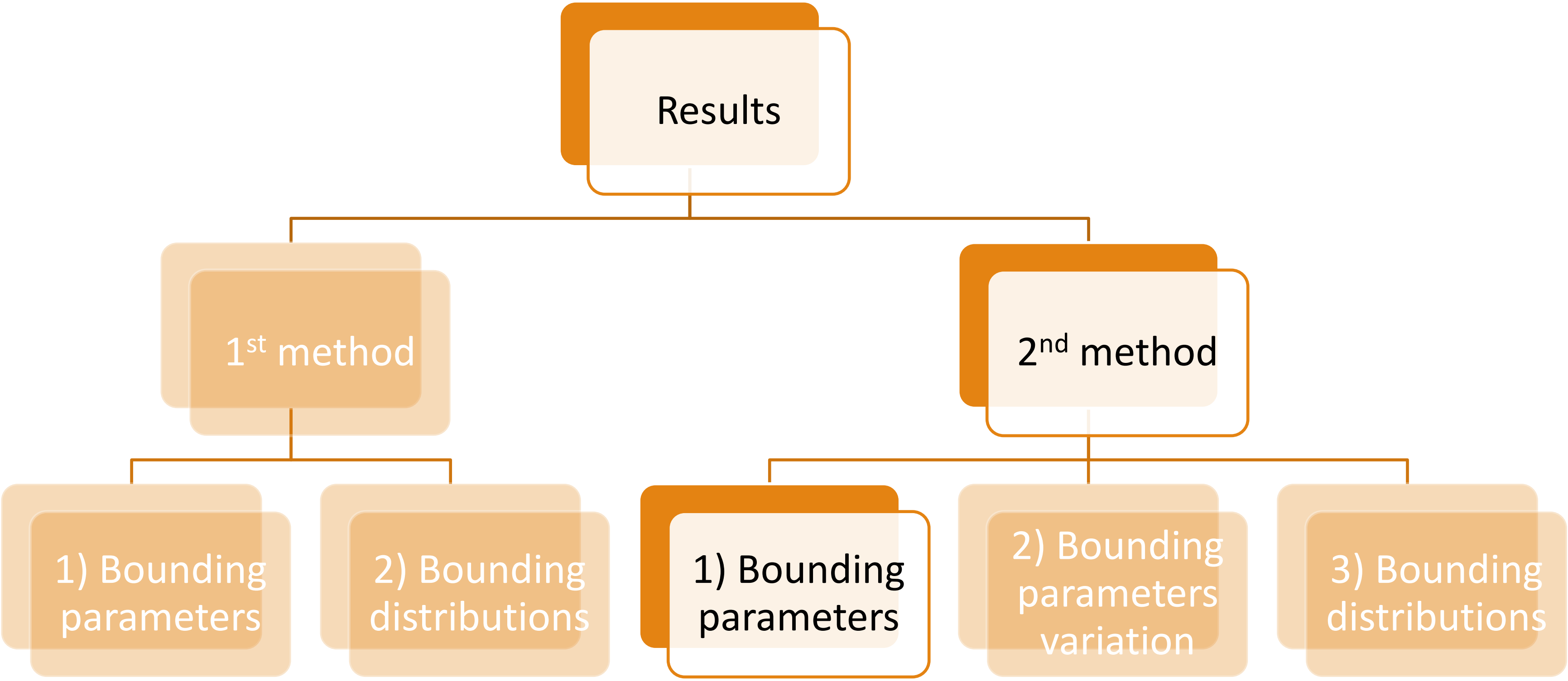






Two-step bounding for the GPS L1 DD errors mapped to zenith. The figure shows on the right the actual data distribution, the symmetric unimodal distribution, and the final Gaussian distribution, and on the left their CDFs

Results: Bounding parameters of the 2nd method



Results: Bounding parameters of the 2nd method

BeiDou - code B1									
OB parameters	Bias (error Mean)								
Pivot EA / Secondary EA	45	50	55	60	65	70	75	80	85
15				0.19466	0.26285	0.32266	0.29991	0.23170	0.40755
20				0.18686	0.10546	0.11396	0.11536	0.14736	0.55807
25				0.10507	0.06350	0.05802	0.15963	0.14006	0.27170
30				0.30927	0.28260	0.23619	0.21581	0.14710	0.48908
35				0.36730	0.16947	0.09088	0.07114	0.05901	1.02202
40				0.08993	0.16974	0.23970	0.32063	0.19173	0.27930
45				0.21434	0.09569	0.07527	0.10195	0.05343	
50				0.03522	0.07254	0.04934	0.13634	0.14650	0.21802
55				0.08351	0.03394	0.04740	0.03993	0.05372	0.32398
60				0.09527	0.03196	0.04228	0.05351	0.04640	0.68766
65					0.07643	0.04163	0.08280	0.09344	0.36900
70						0.06013	1.16393	0.04287	
75							0.11879	0.03069	
80									
85									

Bounding ***mean*** of data collected at different EAs of the reference and the secondary satellites.

