School of Education

Professional Learning for Teaching Mathematics through Problem Solving in Indonesian Primary Schools

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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made.

Signature:

Date :

Abstract

This study focused on teacher professional learning about problem solving in the context of selected Indonesian primary schools. Professional learning was conducted via a Lesson Study Cycle. Teachers' learning was judged in terms of their mathematical content knowledge, and pedagogical content knowledge in relation to teaching using a problem solving approach. Other aims of the project were to identify any changes in beliefs about teaching held by the participant teachers, and to identify and describe elements of Lesson Study that were effective in developing and supporting teacher learning in this Indonesian context.

This project used case study as an analytical method to provide an in-depth perspective based on extensive data collection. The participants in this study were twelve upper grade primary school teachers who came from an inner city area and a suburban area in Bengkulu, Indonesia. Five of the twelve teacher participants acted as volunteer teachers and these were reported and described in-depth as case studies. Data collection in this study involved classroom observations, interviews with teachers, Lesson Study Group Meetings, video recording of classrooms, field notes, and an analysis of documents. Data analysis used grounded theory with inductive analysis to identify emerging themes from the accumulated data that were developed from interview transcripts, field notes, report records, observation, and reflection. Triangulation in data gathering was achieved through the use of multiple data sources.

The findings from this study showed that Lesson Study brought changes in teachers' pedagogical content knowledge and mathematical content knowledge, and also teachers' beliefs about teaching. Reflection during Lesson Study for five volunteer teachers indicated that four teachers were not confident, and only one teacher was confident in teaching with observers in the classroom. Results suggested that the use of manipulatives and open problems were new teaching ideas for both teachers and the students. The teachers had not had much experience in using manipulative problems. They changed to become aware of the fact that manipulative

problems could be used to help students with understanding concepts. Also, more open problems were used as a new approach by all teachers. There was a small change from traditional teaching to the use of less teacher directed work in classrooms. The teachers showed less change in mathematical content knowledge than in pedagogical content knowledge. Nevertheless, they were building mathematical content knowledge by connecting resources with learning. Lesson Study improved the teachers' knowledge of teaching mathematics using a problem solving approach in that they changed from a traditional approach to one where problems were developed. The study has implications for teacher professional learning as it shows that teachers can develop pedagogical content knowledge and mathematical content knowledge through the use of a Lesson Study process that develops learning communities. This in turn aids their planning of programs based on improved problem solving for their students.

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Glossary of Terms

ACARA	Australian Curriculum Assessment and Reporting Authority
BPG	Balai Penataran Guru (Teacher Training Centre)
BPS	Badan Pusat Statistik (Central Statistical Agency)
CCK	Curricular Content Knowledge
Depdiknas	Departemen Pendidikan Nasional (National Department of Education)
D-IV	Diploma Empat (Diploma 4 Year Program)
Ditjend DIKTI	Direktorat Jendral Pendidikan Tinggi (Directorate General of Higher Education)
IMSTEP	Indonesian Mathematics and Science Teacher Education Project
IMSTEP-JICA	Indonesian Mathematics and Science Teacher Education Project-Japan International Cooperation Agency
KTSP	Kurikulum Tingkat Satuan Pendidikan (Curriculum at School Unit Level)
KKG	Kelompok Kerja Guru (Teacher Peer Group)
LPTK	Lembaga Pendidikan Tenaga Kependidikan (Institute of Teacher Education)
MGMP	Musyawarah Guru Mata Pelajaran (Subject Matter Teacher Peer Group)
MCK	Mathematical Content Knowledge
MKKS	Musyawarah Kerja Kepala Sekolah (Principal Peer Group)
NCTM	National Council of Teachers of Mathematics
Puskur	Pusat Kurikululum (Centre for Curriculum)
PCK	Pedagogical Content Knowledge
PGSD	Pendidikan Guru Sekolah Dasar (Primary School Teacher Education)
SPG	Sekolah Pendidikan Guru (School of Teacher Education)
S-1	Strata satu (Under graduate)

SKL	Standar Kompetensi Lulusan (Graduation Standard Competence)
SISTTEMS	Strengthening in-Service Teacher Training of Mathematics and Science Education at Junior Secondary Level
SMCK	Subject Matter Content Knowledge
TIMSS	Trends in International Mathematics and Science Study

CHAPTER ONE

Introduction

Introduction

This chapter introduces the study about professional learning based on problem solving and a Lesson Study approach in an Indonesian Primary School context. The findings provide data to describe changes in teachers' pedagogical content knowledge, teachers' mathematical content knowledge and teachers' beliefs about teaching and learning mathematics. In general, the study sought to evaluate the effects of a Lesson Study approach in bringing about changes in teachers' practices in teaching mathematics. This chapter provides a background of the study, introduces Lesson Study as a model for professional learning, states the research question, the research approach, outlines the significance of the study and gives an overview of the structure of the chapters.

