

1 **A new method for fault identification in real-time integrity monitoring of autonomous**
2 **vehicles positioning using PPP-RTK**

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8 **Abstract**

9 Autonomous vehicles require a real-time positioning system with in-lane accuracy.
10 They also require an autonomous onboard integrity monitoring (IM) technique to verify
11 the estimated positions at a pre-defined probability. This can be computationally
12 demanding. PPP-RTK is a promising positioning technique that can serve this purpose.
13 Since PPP-RTK is developed to process undifferenced and uncombined (UDUC)
14 observations for both network and user sides, it provides the residuals of the individual
15 measurements. This can be exploited to reduce the computational load consumed in the
16 fault detection and exclusion (FDE) process, included in the IM task, without
17 compromising the positioning availability. This research proposes filtering the faulty
18 satellites by the network, then the hardware and location-dependent faults at the user
19 end can be identified. This is achieved by calculating the ratio between the matching
20 UDUC residuals of the user receiver and the nearest reference station observations. This
21 ratio is used to rank the individual observations where the observation with the largest
22 ratio is most likely to be the faulty one. Therefore, it is more likely to identify the faulty
23 observation without generating and testing numerous subsets. In addition, the exclusion
24 can be attempted per observation, which preserves observation availability, unlike the
25 grouping techniques that perform the exclusion per satellite. The method was examined
26 in two test cases where geodetic and commercial receivers were used. Results show that
27 the computational load has been reduced significantly by about 85-99% compared to
28 the solution separation and Chi-squared test methods that are commonly used for FDE.

29

30 **Keywords**

31 Integrity monitoring, fault detection and identification, autonomous vehicles, PPP-
32 RTK, undifferenced and uncombined.

33

34 **Introduction**

35 Autonomous vehicles (AVs) require real-time positioning capabilities. This also
36 necessitates a rigorous integrity monitoring (IM) technique to assure the reliability of
37 the computed positions at a pre-defined probability (El-Mowafy and Kubo 2018;
38 Hassan et al. 2021; Wörner et al. 2016). Although many sensors are involved in
39 operating and monitoring AVs (Li et al. 2022; Sasani et al. 2016), this work is
40 concerned with IM of positioning based on using Global Navigation Satellite Systems
41 (GNSS). One main step of IM is fault detection and exclusion (FDE). It is considered
42 the most computationally demanding process of IM. This can oppose the
43 implementation of IM for real-time applications. Many methods have been introduced
44 to reduce the computational load. Some approaches propose selecting a limited number
45 of satellites among the all-in-view satellites. This selection can be made based on a
46 priori selection algorithms such as those that have a better elevation angle, weighting
47 factors, and satellite health (Gerbeth et al. 2016; Walter et al. 2016). The reduction in
48 the selected satellites will lead, accordingly, to a reduction in the computational load of
49 the IM process. However, this comes at the cost of reducing IM availability. In addition,
50 it removes satellites that may be fault-free and keeps others that may contain faults that
51 will be removed later, hence, reducing the availability even further.

52 Some other approaches suggest performing what is called fault consolidation,
53 known as Clustered Advanced receiver autonomous integrity monitoring (ARAIM). In
54 such approaches, many satellites are combined and attempted for exclusion together to
55 cover different fault modes in one processing. Different satellite grouping techniques
56 were described by (Orejas and Skalicky 2016; Orejas et al. 2016). For instance, (Ge et
57 al. 2017) attempted clustering and excluding the satellites of the same orbit plane.
58 However, this cannot be conducted with all constellations since only the BDS
59 constellation has different orbits. (Walter et al. 2014) clustered the satellites of the same
60 constellation ignoring some fault modes that are less likely to happen; thus, reducing
61 the number of the tested subsets and the computational complexity. Unlike the previous
62 two approaches, where the number of the generated subsets changes based on the

63 number of satellites and the fault probabilities of both the satellites and constellations,
64 (Blanch et al. 2018) sought a further reduction in the computational load by fixing the
65 number of the tested subsets regardless the fault probabilities of both the satellites and
66 constellations. Although these methods managed to reduce the computational burden,
67 this comes at the expense of compromising the availability, conservatism, and precision
68 of the protection level (PL).

69 Other approaches examined the reduction of the computational load through
70 saving in the mathematical process of estimating the solution of the tested subsets. For
71 example, (Gunning et al. 2018) suggested that all the calculated models and corrections
72 for the all-in-view situation can be used to estimate the solution of the generated subsets
73 during the solution separation (SS) test, given that they are close to each other. While
74 this included some saving in the computational load, it also included some
75 approximation and can be only applicable, with some concerns on its impact in different
76 situations, in some positioning techniques such as precise point positioning (PPP).
77 Similarly, (Blanch et al. 2019) tested the replacement of the residuals covariance matrix
78 of each subset solution, which is very computationally demanding as it requires two
79 matrix inversions by another matrix obtained in the all-in-view case. No full matrix
80 inversion is needed in that case. Whereas this can reduce the computational load
81 significantly, it degrades the PL to a great extent as well. This may be accepted for
82 some applications where up to several meters of accuracy is authorized, but this is not
83 the case for autonomous vehicles where only in-lane accuracy is of main concern.
84 Furthermore, (El-Mowafy and Wang 2022) proposed a method where the inverse of the
85 covariance matrix of the state vector for any generated subset, considering single or
86 multiple faults, is computed without inversion from the single, all-in view, normal
87 matrix without any further inversion. It proved that this could reduce the complexity of
88 the calculations substantially without compromising the solution availability or quality.

89 All of the aforementioned approaches managed to provide means to reduce the
90 computational load. In most cases, this has adversely affected other parameters, which
91 might be acceptable for some applications. The shared part among all of these research
92 works is that the user still needs to test all the generated subsets to identify the faulty
93 observation(s)/satellite(s).

94 We present a new process for FDE that potentially reduce the computational
95 time compared to the current methods. The PPP-RTK approach is selected as the most

96 suitable to provide the needed in-lane accuracy for AVs in real-time, as will be
97 discussed in the next section. In the proposed FDE method, faults due to satellite errors
98 will be checked by the network processing center exploiting the known ground
99 positions of the network stations. This shall reduce the risk of having a fault due to
100 observed satellites at the user end. While the atmospheric, location and receiver-related
101 errors at the user end will be checked using a ratio between the residuals of the user
102 position solution and their counterparts of the nearest reference station from the
103 network, assumed to be fault-free. This ratio shall provide the user with an indicator of
104 which observation(s) could be faulty and worth attempting testing for exclusion in case
105 of a fault is detected in the overall solution. Therefore, the computational load is
106 expected to be significantly reduced as there will be no need to form all possible subsets
107 to identify the faulty observation(s). In addition, the availability will be maintained
108 since the exclusion is based on the observations not satellite(s), thanks to using UDUC
109 PPP-RTK as a positioning technique. For example, if dual frequency receiver is used
110 where each satellite offers two code and two phase observations, the exclusion will
111 suspect all four individual observations. This is unlike the current grouping technique
112 where a faulty observation causes the exclusion of the satellite including all its four
113 observations (or more in case more frequencies are observed). The method can also be
114 combined with any of the previously stated methods to reduce the computational load
115 even further.

116 The next section briefly discusses the used PPP-RTK technique and explains
117 the rationale for its selection for the positioning of AVs. A full description of the newly
118 proposed method, including the criteria and advantages, as well as the testing examples,
119 is provided in the following section. In the fourth section, the results of applying the
120 proposed new method in two different test cases are presented and discussed. The
121 conclusion and the future work are given in the last section.