BeiDou code B1 data

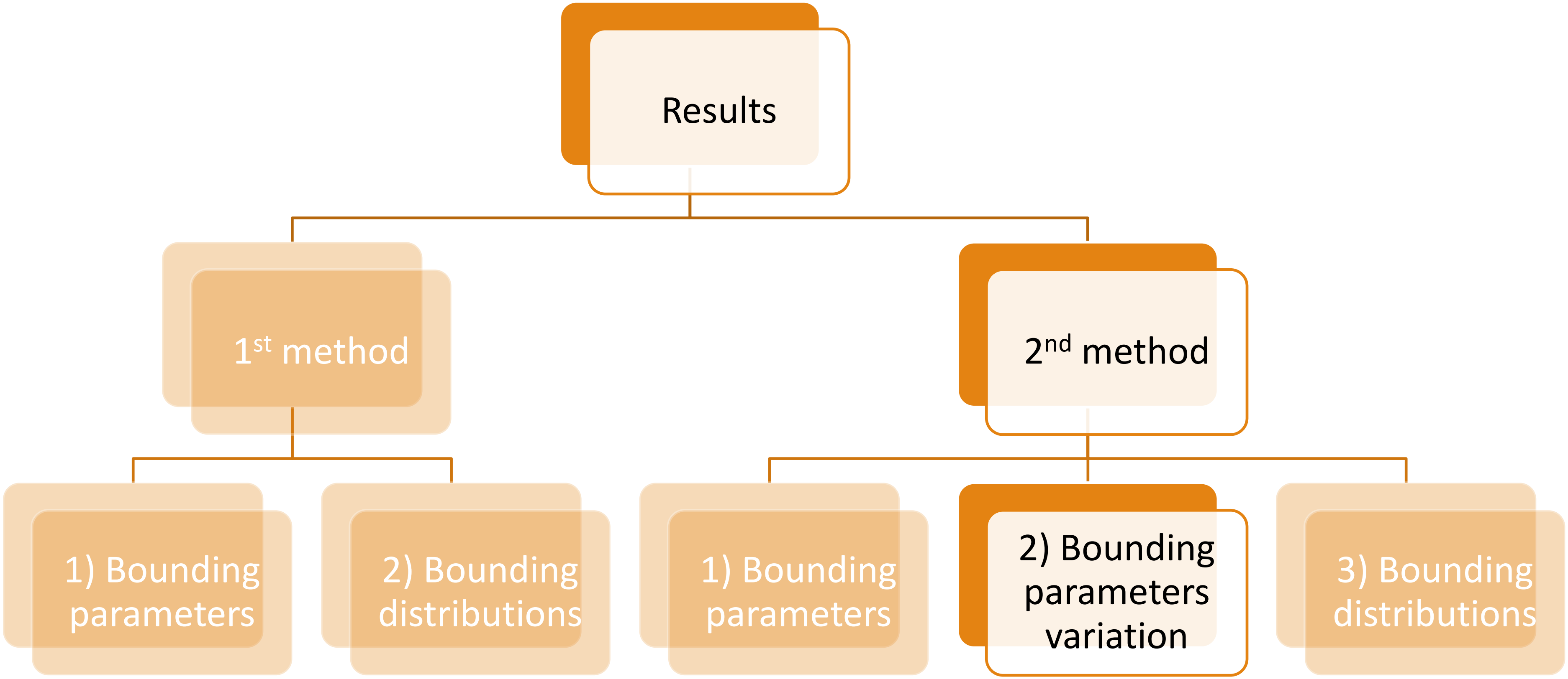
Results: Bounding parameters of the 2nd method

OB parameters	Standard deviation								
Pivot EA / Secondary EA	45	50	55	60	65	70	75	80	85
15				1.26271	1.58852	2.32259	2.49742	1.27451	1.27737
20				1.15377	1.06690	1.34817	1.29533	1.11054	1.28488
25				1.15291	1.07174	1.05592	1.06467	1.05496	1.02945
30				0.89072	0.98765	0.90792	0.91941	1.06465	1.20275
35				0.98070	1.04903	0.97389	1.27317	1.12253	1.21600
40				1.00239	1.10752	1.16926	1.14126	1.06088	1.09598
45				1.11023	1.06245	0.94887	1.01617	0.94328	
50				0.80934	0.72393	0.74070	0.72991	0.84436	0.77159
55				0.67239	0.74650	0.62601	0.62805	0.60672	0.67492
60				0.68487	0.64432	0.65048	0.74803	0.62477	0.74399
65					0.59777	0.60996	0.65351	0.64201	0.63855
70						0.65612	0.61696	0.56838	
75							0.60486	0.57032	
80									
85									