Background

The study aims to describe and identify the effects of professional learning of a small group of Indonesian primary school teachers based on the Lesson Study Cycle. This section describes the issues and policies in regards to Indonesia in two categories: primary school mathematics education and primary school teachers in Indonesia.

Primary school mathematics education in Indonesia

Education involves a conscious effort to cultivate the full potential of human resources through teaching activities, including efforts to improve the quality of teaching and learning. In the Indonesian educational system, prior to attending university, students must complete six years in primary school, three years in secondary school and three years in senior high school. At the primary school level students are required to take at least eight subject areas including mathematics. In general, the subject of mathematics within primary schools is taught by a classroom teacher, but in a few schools mathematics is taught by a specialist mathematics teacher in the upper classes (Year Four to Six).

Mathematics occupies an important position, as important as the development of science and technology, because mathematics is the language of science, and helps develop thinking, reasoning and problem solving in real life. However, mathematics, as a subject area, is often considered by most students in Indonesia to be difficult when comparing it to other subjects, both at primary and secondary school levels.

One of the central issues in Indonesia is low mathematics achievement when children make the transition from primary to secondary school. Students' achievement in learning mathematics, both nationally and internationally, has been considered to be unsatisfactory. The Trends in International Mathematics and Science Study (TIMSS) (Tim TIMSS Indonesia, 2011) found that the Indonesian position for Grade 8 mathematics was 34th out of 38 countries in 1999, 35th out of 46 countries in 2003, and 36th out of 49 countries in 2007 (See http://litbang.kemdikbud.go.id/detail.php?id=214). This position shows that Indonesian students have a very low performance in mathematics in comparison to other countries.

An Indonesian national examination offers an indication of student achievement in mathematics at the end of the primary school stage. The results are used as a prerequisite for entry into secondary school. Places are offered on a competitive basis. The national examination tests are set with more emphasis on the memorisation of facts and skills than on students' critical thinking. Consequently, to achieve a high score in the national examination, the students need more time to learn the facts outside of school time. All students take the national examination, so they will be given additional hours to study mathematics out of school time. It is believed that the main reason students follow the extra course is to get the best score for their final examination, instead of wishing to gain a deeper understanding into mathematics. As a result, what the students learn is quickly forgotten.

Another prevalent issue in Indonesia is that mathematics in primary schools is generally taught by most primary teachers using a traditional teaching approach. Koseki (2007) said that teaching methods included:

> … the copy method and the lecture method. In the problem solving approach, the teacher would give the students a problem, have the students think about the answer, and then, instead of walking around the classroom and see how they worked out their answers, would simply announce the answer after a certain period of time. During this time, the students copied the problem written on the chalkboard into their notebooks. This does not allow the students to appreciate various methods of solving problems. (p. 214)

Teachers use textbooks as their main resource when explaining mathematical subjects. In solving problems, the teacher works out the problems, while talking and writing simultaneously on the white board. Then, the students start working on mathematics individually. They work in silence, and follow the teacher’s example, and then they practice the procedure with exercises in the same sort of problems. The students copy the material from the white board or textbook and offer little in writing about their own opinion or thinking. Most spend a lot of time listening to the teacher’s explanation, doing mathematics from the white board, and doing similar questions for practice. Then, the teacher gives a mark on their exercise book. Some students worry about their answers, because if they do not get the correct answer, the teacher will give them a low score and set extra homework. Furthermore, there is little communication between students and the teacher, because the teacher gives instructions to follow the teacher’s worked example. This traditional way of teaching has a negative influence on the students’ attitudes towards mathematics, which means that most students do not like to learn mathematics, and that some of them are even afraid of mathematics. So, some students believe that mathematics is a difficult subject.

Since 2006, Indonesia has implemented a curriculum at the educational unit level (Kurikulum Tingkat Satuan Pendidikan) (KTSP) for primary and secondary schools. The schools were authorised to develop a curriculum that corresponded with local culture and society, and met the content standards and competency for graduation standards of the National Educational Standard. The basic competence level was seen as a minimal competence to be developed by each school year level. To implement KTSP curriculum, at the school level, the primary school teachers were expected to design and implement lesson plans, use teaching aids and a student active approach. However, most schools had difficulty in developing a curriculum and the teaching materials to match this outline. Department of National Education Centre for Curriculum (Departemen Pendidikan Nasional Pusat Kurikulum) (2007) found that most teachers used only textbooks as instructional resources and lacked an understanding of evaluation for conceptual understanding. In addition, the schools generally had no specialist teacher who is competent to help teachers to implement KTSP.