122

123 **PPP-RTK for real-time positioning of autonomous vehicles**

124 Not all GNSS-based positioning methods are suitable for AVs. For example,
125 conventional PPP requires a long initialization time before providing a valid position
126 (Du et al. 2021). Traditional RTK requires a dense infrastructure of base stations and a
127 radio connection that may sometimes be interrupted (El-Mowafy and Kubo 2017). The
128 use of networks provides redundancy and consistency of the positioning output,

129 therefore, operating a reference network rather than multiple single reference stations
130 would provide a more efficient solution in terms of cost against the covered area
131 (Landau et al. 2003; Vollath et al. 2002). The methodologies of the network-RTK
132 (NRTK) and its corrections transmitted to users differ, which include the virtual
133 reference station (VRS) (Wanninger 2003), area broadcast (FKP), and master-auxiliary
134 (Mac) methods (Janssen 2009; Takac and Zelzer 2008). Based on the utilized protocol,
135 the infrastructure of the network as well as the user software can be defined. Unlike
136 these methods, where processing is based on differencing techniques, PPP-RTK
137 represents another method where both the network and the user process undifferenced
138 and uncombined (UDUC) observations (Zha et al. 2021).

139 The corrections sent to users to deal with the observation errors are classified
140 into the observation-state representation (OSR) and the state-space representation
141 (SSR) (Wabben et al. 2005). While the first provides the corrections for the combined
142 errors, the second provides them for each error source individually, as shown in Fig. 1.
143 Hence, differential GPS (DGPS), traditional RTK, and traditional NRTK (where a
144 differencing technique is used) employ the OSR protocol, whereas PPP, satellite-based
145 augmentation systems (SBAS) (Chen et al. 2022), and PPP-RTK usually use the SSR
146 protocol. PPP-RTK has the following practical advantages compared to the rest of the
147 methods (Zhang et al. 2019): 1) the ability to study the impact of each error source; 2)
148 since it processes the UDUC observations, the calculated residuals are obtained for each
149 UDUC observation, which provides the possibility of better screening of individual
150 observations; 3) unlike most traditional NRTK methods (e.g., VRS, Mac) that require
151 two-way communications between the network and the user(s), PPP-RTK only requires
152 a one-way communication system with all users within the coverage area of the
153 network; hence, reducing security and bandwidth hazards. Due to these advantages, this
154 work proposes PPP-RTK as the most convenient method for AVs real-time positioning.
155 The following section provides more details concerning the PPP-RTK at both the
156 network and the user sides, based on which our method is presented.

157

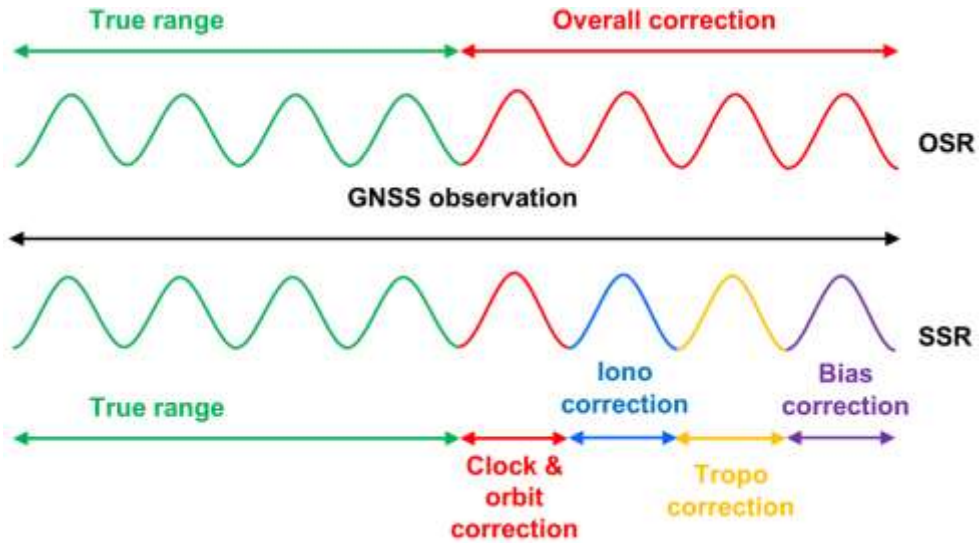


Fig. 1 Sketch of the difference between the observation-state representation (OSR) and the space-state representation (SSR)

158

159 Network Processing in the SSR mode (UDUC PPP-RTK)

160 PPP-RTK processing takes place on both the network and the user sides. The
 161 observation equations at the network end can be expressed as (Odijk et al. 2017):

$$E(C_R^s) = c(\bar{t}_R - \bar{t}^s) + \bar{T}_{RW} + \mu_i \bar{I}_{R_i}^s \quad (1)$$

$$E(\varphi_R^s) = c(\bar{t}_R - \bar{t}^s) + \bar{T}_{RW} - \mu_i \bar{I}_{R_i}^s + \frac{c}{f_i} \bar{N}_{R_i}^s + \bar{\delta}_{R_i} - \bar{\delta}_i^s \quad (2)$$

162 where $E(\cdot)$ is the expected value of the observed minus computed terms; C_R^s , φ_R^s are
 163 pseudorange code and phase observations (in m); s , R refer to the observed satellite and
 164 the receiver, respectively; c represents the speed of light; t_R , t^s are the satellite and
 165 receiver clock offsets, respectively; T_{RW} is the wet part of troposphere delay at the slant
 166 angle; $I_{R_i}^s$ represents the ionospheric delay on the first frequency; f_i is the frequency of
 167 the observed signal i ; $N_{R_i}^s$ is the phase ambiguity; δ_{R_i} is the phase bias; $\mu_i = f_1^2 / f_i^2$ is
 168 a multiplier factor based on the frequency; $(\bar{\cdot})$ denotes a certain representation that has
 169 been used to eliminate the rank deficiency using the S-system theory (Teunissen 1985)
 170 as shown in Table 1:

171

Table 1 PPP-RTK parameters representation of the network model

| Parameter | Definition |
|----------------------|---|
| \bar{t}^s | $t^s - t_M + [d_{IF}^s - d_{MIF} - T_{Mw}]/c$ |
| $\bar{\delta}_i^s$ | $\delta_i^s - d_{IF}^s + \mu_i d_{GF}^s - \delta_{M_i} + d_{MIF} - \mu_i d_{M_{GF}} - \lambda_i N_{M_i}^s$ |
| $\bar{N}_{R_i}^s$ | $(N_{R_i}^s - N_{M_i}^s) - (N_{R_i}^{SP} - N_{M_i}^{SP})$ |
| \bar{T}_{Rw} | $T_{Rw} - T_{Mw}$ |
| \bar{t}_R | $t_R - t_M + (d_{RIF} - d_{MIF})/c$ |
| $\bar{\delta}_{R_i}$ | $(\delta_{R_i} - d_{RIF} + \mu_i d_{R_{GF}} + \lambda_i N_{R_i}^{SP}) - (\delta_{M_i} - d_{MIF} + \mu_i d_{M_{GF}} + \lambda_i N_{M_i}^{SP})$ |
| $\bar{I}_{R_i}^s$ | $I_{R_i}^s + d_{R_{GF}} - d_{GF}^s$ |

172

173 where M is the master station among the reference stations; $d_{M,RIF,GF}$ represents the
174 ionospheric-free (IF) and geometry-free (GF) linear combinations of the code bias for
175 reference station (R) or master station (M); $d_{IF,GF}^s$ is the ionospheric-free (IF) and
176 geometry-free (GF) linear combinations of the code bias for satellite; $\lambda_i = c/f_i$; s_P
177 denotes the pivot satellite among the observed satellites.