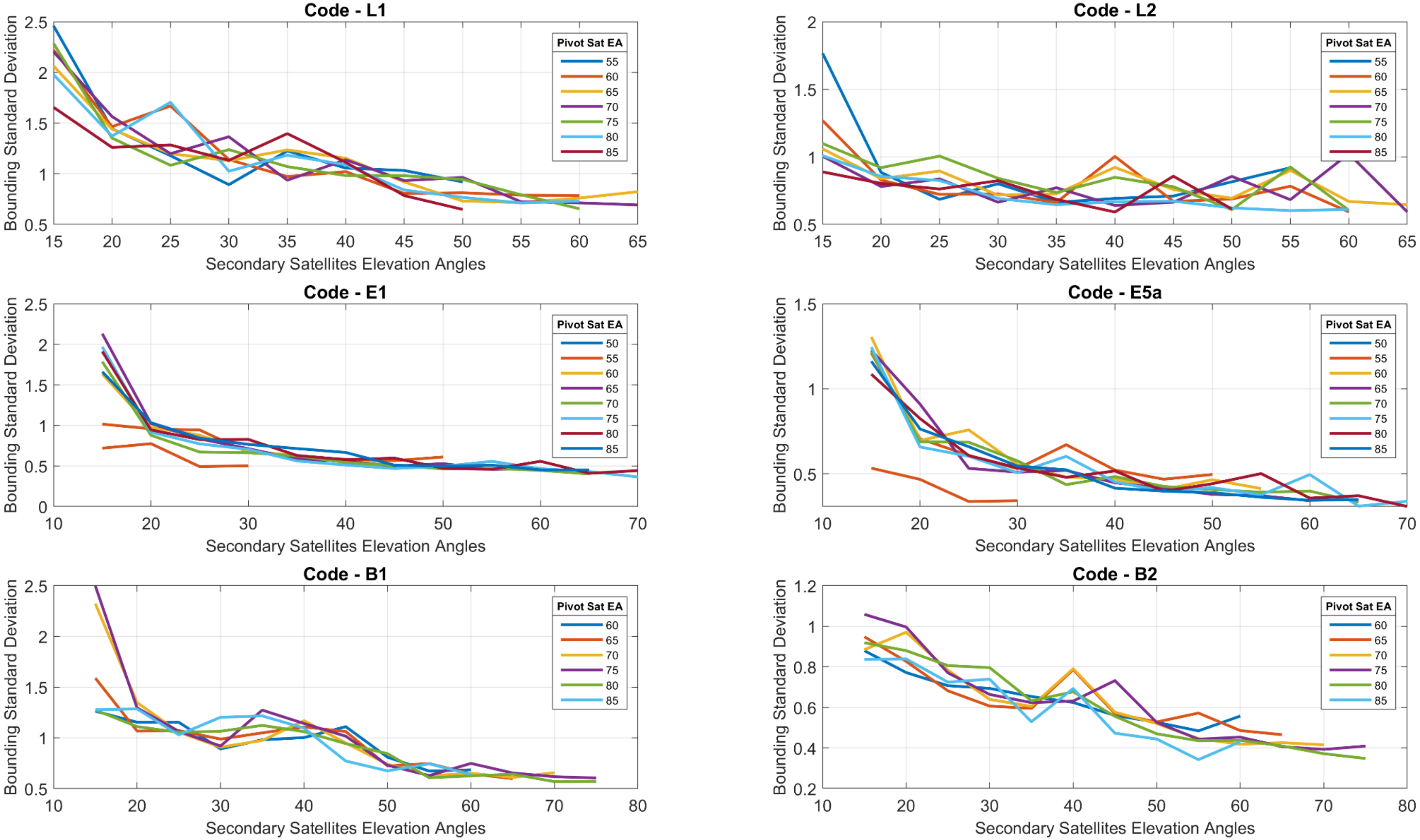
Bounding ***standard deviation*** of data collected at different EAs of the reference and the secondary satellites.