Although there were many problems in the teaching and learning of mathematics at primary school, the Indonesian government set the content standard for primary school mathematics. National Education Standard Agency (Badan Standar Nasional Pendidikan) (BSNP, 2006) noted the goals of mathematics in the content standard for primary school are the following:

- Understanding mathematical concepts, explaining mathematical concept relationships and applying the concept or algorithms flexibly and efficiently in problem solving;

- Using logical thinking, including patterns and properties, making generalisations, constructing proof, or explaining a mathematical statement;

- Solving a problem that includes understanding the problem, designing a mathematical model, solving with the model, and interpreting the result;

- Communicating ideas with symbols, tables, diagrams or other equipment to aid explanation;

- Having a positive attitude and valuing the application of mathematics in real life, namely having sense of curiosity, engagement, and interest in learning mathematics, also having confidence in problem solving. (BSNP, 2006, p. 148)

In primary schools (BSNP, 2006, p. 147) noted the curriculum emphasised that a problem solving approach should be the focus of attention in teaching and learning mathematics. This approach included using closed problems with a single solution, open problems, and problems with various solutions. Teaching with a problem solving approach was not receiving much teacher attention due mainly to the lack of content knowledge and pedagogical knowledge of this approach by many teachers. Developing good teachers was one of the main factors in an effort to raise the quality of the teaching process and improve students' learning.

Primary school teachers in Indonesia

One of the factors relating to the low quality of national education achievements is the low quality of teachers. The issue of developing teacher quality involved many aspects, such as qualifications, professional development, protection of the profession, certification, and welfare.

At the beginning of 1991, the Indonesian government decided to improve the qualification level of primary school teachers. Anyone who wanted to be a teacher in a primary school had to complete two years study at the institute of teacher training of Primary School Teacher Education, (Pendidikan Guru Sekolah Dasar) (PGSD) instead of three years education after junior high school (Sekolah Pendidikan Guru) (SPG). Then in 2003, to become a primary teacher, it was necessary to complete a four years program for undergraduate students. This program was followed up by the government policy (Peraturan Pemerintah Republik Indonesia No 19, 2005) on National Educational Standards that required Primary school teachers to have a minimum academic qualification of an undergraduate program (D-IV for diploma or S-1 for a bachelor degree) with an educational background in primary education, psychology, or a primary teacher certificate.

The goal of primary teachers' professional learning referred to the government policy Permen Diknas No 16, 2007, included academic qualifications and a teachers' competency standard. The Teachers' Competency Standard was divided into four main competencies: pedagogical, personal, social, and professional competence (BSNP, 2007). The two of the four teachers' competency standards that have most influence in teaching mathematics for primary schools are pedagogical and professional competence.

The pedagogical competence of a primary school teacher had the following characteristics:

- Knowing the characteristics of learners from physical, moral, social, cultural, emotional, and intellectual points of view;
- Knowing the theory of learning and principles of learning in education;
- Developing curriculum related to the subject area;
- Organising an educational learning environment;
- Utilising information and communications technology for supporting learning;
- Facilitating the development of potential learners to actualise different potentials of students;
- Communicating effectively and being empathetic and courteous with students;
- Organising assessment and evaluation processes suitable for the learning outcomes;
- Utilising the result of assessment and evaluation in planning;
- Being reflective to improve the quality of learning.

Professional competence of primary teachers was as follows:

- Knowing the content, structure, concepts, and scientific mindset that supports the subject area;
- Knowing the basic standards of competence and the competence of the subject area;
- Developing the content of the subject area;
- Developing professionalism in a sustainable manner by performing reflective acts;
- Utilising information and communications technology to communicate and for self-development. (BSNP, 2007)

Although the government had given more attention towards issues in education and many efforts had been undertaken, the quality of education was not yet in accordance with expectations. The demand of educational quality was very closely related to improving academic competence and professional competence.

Sembiring (2010) found that there were no differences in the academic standard and pedagogical competence of teachers who were certified and teachers who were not certified. It showed the complexity of teachers' competence. Theoretically the teachers who were certified would be more competent in academic qualifications, pedagogical knowledge and professional competence than teachers who were not certified.

The results of a study by the Direktorat Tenaga Kependidikan (Directorate of Educational Human Resources) (Kuntadi, 2009) based a professional competence examination of 15,186 primary school teachers in all subjects, in 16 provinces found that:

- 146 teachers or 0.096% were classified A as having the highest competence;
- 3,422 teachers or 2.25% were classified B;

- 4,9514 teachers or 32.61% were classified C; and
- 9,582 teachers or 63.105% were classified D as having the lowest competence.