178 The above equations are given for networks using dual-frequency observations,
179 which is the case we considered. The form of the estimable parameters would be
180 similarly modified in case more frequencies are involved.

181

182 User Processing in the SSR mode (UDUC PPP-RTK)

183 The user can apply the PPP-RTK processing technique by exploiting the corrections
184 provided by the network, where at a user end:

$$E(C_U^s + c(\bar{t}^s)) = r_U^s + c(\bar{t}_U) + \bar{T}_{Uw} + \mu_i \bar{I}_{U_i}^s + \mu_i \bar{d}_{U_{GF}} + e_{U_C}^s \quad (3)$$

$$E(\varphi_U^s + c(\bar{t}^s) + \bar{\delta}_i^s) = r_U^s + c(\bar{t}_U) + \bar{T}_{Uw} - \mu_i \bar{I}_{U_i}^s + \lambda_i \bar{N}_{U_i}^s + \bar{\delta}_{U_i} + \epsilon_{U_\varphi}^s \quad (4)$$

185 where U is the user receiver; r_U^s represents the range between satellite s and the user;
186 $e_{U_C}^s$, $\epsilon_{U_\varphi}^s$ are the code and phase observation noises that may include multipath and
187 other location-dependent errors; $(\bar{\cdot})$ denotes a certain representation that is used to
188 eliminate the rank deficiency using the S-system theory, as shown in Table 2.

Table 2 PPP-RTK parameters representation of the user model

| Parameter | Definition |
|----------------------|---|
| \bar{t}_U | $[t_U + (d_{U_{IF}}/c)] - [t_M + (d_{M_{IF}}/c)]$ |
| $\bar{I}_{R_i}^s$ | $I_{U_i}^s + d_{M_{GF}} - d_{GF}^s$ |
| $\bar{d}_{U_{GF}}$ | $d_{U_{GF}} - d_{M_{GF}}$ |
| $\bar{\delta}_{U_i}$ | $(\delta_{U_i} - d_{U_{IF}} + \lambda_i N_{U_i}^{SP}) - (\delta_{M_i} - d_{M_{IF}} + \lambda_i N_{M_i}^{SP})$ |

190

191 **IM of real-time positioning**

192 ARAIM is an efficient method (Blanch et al. 2012; Blanch et al. 2011) that can be used
 193 for IM of the positioning of AVs. However, the current ARAIM methods were basically
 194 developed for aviation applications, which have many differences compared to ground
 195 applications, as discussed in (Elsayed et al. 2023). This encouraged many researchers
 196 to try to adapt ARAIM to different positioning techniques such as PPP (Du et al. 2021),
 197 RTK (Wang and El-Mowafy 2021), and PPP-RTK (Zhang et al. 2023). One main step
 198 of ARAIM is FDE. It is considered a computationally expensive step, due to the need
 199 to test a huge number of possible observation faults, which represents a hurdle for the
 200 implementation of IM in real-time applications. The SS and Chi-squared test methods
 201 are traditionally used for FDE in ARAIM (Joerger and Pervan 2014; Joerger and Pervan
 202 2016). In the SS test, the position solution of different subsets, excluding the
 203 observation(s) that is/are checked for being faulty, one at a time, is estimated and
 204 compared against the position solution of the all-in-view observations solution at each
 205 epoch. The test statistic is expressed as:

$$\Delta \hat{x}^{k_{1...n}} = \hat{x}^{k_{1...n}} - \hat{x}^0 \quad (5)$$

206 where \hat{x}^0 is the all-in-view solution, while \hat{x}^k is the solution of the subset $k_{1...n}$ where
 207 n is the number of the tested subsets. This number is based on the selected fault modes
 208 that define how many suspected faulty observations should be considered for exclusion.
 209 Then, the solution difference $\Delta \hat{x}^{k_{1...n}}$ of each subset is compared against a statistical
 210 threshold value as per equations (6) & (7):

$$\tau_{k_1\dots n} = \frac{|\Delta\hat{x}^{k_1\dots n}|}{T_{k_1\dots n}} \leq 1 \text{ or } |\hat{x}^{k_1\dots n} - \hat{x}^0| \leq T_{k_1\dots n} \quad (6)$$

$$T_{k_1\dots n} = H_{fa} \sigma_{k_1\dots n} \quad (7)$$

211 where assuming that the fault-free observation errors will have a normal distribution;
 212 H_{fa} is the quantile of CDF normal distribution, which is calculated based on the
 213 assigned probability for false alert (fa) and the total number of the considered fault
 214 modes, whereas $\sigma_{k_1\dots n}$ is the standard deviation. The number of the required tests would
 215 be numerous when considering the possibility of concurrently multiple faulty
 216 measurements, not only single faults as mostly considered in the literature, and
 217 accordingly, the computational load would be huge.

218 In the case of the Chi-squared test, the residuals of the estimated position
 219 solution are scanned every epoch for faults as follows:

$$\hat{r}_U^T Q_{y_U}^{-1} \hat{r}_U \leq \chi_\alpha^2(df, 0) \quad (8)$$

220 where \hat{r}_U is the residuals vector of the position solution; Q_{y_U} is the covariance matrix
 221 of the observations; α is the significance value that is decided based on the design of
 222 the IM process and the application on hand, e.g., 0.001; df is the degree of freedom,
 223 i.e., the difference between the number of observations and unknowns at the tested
 224 epoch. If the test fails at any epoch, a fault is assumed present, and exclusion must be
 225 attempted to identify that fault. The identification starts by reapplying the test on all
 226 possible subsets based on the defined fault modes. Once the test passes for a certain
 227 subset, the excluded satellite(s) is considered to be faulty. Unlike the SS test that is
 228 performed at every epoch for all possible subsets, the Chi-squared only starts the
 229 exclusion attempts when the all-in-view satellite observations solution does not pass.
 230 However, this represents a computational burden for real-time positioning when a fault
 231 is suspected. Another drawback that both the SS and Chi-squared methods share in the
 232 commonly used grouping approaches is that the subsets formation is based on testing
 233 satellites, grouping all their observations, not individual observations. Although this
 234 helps in reducing the computational load, it –unnecessarily– removes healthy
 235 observations (i.e. results in loss of information, which may be vital since real-time
 236 positioning involves a finite number of observations) and compromises the availability

237 since all observations of the suspected satellite(s), not specifically the faulted
238 observations, are excluded when a fault is detected.

239

240 **Proposed Methodology**

241 We classified faults into two main types according to their source. The first type is the
242 faults due to satellite errors. This kind of fault risks both the network stations and user
243 receivers altogether. Therefore, it is proposed that the faulty satellites due to satellite
244 errors are checked by the network reference stations exploiting their known positions,
245 where testing is performed in real-time in the PPP-RTK scheme. An RTCM message
246 is proposed to be transmitted in near-real time to the network subscribers that contains
247 a list of these faulty satellites. Users shall exclude the listed satellites before the
248 positioning process.