BeiDou code B1 data

Results: Variation of bounding parameters of the 2nd method

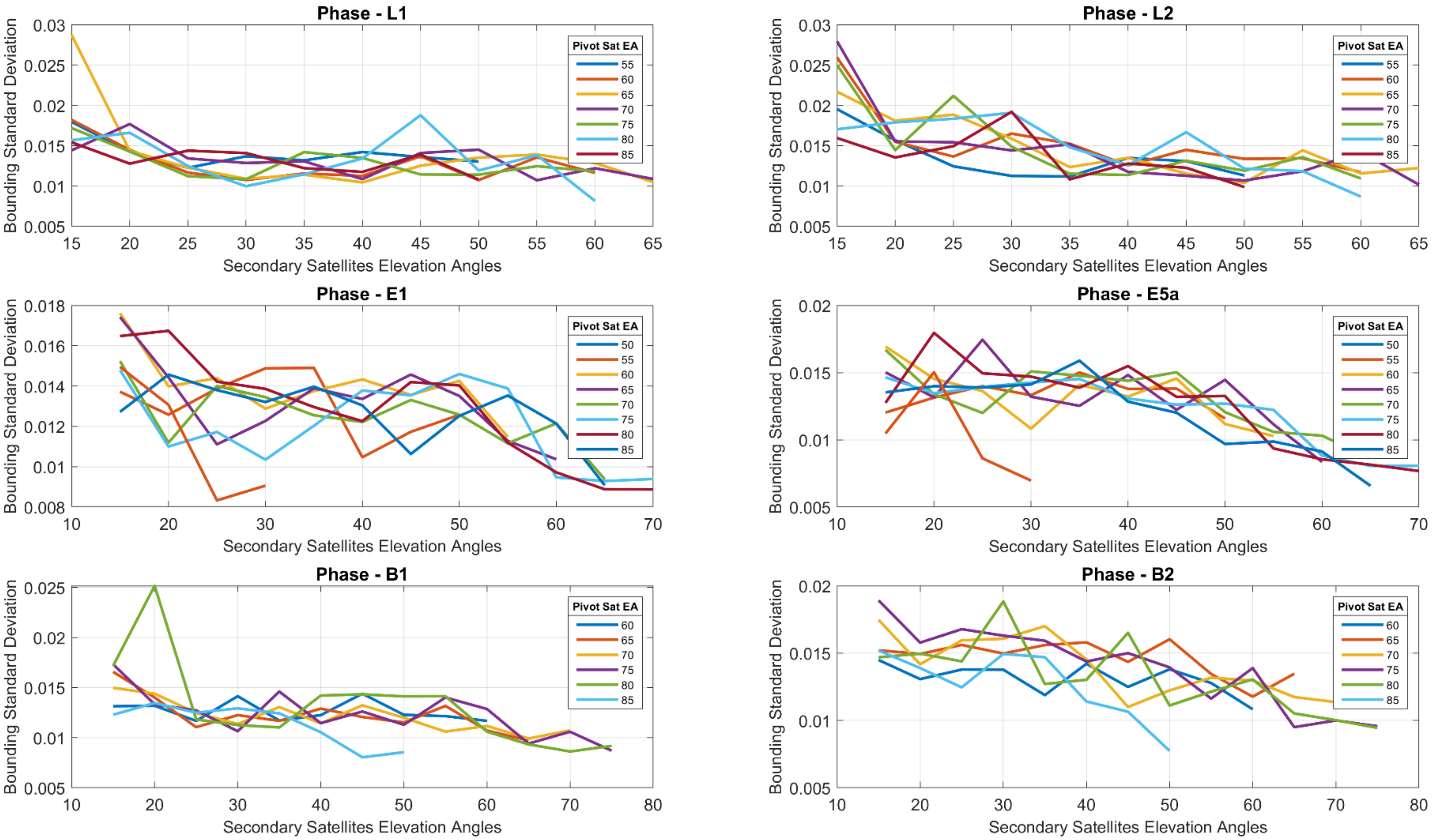


Results: Variation of bounding parameters of the 2nd method