It showed that 9,582 teachers or 63.10% did not have the necessary competence and needed to improve their professional learning (Kuntadi, 2009, p. 3). The result of this professional competence examination suggested that primary school teachers lacked knowledge, which included subject matter knowledge and pedagogical knowledge.

Achieving these standard competences hinged on the capabilities of teachers. In practice teacher professional development was done through pre-service and in service training. Activities for in service training were implemented because the government had proposed programs in the form of projects, which according to the program policy makers were important to be conducted. Meanwhile, the training of primary school teachers, especially in improving the competence of mathematics at primary schools, was conducted by the government through the Teacher Peer Group (Kelompok Kerja Guru) (KKG). The KKG is a form of professional learning based upon the guiding principle, "from teachers to teachers and by the teachers". The objectives of KKG were to:

- Solve teaching-learning problems;
- Test and develop new ideas to improve the quality of teaching and learning; and
- Improve teachers' professionalism. (Evans, Tate, Navarro, & Nicolls, 2009)

Some models of professional learning stressed how to teach rather than focussing any attention on how students learn. As a result, the professional development did not necessarily contribute towards teaching and learning processes that enhanced student learning outcomes. Although teacher professional learning had been developed, and included models like contextual teaching, Realistic Mathematics

Education, and classroom action research, most teachers tended not to change from their traditional approach. Professional learning in schools was intended to assist teachers to change but for many reasons it did not do so. Lesson Study had not yet been widely used for teachers in Indonesian primary schools. In this research, Lesson Study will be used as a professional development model for the professional learning of teachers for teaching problem solving in mathematics.

Lesson Study as Professional Learning

The term Lesson Study as a teacher professional learning model originated in Japan. Lesson Study is a collaborative process with a group of teachers who identify and work towards solving teaching and learning problems. A group of teachers meets to design lessons, implement lesson plans, observe lessons, and then revise lesson plans based on their observations, experience and discussion. One member of the group carries out the lesson while the others observe. After the lesson is carried out the group meets again to discuss the lesson. This is followed by a revised lesson plan that another member implements in his/her own class while being observed by the other teachers. Successful Japanese Lesson Study has been adopted by some countries such as the USA, Indonesia, and Australia (Marsigit, 2006; Pierce & Stacey, 2009; Liberman, 2009).

Lesson Study, as a form of professional learning, is still relatively new for primary school teachers in Indonesia. In the fiscal years 2001-2003, a pilot project of a Teaching Learning Model of secondary mathematics and sciences through Lesson Study was carried out by Pursuing Good Practice of Secondary Mathematics Education Through Lesson Studies in Indonesia IMSTEP-JICA in collaboration with three universities: UPI Bandung, UNY Yogyakarta, and UM Malang. Here the Japanese Government supported facilities with training as well as Educational Experts (Marsigit, 2006). In 2003-2005 as a follow up to the IMSTEP project, three universities collaborated with the Principal Peer Group (Musyawarah Kerja Kepala Sekolah) (MKKS) and Subject Matter Teacher Peer Group (Musyawarah Guru Mata

Pelajaran) (MGMP) and conducted Lesson Studies. In addition, Karim (2006) concluded that the implementation of Lesson Study had some impact as follows:

- Collaboration, collegiality, and communication among teachers and lecturers were formed;
- Implementations of the research lesson was started by others;
- Mathematics lecturers were directly involved in mathematics instruction in schools;
- The mathematics teachers association was more empowered.

In 2006-2008 the IMSTEP project was extended to become SISTTEMS (Strengthening in-Service Teacher Training of Mathematics and Science Education at Junior Secondary Level). The implementation of the project was limited to three districts in Java. The goal of SISTTEMS was to develop a model of MGMP's activity through the implementation of Lesson Study to improve teacher quality in mathematics and science. To achieve their goals, some activities were designed for Lesson Study based on MGMP at a Lesson Study based school. In an effort to improve the sustainability of the Lesson Study program at university and school level, the Directorate General Higher Education (Ditjen DIKTI) funded university teachers (LPTK) and provided school level funding, starting in 2009. The Indonesian Mathematics and Science Teacher Education Project (IMSTEP) had introduced 'Lesson Study' as a part of in-service teacher training at secondary schools since 2001.