249 The second type of fault is due to the user receiver, anomalies in the atmosphere
250 corrections, and location-related errors such as anomalous multipath. This kind of fault
251 emerges only on the user side and cannot be detected by the network. Therefore, a
252 receiver autonomous integrity monitoring protocol is required by the user. In this
253 approach, the calculated residuals of every position solution are scanned for faults using
254 Chi-squared testing (equation (8), assuming that the squared residuals in the unbiased
255 fault-free case would follow a Chi-squared distribution). In the case of detecting a fault,
256 an identification process is required to identify which observations are faulty. As
257 discussed earlier, this is a computationally expensive step that may affect the real-time
258 positioning performance. Therefore, it is proposed, in this new method, to calculate the
259 absolute values of the ratio between the residuals of the individual observations of the
260 user position solution and their counterparts of the nearest reference station, assuming
261 that the latter, after elimination of the faulty satellites, are fault-free. There is no need
262 to use normalized values, where the elevation-angle-weighted model is typically used,
263 and the distances between the network stations are relatively short, i.e., < 100 km, such
264 that the standard deviations that are used to normalize the residuals would be almost
265 the same at the rover and the reference station. The procedure can be expressed as
266 follows:

$$\hat{r}_{Uc,\varphi}^{S_{1...m}} = y_{Uc,\varphi} - G_U \hat{x}_U \quad (9)$$

$$ratio = \begin{bmatrix} \hat{r}_{UC_{i=1}}^{S_{1...m}} / \hat{r}_{MC_{i=1}}^{S_{1...m}} \\ \hat{r}_{UC_{i=2}}^{S_{1...m}} / \hat{r}_{MC_{i=2}}^{S_{1...m}} \\ \hat{r}_{U\varphi_{i=1}}^{S_{1...m}} / \hat{r}_{M\varphi_{i=1}}^{S_{1...m}} \\ \hat{r}_{U\varphi_{i=2}}^{S_{1...m}} / \hat{r}_{M\varphi_{i=2}}^{S_{1...m}} \end{bmatrix} \quad (10)$$

267 where m is the number of the observed satellites by the user after excluding the
 268 suspected satellites by the network; $y_{UC,\varphi}$ is the user code and phase observations
 269 vector; and G_U is the design matrix.

270 Based on the calculated ratios, the observations can be ranked in descending
 271 order for their likelihood of containing faults. The observation of the largest ratio is
 272 considered the most suspected to be faulty. Fig. 2 shows an example of the ranking
 273 criterion. It shows that in the case of ten satellites in view where two frequencies are
 274 tracked, the observations shaded in red refer to the observations with the largest ratios,
 275 while those shaded in green have the lowest ratios. The exclusion shall be attempted
 276 with the observation with the largest ratio. In addition, their highly correlated
 277 observations, if any, from the same satellite should also be considered for exclusion.
 278 The correlation coefficient is calculated as follows:

$$\rho_{A,B} = \frac{D_A^T Q_{\hat{r}_U}^{-1} D_B}{\sqrt{D_A^T Q_{\hat{r}_U}^{-1} D_A} \sqrt{D_B^T Q_{\hat{r}_U}^{-1} D_B}} \quad (11)$$

$$Q_{\hat{r}_U} = Q_{y_U} - G_U (G_U^T Q_{y_U}^{-1} G_U)^{-1} G_U^T \quad (12)$$

279 where ρ is the correlation coefficient between errors in the observations A, B ; D is a
 280 zero column vector with ones at A and B entries only; $Q_{\hat{r}_U}$ is the covariance matrix of
 281 the user residuals; and Q_{y_U} is the covariance matrix of the user observations.

282

| Observed satellites | | | | | | | | | | |
|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Satellite PRN | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Observations | $\varphi 1$ | P2 | $\varphi 2$ | C1 | $\varphi 1$ | $\varphi 2$ | $\varphi 1$ | C1 | C1 | $\varphi 2$ |
| | C1 | $\varphi 2$ | C1 | $\varphi 1$ | $\varphi 2$ | P2 | C1 | $\varphi 2$ | $\varphi 1$ | P2 |
| | $\varphi 2$ | C1 | $\varphi 1$ | P2 | C1 | $\varphi 1$ | P2 | P2 | $\varphi 2$ | $\varphi 1$ |
| | P2 | $\varphi 1$ | P2 | $\varphi 2$ | P2 | C1 | $\varphi 2$ | $\varphi 1$ | P2 | C1 |

Fig. 2 Example of the ranking criterion of the observations based on the ratio of the residuals with the reference station where red-shaded observations are the most likely to be faulty

283

284 After an initial exclusion of the suspected observation(s), the position solution
 285 is re-estimated and the Chi-squared test is repeated. If the detection test continues to
 286 fail, the exclusion of the second most vulnerable observation and its highly correlated
 287 observations is attempted with and without re-inserting the previously excluded
 288 observations back into the model. These procedures are repeated until the test passes,
 289 and the faulty observation is identified. The exclusion of the faulty satellites reported
 290 by the network and the exclusion of the highly correlated observations shall
 291 significantly minimize the risk of having more than one faulty observation, and the
 292 method can be applied for the exclusion of single or two simultaneous observations.
 293 The method can be applied in the case of suspecting two simultaneous faulty
 294 observations by selecting the two observations with the largest ratio values, whether
 295 they are of the same satellite or from two different satellites. The flow chart presented
 296 in Fig. 3 shows the overall process.

297

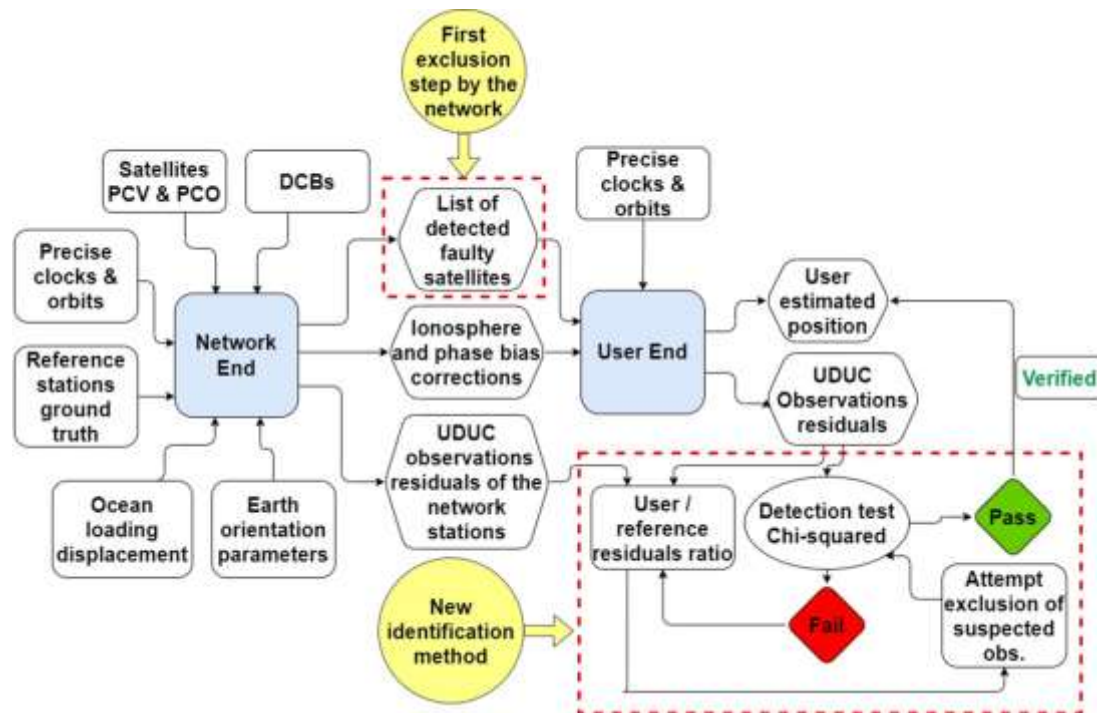


Fig. 3 Flow chart of the proposed method for FDE using PPP-RTK

298

299 Table 3 shows the main merits of the proposed method. It makes the
 300 identification process much faster due to the statistically-based selection process of a
 301 limited number of suspected observations, thereby, avoiding testing of all subsets. In
 302 addition, the identification is performed for the individual observations, maintaining
 303 the information availability, since the exclusion is performed per individual
 304 observations, not per satellites, as the case in the traditional grouping techniques. This
 305 is facilitated because the UDUC PPP-RTK offers the ability to calculate the residuals
 306 of the individual observations.