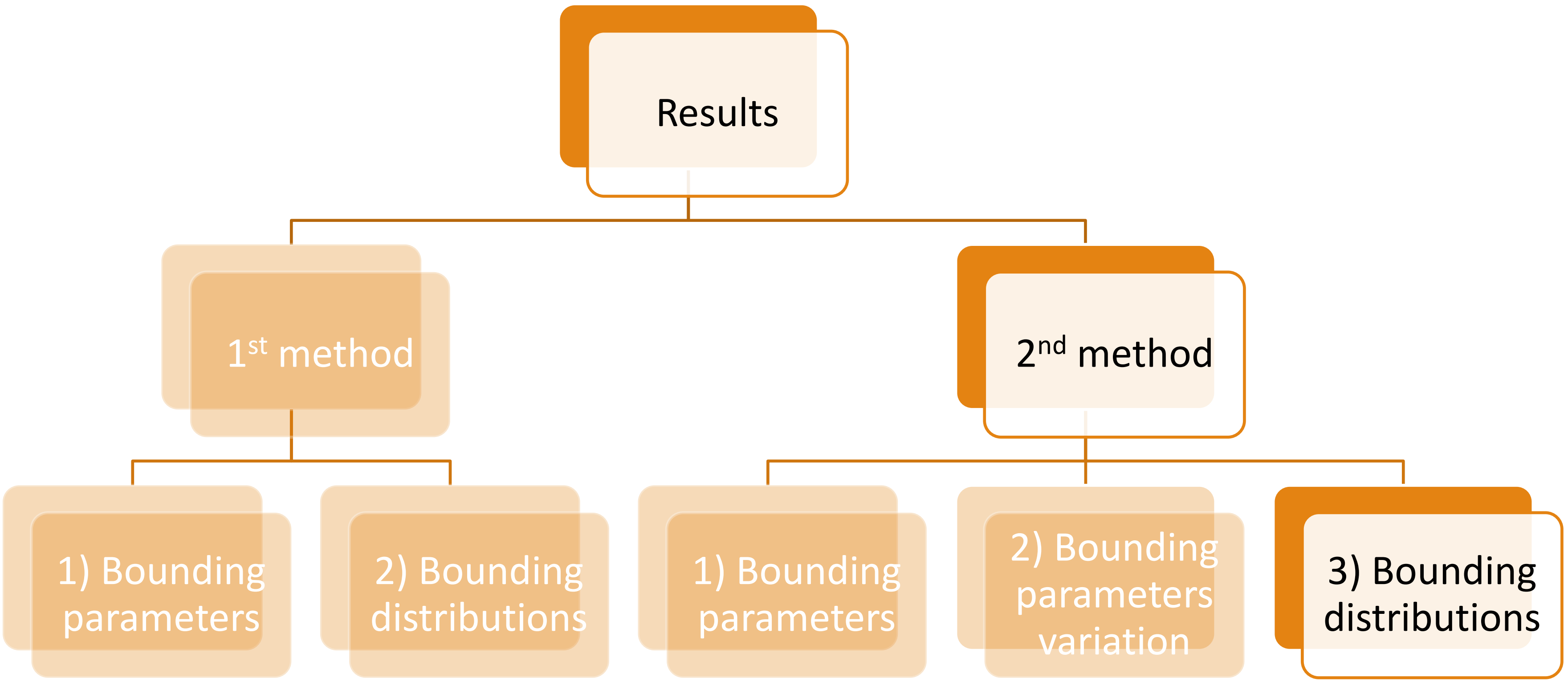
Variation of the bounding standard deviation of the DD ***code*** errors with respect to the EA of the observed satellites