307

Table 3 Main advantages of the proposed method for fault identification

| Advantages of PPP-RTK | Risk reduction at the user end | Improving identification process speed | Better availability maintaining |
|--|--|--|--|
| <ul style="list-style-type: none"> • Precise and real time position method; • Ability to analysis error sources; • Reduces communication risks and the required bandwidth as one-way communications are used; • UDUC processing allows better analysis of individual observations. | <ul style="list-style-type: none"> • The satellites are checked by the network for any faulty satellites due to satellite faults and a list of the faulty satellites is sent to user(s) to exclude. | <ul style="list-style-type: none"> • The suspected observations are selected based on the ratio value without the need to generate all possible subsets; • The ratio is calculated based on the real values of the residuals | <ul style="list-style-type: none"> • Exclusion is attempted on observations not on satellites as it is the case in grouping techniques; • The identification only takes place when the detection test fails unlike SS where it is performed every epoch and for all subsets. |

308

309 Experimental test cases

310 The proposed method has been tested in two different situations with different
311 parameters to verify the outcome of the proposed approach. A geodetic receiver that
312 observes GPS legacy frequencies only is used in the first case. In the second example,
313 a low-cost receiver was used to observe two frequencies from multi-constellation
314 GNSS. The latter kind of receivers, due to their low cost, is anticipated to be onboard
315 most cars, etc. The variations also extended to include the testing dates and the number
316 of reference stations of the network, and their distances to the user. Table 4 summarises
317 the experiments' strategy, and Fig. 4 shows a map of the receivers' distribution of the
318 network and the user.

319

Table 4 Testing strategy and parameters of the two applied test cases

| Parameter | Value | |
|-----------|---------------|---------------|
| | Test case (1) | Test case (2) |
| | | |

| | | |
|--------------------------------------|-----------------------------------|---|
| # of network stations | 10 CORS* | 7 CORS* |
| User receiver | Geodetic receiver (Trimble NETR9) | Commercial receiver (u-blox F9P) |
| Sampling interval/test period | 30 sec / 10 hr | 1 sec / 12 hr |
| Date of testing | 1 July 2022 | 13 Oct 2022 |
| GNSS/frequencies | GPS: C1C, L1C, C2L, L2L | GPS: C1C, L1C, C2L, L2L Galileo: C1C, L1C, C7Q, L7Q BDS: C2I, L2I, C7I, L7I |
| Distances to the user (km) | 3 ~ 32 | <1 ~ 20 |

* Continuously Operating Reference Stations (CORS)

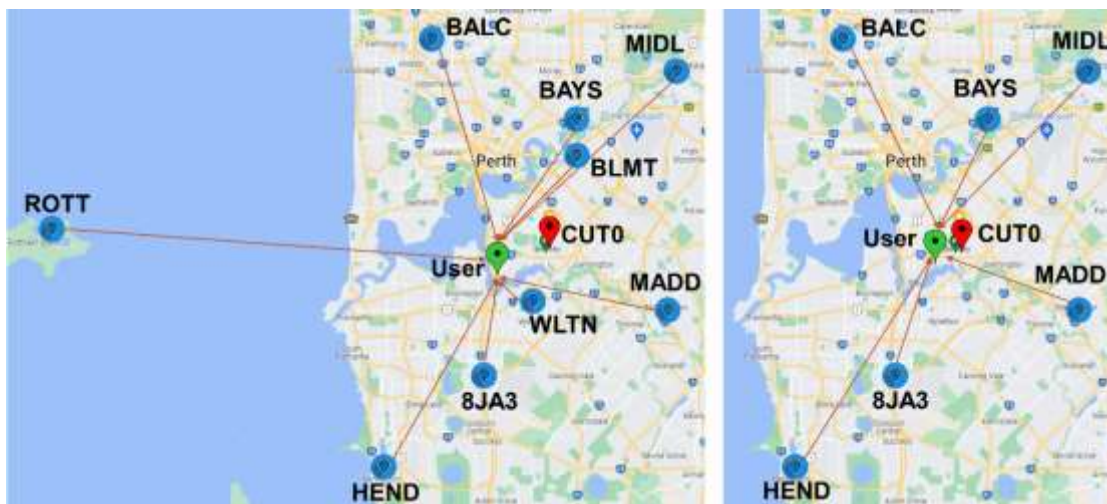


Fig. 4 Layout of the network stations (in blue), including the reference station (in red) and the user receiver (in green) for test case (1) on the left panel and test case (2) on the right panel

321 **Results and discussion**

322 In the two testing examples, the network processes the data and provides the error
323 corrections as well as a list of faulty satellites to the user. The network also provides
324 the user with the residuals of the observations of the reference stations. At the user end,
325 the faulty satellites reported by the network are excluded, and the residuals of remaining
326 individual observations are calculated. Fig. 5 and Fig. 6 show the residuals of the four
327 observations of each satellite in the first test case tracked by the selected reference
328 station (nearest to the user) and the user, respectively, during the test period. The
329 residual behaviour was very much the same in the second test case. From the two
330 figures, it is evident that the overall values of the user residuals are larger than their
331 counterparts at the reference station, in particular for the code observations. This is
332 expected since both the precautions taken in the setup of a reference station, e.g.
333 minimizing the impact of multipath, and the processing that exploits the known position
334 of the reference station shall help in reducing the level of the computed residuals. This
335 is reflected in their RMS values given in Table 5.
336

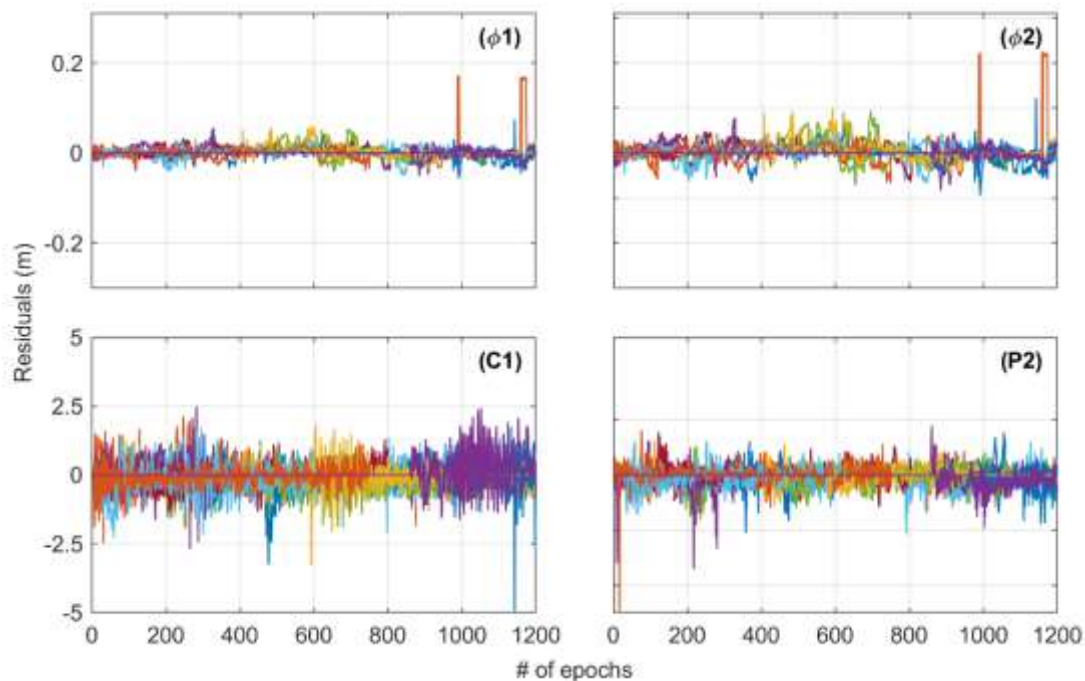


Fig. 5 Residuals of phase (top) and code (bottom) observations of L1 (left) and L2 (right) frequencies for all GPS-tracked satellites by the *reference station* during the first test where different colours represent different satellites

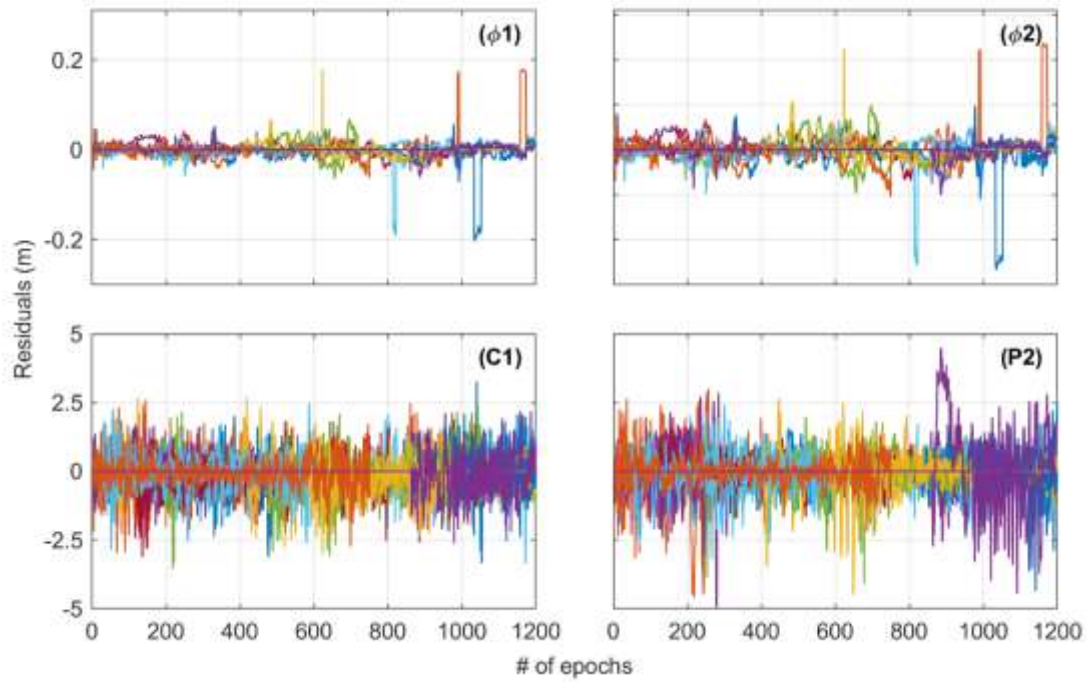


Fig. 6 Residuals of phase (top) and code (bottom) observations of L1 (left) and L2 (right) frequencies for all GPS-tracked satellites by the *user* during the first test where different colours represent different satellites

Table 5 RMS of the observation residuals of each signal type of the tracked GPS satellites for both reference and user stations over 1200 epochs period, the total length of test case (1)

| RMS (m) | | |
|----------|-------------------|---------------|
| | Reference station | User receiver |
| ϕ_1 | 0.0139 | 0.018 |
| ϕ_2 | 0.0208 | 0.0264 |
| C1 | 0.4852 | 0.6756 |
| P2 | 0.3498 | 0.736 |

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For Fig. 7 and Fig. 8, they represent test case (1) where GPS only was used. The four plots in each figure refer to four different and independent epochs (Fig. 7 and Fig. 8 show eight different epochs) in which a fault has been detected in each of them. These epochs are representative examples among many other epochs where faults have been

342 detected. The graph on each plot represents the absolute value of the ratio between the
343 residuals of the observations of the user and their counterparts of the nearest reference
344 station of the network at that epoch. For the four epochs presented in Fig. 7, the
345 identification of the faulty observation was successful after the first iteration as per the
346 criterion described earlier in the methodology section. In brief, in these four epochs,
347 Chi-squared test has detected a fault. To identify which observation is the faulty one in
348 each epoch, the ratio has been calculated and the observation with the largest ratio has
349 been excluded. The position solution was estimated, and the residuals were computed
350 after that exclusion. The detection test (Chi-squared test) was performed on the new
351 residuals, and it passed, meaning that the excluded observation (the one with the largest
352 ratio that is encircled in red in Fig. 7) was the faulty one. The four epochs presented in
353 Fig. 8 follow the same explanation described for Fig. 7. The difference is in these four
354 epochs the detection test did not pass after the exclusion of the observation of the largest
355 ratio. Therefore, the observation of the second largest ratio (i.e., the observation
356 encircled in red in Fig. 8) was excluded and the detection test passed. The eight epochs
357 depicted in Fig. 9 and Fig. 10 are similar to those in Fig. 7 and Fig. 8, respectively, but
358 for test case (2) where multiple constellations were observed using a commercial low-
359 cost receiver.

360

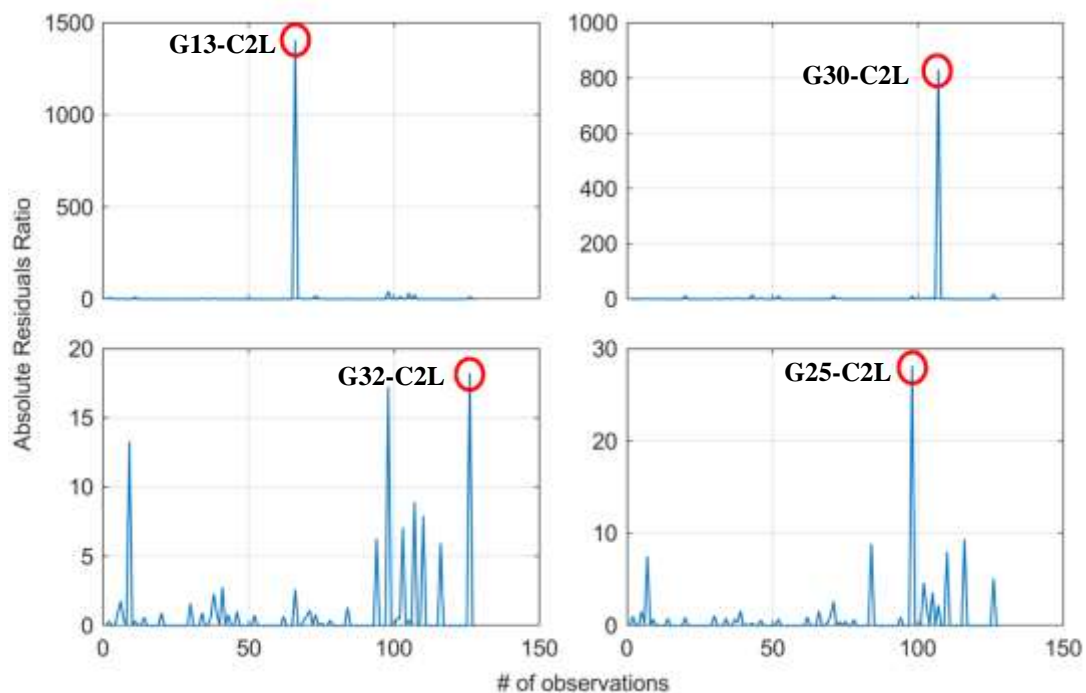


Fig. 7 Absolute ratios between the residuals of the user receiver and reference station observations at four different epochs as representative examples in *the first test* where a fault has been detected among GPS observations only. The encircled observations, with *the largest ratio*, are the faulty observations. The satellite PRN and the observation type of each faulty observation are mentioned in the text within each respective plot