Results: Variation of bounding parameters of the 2nd method

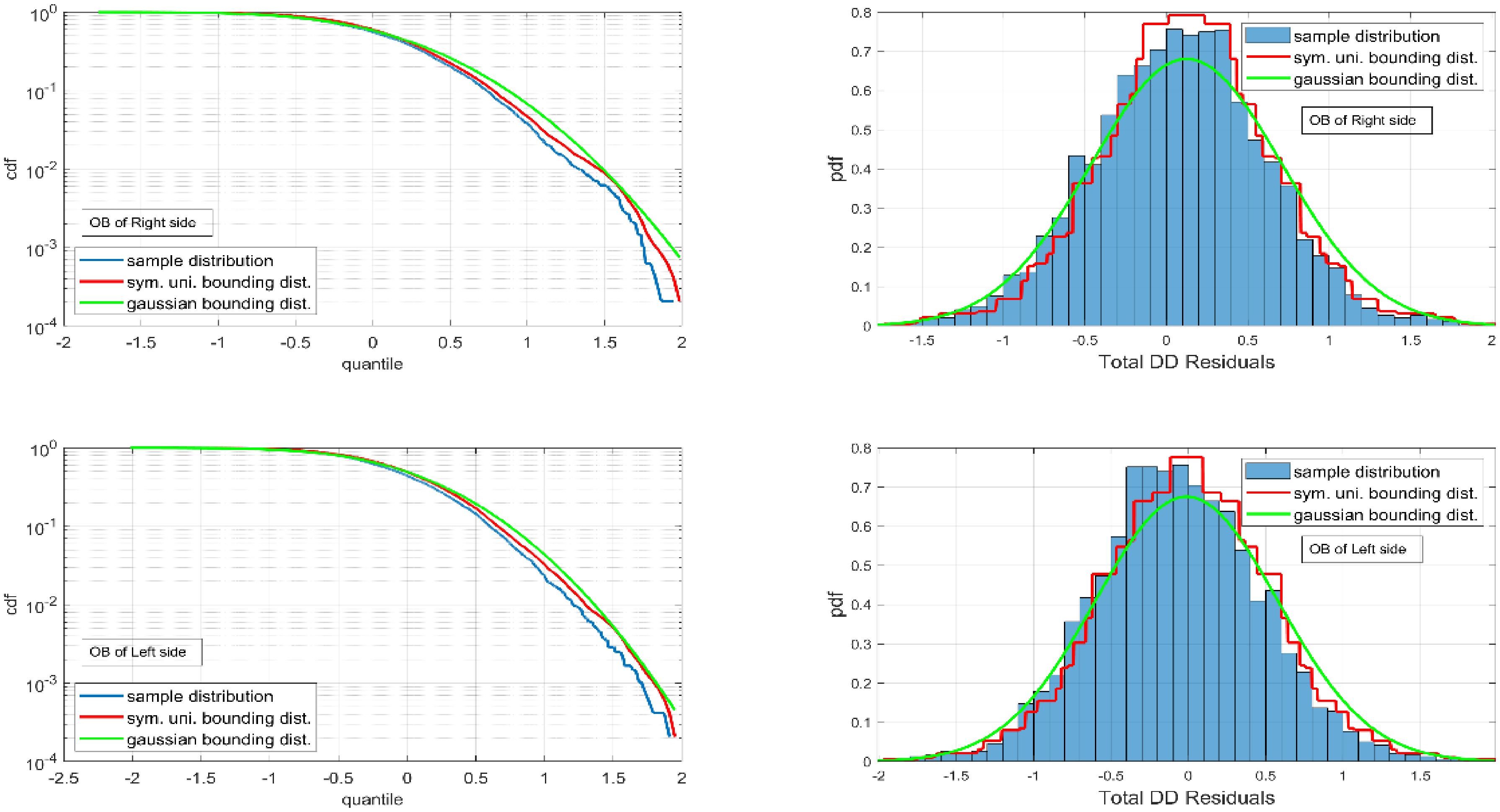


Variation of the bounding standard deviation of the DD ***phase*** errors with respect to the EA of the observed satellites

Results: Bounding distributions of the 2nd method



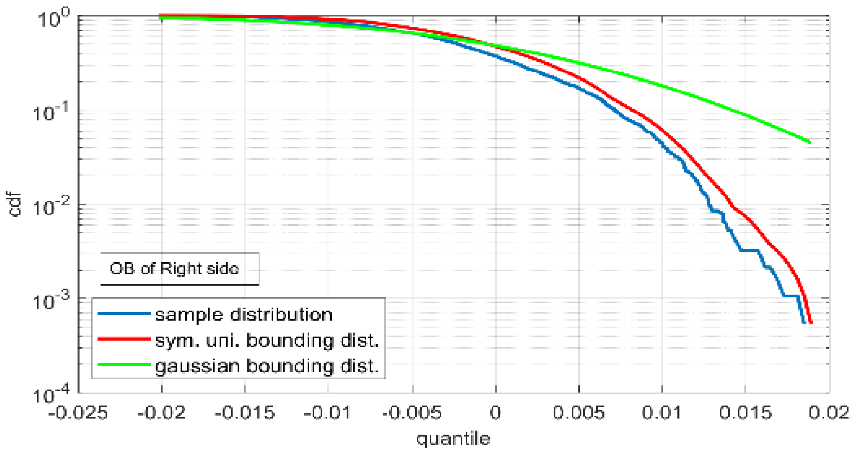
Results: Bounding distributions of the 2nd method

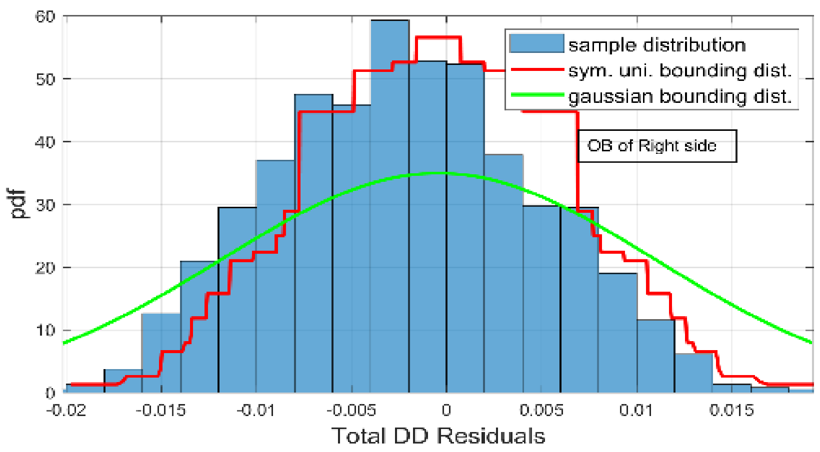


Two-step bounding for the GPS L2 DD code errors collected at 85 and 45 degrees as an EA for the pivot and secondary satellites respectively.