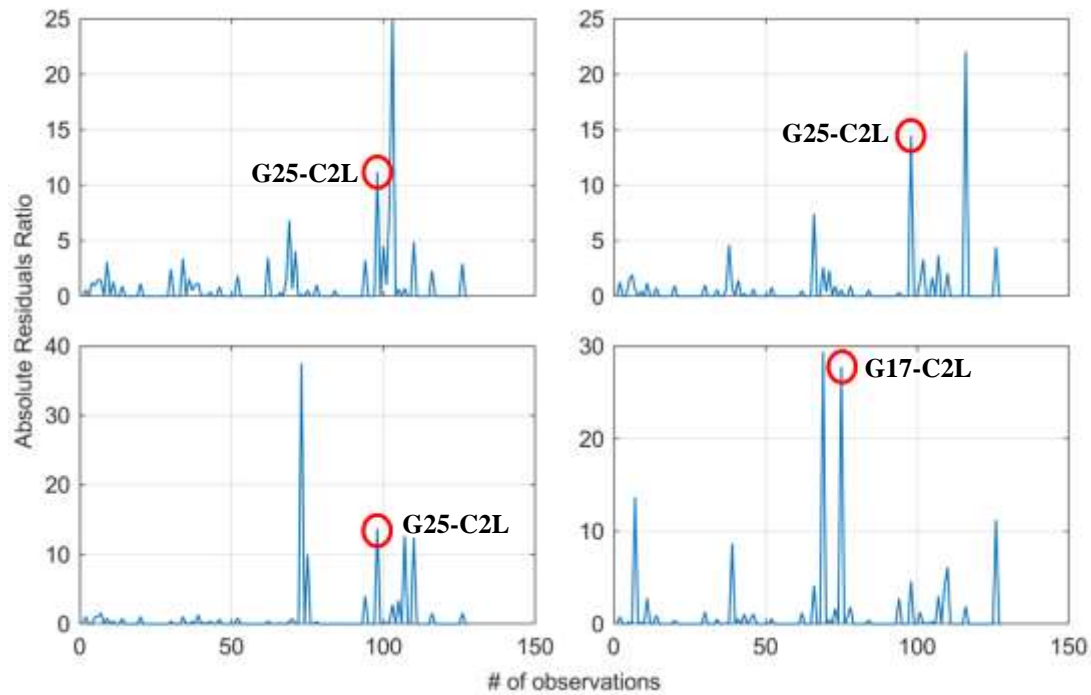


Fig. 8 Absolute ratios between the residuals of the user receiver and reference station observations at four different epochs as representative examples in *the first test* where a fault has been detected among GPS observations only. The encircled observations with *the second largest ratio*, are the faulty observations. The satellite PRN and the observation type of each faulty observation are mentioned in the text within each respective plot

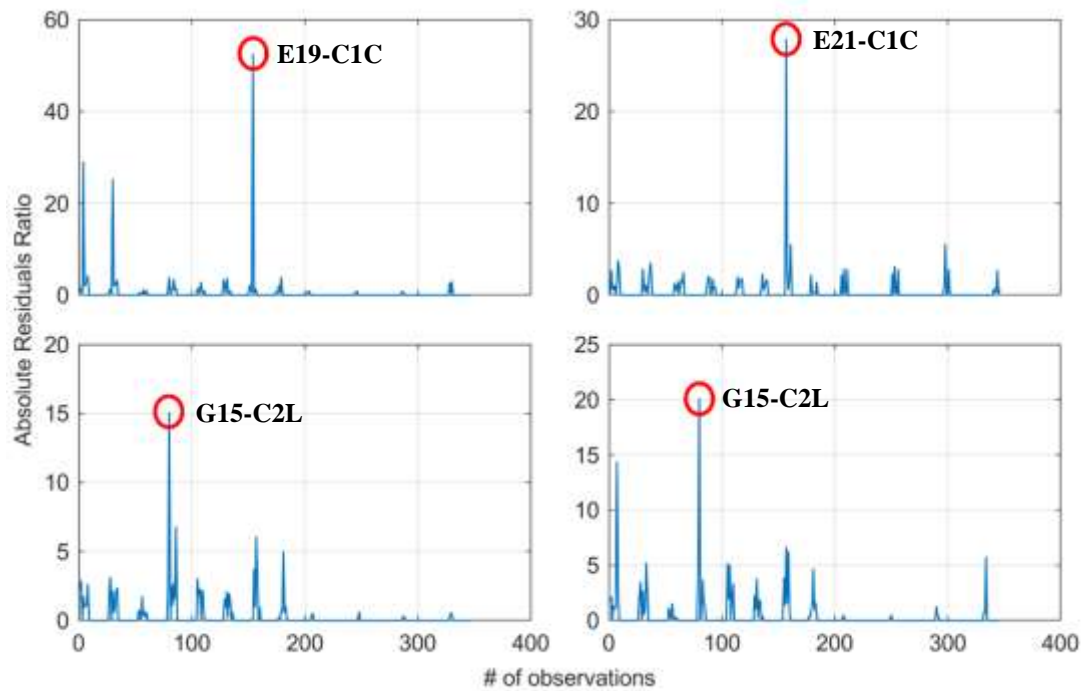


Fig. 9 Absolute ratios between the residuals of the user receiver and reference station observations at four different epochs as representative examples in *the second test* where a fault has been detected among GPS, Galileo and BDS observations. The encircled observations with *the largest ratio*, are the faulty observations. The satellite PRN and the observation type of each faulty observation are mentioned in the text within each respective plot

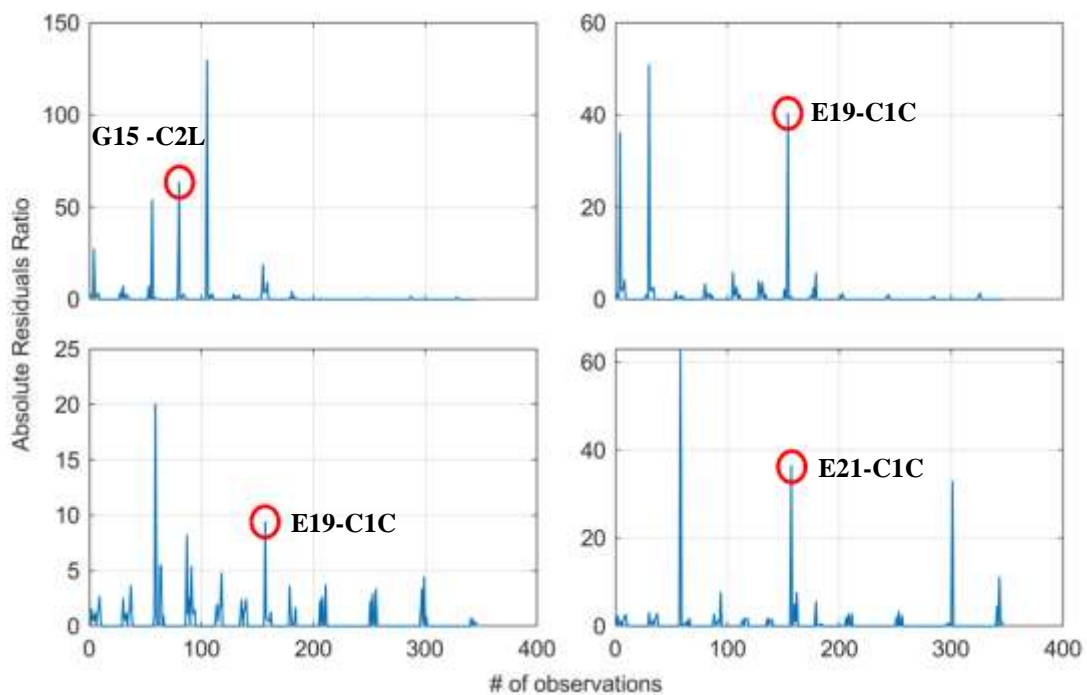


Fig. 10 Absolute ratios between the residuals of the user receiver and reference station observations at four different epochs as representative examples in *the second test* where a fault has been detected among GPS, Galileo and BDS observations. The encircled observations with *the second largest ratio*, are the faulty observations. The satellite PRN and the observation type of each faulty observation are mentioned in the text within each respective plot