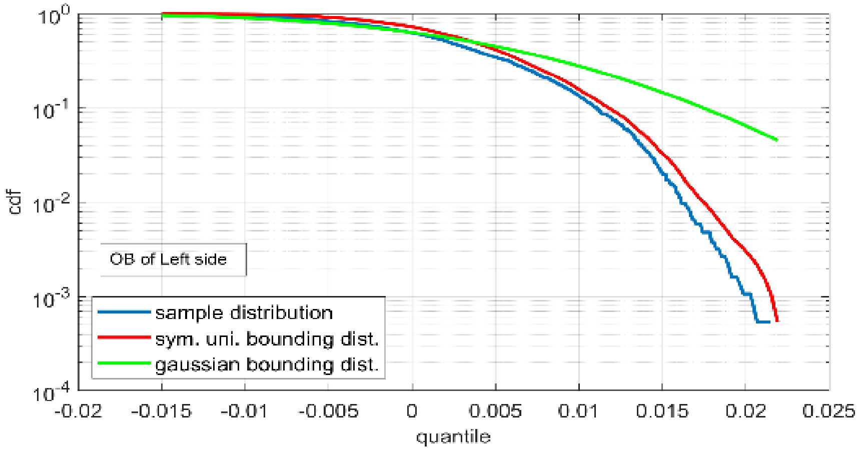
It shows the actual data distribution, the symmetric unimodal distribution, and the final Gaussian distribution (right), and CDFs (left)

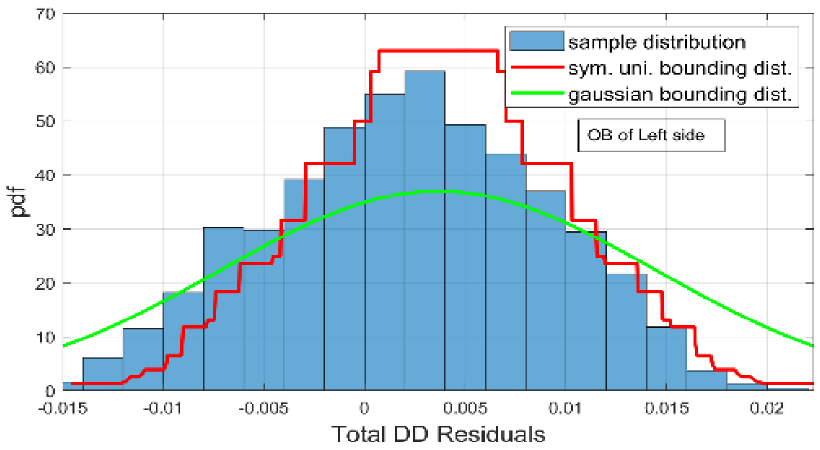
Results: Bounding distributions of the 2nd method





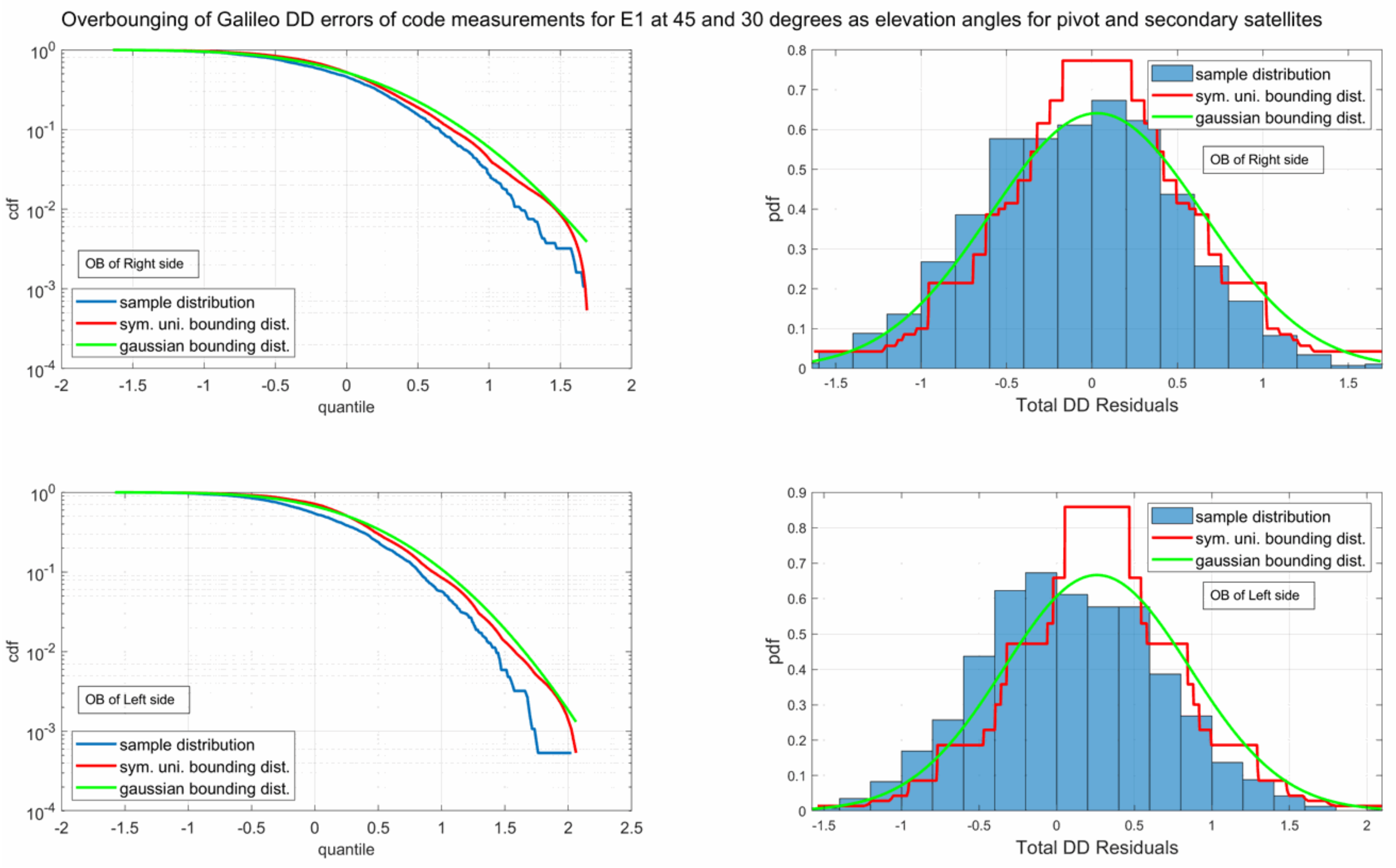
Two-step bounding for the Galileo E1 DD phase errors collected at 45 and 30 degrees as an EA for the pivot and secondary satellites respectively.





It shows the actual data distribution, the symmetric unimodal distribution, and the final Gaussian distribution (right), and CDFs (left)

Results: Bounding distributions of the 2nd method



Curtin University

Conclusion

This research has proven the possibility to obtain a representative stochastic model that accounts for observations correlation based on empirical data of a relatively short period (one year) using a bounding technique. This weighting function can be used for IM of multi-GNSS precise positioning for autonomous vehicles.

$$
\begin{bmatrix}
\sigma^2_{G1,2} & q_{G1,2,3}\,\sigma_{G1,2}\,\sigma_{G1,3} & q_{G1,2,4}\,\sigma_{G1,2}\,\sigma_{G1,4} & 0 & 0 & 0 & 0 & 0 & 0 \\
q_{G1,2,3}\,\sigma_{G1,2}\,\sigma_{G1,3} & \sigma^2_{G1,3} & q_{G1,3,4}\,\sigma_{G1,3}\,\sigma_{G1,4} & 0 & 0 & 0 & 0 & 0 & 0 \\
q_{G1,2,4}\,\sigma_{G1,2}\,\sigma_{G1,4} & q_{G1,3,4}\,\sigma_{G1,3}\,\sigma_{G1,4} & \sigma^2_{G1,4} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma^2_{E1,2} & q_{E1,2,3}\,\sigma_{E1,2}\,\sigma_{E1,3} & q_{E1,2,4}\,\sigma_{E1,2}\,\sigma_{E1,4} & 0 & 0 & 0 \\
0 & 0 & 0 & q_{E1,2,3}\,\sigma_{E1,2}\,\sigma_{E1,3} & \sigma^2_{E1,3} & q_{E1,3,4}\,\sigma_{E1,3}\,\sigma_{E1,4} & 0 & 0 & 0 \\
0 & 0 & 0 & q_{E1,2,4}\,\sigma_{E1,2}\,\sigma_{E1,4} & q_{E1,3,4}\,\sigma_{E1,3}\,\sigma_{E1,4} & \sigma^2_{E1,4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{C1,2} & q_{C1,2,3}\,\sigma_{C1,2}\,\sigma_{C1,3} & q_{C1,2,4}\,\sigma_{C1,2}\,\sigma_{C1,4} \\
0 & 0 & 0 & 0 & 0 & 0 & q_{C1,2,3}\,\sigma_{C1,2}\,\sigma_{C1,3} & \sigma^2_{C1,3} & q_{C1,3,4}\,\sigma_{C1,3}\,\sigma_{C1,4} \\
0 & 0 & 0 & 0 & 0 & 0 & q_{C1,2,4}\,\sigma_{C1,2}\,\sigma_{C1,4} & q_{C1,3,4}\,\sigma_{C1,3}\,\sigma_{C1,4} & \sigma^2_{C1,4}
\end{bmatrix}
$$

Future work

- Investigating the efficiency of the designed empirical weighting functions.
- Carrying out sensitivity analysis to examine different mapping functions or analysing longer periods of GNSS observations.
- Using adaptive Kalman Filter in the FDE step.
- Testing the developed methods by GNSS data collected by low-cost receivers.

Questions?