361

362 It is noted that the new approach using the user/reference residuals ratio was not
 363 *always* able to identify the faulty observation(s) from the first exclusion attempt and
 364 more testing cycles were needed. This can be explained as follows: from equation (10),
 365 it can be shown that the more the reference station residuals are accurate, the more the
 366 ratio will be sensitive to identify a fault from the first attempt. However, in some cases,
 367 the best fit of the observations with the final solution performed at the reference station
 368 can produce large residuals for some observations due to their specific errors such as
 369 multipath, imperfection of the bias model, etc., which are reflected in their residuals.
 370 This can cause an increase in the value of some residuals of the reference stations, and
 371 as a result, the ratio related to these observations may not be the highest in case their
 372 counterparts are the faulty ones at the user end. However, the case, where the largest
 373 ratio is unrepresentative of the faulty observation after a few exclusion attempts, was
 374 infrequent during the test cases.

375 Table 6 shows the overall statistics of the two testing cases in terms of the
 376 identification potentials of the new approach. 112 and 110 faulty epochs were found in
 377 testing examples one and two, respectively. The ratio method identified the faulty
 378 observation(s) at the first exclusion attempt in about 16 -19% of the number of faulty
 379 epochs in the two test cases increased to 76 – 81%, after six exclusion attempts.

380

Table 6 Percentage of identification success in the two testing examples

| # of exclusion attempts | Test case (1)% | Test case (2)% |
|--|-----------------------|-----------------------|
| First exclusion attempt | 19.64 | 16.36 |
| First and second exclusion attempts | 39.29 | 30 |
| First to third exclusion attempts | 50.89 | 46.36 |

| | | |
|---|-------|-------|
| First to fourth exclusion attempts | 60.71 | 61.81 |
| First to fifth exclusion attempts | 71.43 | 69.1 |
| First to sixth exclusion attempts | 81.25 | 76.36 |

381

382 Table 7 presents a comparison between the proposed identification method and
383 the SS as well as Chi-squared methods in terms of their ability to reduce the
384 computational load as a factor in the number of the required observation subsets in the
385 two tests. The new method saved around 85% and 98% of the computational load of
386 the FDE process in the first test case compared to Chi-squared and SS, respectively,
387 while it saved about 94% and 99.999% in the second test compared to the two methods.
388 The percentage of reducing the computational load is proportional to parameters such
389 as observation period, sampling interval, number of observations at each epoch, and the
390 number of detected epochs with faulty observations. This is because the number of the
391 required subsets for testing increases significantly with the increase of these parameters
392 in the case of using SS and Chi-squared methods. Whereas the number of the generated
393 and tested subsets when using the new ratio method is only dependent on the number
394 of performed iterations needed to identify the faulty observations. This example shows
395 how the new ratio method is effective in significantly reducing the computational load
396 of the FDE process especially when a high sampling rate is required.

397

Table 7 Comparison between the proposed identification method and the solution separation and Chi-squared identification methods concerning the ability to save in the computational load

| Testing example | Test case (1) | | | Test case (2) | | |
|------------------------------------|----------------------|------|-------|----------------------|------|-------|
| Observation period (hr) | 10 | | | 12 | | |
| Sampling interval (sec) | 30 | | | 1 | | |
| Average No. of observations | 32 | | | 76 | | |
| No. of faulty epochs | 112 | | | 110 | | |
| Identification method | SS | Chi- | Ratio | SS | Chi- | Ratio |
| | | 2 | | | 2 | |

| | | | | | | |
|---|---------|------|-----|--------|------|-----|
| No. of tested subsets in the one-satellite-out fault mode | 38400 | 3584 | 507 | ~ 3 | 8360 | 493 |
| | million | | | | | |
| % of computational load saving the FDE process when using the ratio method | ~ 98 | ~ 85 | | ~99.99 | ~ 94 | |

398

399 **Conclusion**

400 Autonomous navigation of vehicles, drones, and others requires real-time precise
401 positioning with efficient integrity monitoring capability. PPP-RTK positioning
402 method can cover a wide area and provide precise corrections with fast solution
403 initialization and has the additional advantage of IM, i.e., providing the residuals for
404 the individual observations by processing the UDUC observations. Current FDE
405 methods represent a major challenge for real-time applications as it encompasses the
406 generation and testing of numerous observation subsets to identify faulty observations
407 when detected, especially when multiple faults take place concurrently. To reduce the
408 computational load, some suggested methods, such as the grouping technique, result in
409 the loss of valuable information from the observations of the removed satellite.

410 We propose a new approach that can reduce the computational load of the FDE
411 process without affecting the observation availability. We suggest excluding faulty
412 satellites at the network station exploiting the known positions of the stations, and
413 sending this information to users. For errors due to the user environment or due to
414 imperfect error treatment, when a fault has been detected at a certain epoch, the ratio
415 between the observation residuals of the user receiver and the closest reference station,
416 assuming that the latter is fault-free, is to be calculated. The highest ratio can indicate
417 the faulty observation(s) so that their exclusion is attempted to avoid checking for
418 solutions from all possible numerous observation subsets to identify the fault as done
419 by the traditional methods. Moreover, the exclusion will be based on screening the
420 individual observations, not the whole satellite, which maintains the observation
421 availability due to processing UDUC observations.

422 Two representative tests were performed to demonstrate the performance of the
423 proposed method. The first included a geodetic receiver that tracked GPS observations
424 only, and the second test comprised a low-cost receiver that is most likely to be used in

425 AVs observing multi-GNSS constellations measurements. In the two tests, the new
426 ratio method provided consistent performance where the faulty observations have been
427 identified from the first exclusion attempt in 16 – 19% of the epochs where a fault has
428 been detected, while it took up to six exclusion attempts to identify the faulty
429 observation in around 76 – 81% of the faulty epochs. When compared to the commonly
430 used FDE methods, such as the SS test and conventional Chi-squared test, it takes only
431 <1% and 15%, respectively, of the time required for detection and identification. This
432 is based on the observing period and interval, the number of the faulty epochs, i.e., 112
433 and 110, and the average number of observations at each epoch, i.e., 32 and 76, in the
434 two test examples, respectively. The future work plans include testing in a kinematic
435 mode where the receiver is mounted on top of a moving vehicle. Also, it includes
436 involving testing more frequencies, and for more challenging environments.
437

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447

448 **Data Availability**

449 Data used, generated and analysed in this study will be made available upon reasonable
450 request from the corresponding author.
451

452 **Authors contributions**

453 **Hassan Elsayed:** Conceptualization, Methodology, Software, Validation, Formal
454 analysis, Investigation, Resources, Data Curation, Writing - Original Draft; **Ahmed El-**
455 **Mowafy:** Conceptualization, Methodology, Resources, Writing - Review & Editing,
456 Supervision; **Kan Wang:** Writing - Review & Editing, Supervision.

457

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558

559 **Authors Autobiography**



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575