Marsigit (2006) noted that Lesson Study could improve the practice of secondary mathematics teaching and learning processes, in terms of teaching methodology, teacher competencies, student achievements, alternative evaluation methods, use of teaching and learning resources and the syllabus. In addition, Karim (2006) said the impact of implementing Lesson Study led to mathematics educators being involved in mathematics instruction in school, working together in a group to plan, implement, and observe, as well as reflect on their lessons.

The Indonesian educational system has been changed from a centralised to a decentralised system since 2002. The system had introduced new challenges to improve the quality of teaching. The teachers at school level had to develop their own curriculum from the National Curriculum that contained the outline of competency standards, basic competency, and achievement indicators. The teachers needed to be active in using their knowledge to develop their instruction. This situation required teachers to be professional in their knowledge of teaching in their classroom.

In Indonesia, the content standard of primary school mathematics (BSNP, 2006) stated that problem solving was to be the focus in learning mathematics. This includes using closed problems with a single solution, open problems with no single solution, and solving problems in different ways. Meanwhile, one of the goals of the content standard of primary school mathematics was that students should able to solve mathematical problems that include the ability to understand the problem, design mathematical models, solve mathematical models and interpret the obtained solution. Consequently, primary school teachers needed to understand the purpose of solving mathematical problems. They needed to improve their skills to understand the problem, create mathematical models, solve problems and interpret the solution.

Research Questions

This study was designed to investigate the effect of professional learning through the use of a Lesson Study Cycle in an Indonesian primary school context. The objectives of this study were, in particular, to describe changes in teachers' pedagogical content knowledge, teachers' mathematical content knowledge and teachers' beliefs about teaching and learning mathematics using problem solving. In general, the study sought to evaluate the effect of Lesson Study in bringing about change in primary teachers' practices in teaching mathematics.

The main research question in this study was: *To what extent does professional learning based on problem solving and a Lesson Study approach effect: teachers'* *pedagogical content knowledge, teachers' mathematical content knowledge, and teachers' beliefs about teaching and learning mathematics?*

This is supported by a subsidiary question: *In what ways does Lesson Study bring about change in teachers' practice in teaching mathematics through problem solving in Indonesian primary school context?*

Research Approach

To address the research question, a Quasi-experimental approach was been chosen. Quasi-experiments are experimental situations in which the researcher assigns, but not randomly, participants to groups because the experimenter cannot artificially create groups for the experiment (Creswell, 2005). In this case, groups consisted of teachers who continued to teach in their regular class situations.

Qualitative research methods were used in this study. Creswell (2005) stated that qualitative research is a type of educational research in which the researcher relies on the views of participants, asks broad general questions, collects data consisting largely of words (or text) from participants, describes and analyses these words for themes and conducts the inquiry in a subjective manner.

The study is about teacher professional learning, involving 6 primary school teachers from an inner city area and 6 primary teachers from a suburban area, to enhance teaching mathematics through problem solving in primary schools. Throughout the study, data were collected using a range of methods: teacher interviews, in-class observations of teaching practice, small group discussions of Lesson Study meetings and analysis of documents. These data allowed the researcher to focus on insight, discovery and interpretation. The study used case studies as an analytical method to provide an in-depth perspective based on extensive data collection. Sowder (2007) noted case studies have become a useful way to understand and evaluate the effectiveness of professional development or intervention in the teacher preparation process.

Significance

The study aspires to provide a better understanding of what happens in classroom practice when the teacher teaches mathematics using a problem solving approach. This essentially involves beginning a lesson with a problem. Then students solve the problem according to their own methods and the teacher guides the construction of the new concepts in a way that is different from traditional teaching. So this study may lead to more information in using a problem solving approach in an Indonesian context.

This study will provide information about professional learning for primary school teachers that may help the teachers develop their knowledge via Lesson Study while they are teaching in their classroom. A Lesson Study approach may help to create professional learning communities in schools. At the school level, this study could help teachers to develop teaching materials based on competencies that are appropriate to local culture and society and which relate to the development of the KTSP curriculum, where teachers have opportunities to create and develop curriculum related to the school need.

The other significance is that educators, who have been involved in Lesson Study through observation of teaching practices and Group Meetings are now able to recognise the difficulties of teaching methods in problem solving and to understand the complexity of changing the process. Ultimately, this study can contribute to the collective knowledge of models for teacher professional learning and help to inform teachers, educators, schools and policy makers in the department of education. As a result of informing authorities of these outcomes it may be possible to redefine links between teachers' knowledge and implementation of professional learning and to address the need to improve teachers' professional competency, as described earlier.

Structure of the Chapters

There are six chapters in this study, and they are briefly described here:

Chapter One presents the introduction to the background of the study on professional learning for mathematics in Indonesian Primary Schools. This chapter provides also a background on Lesson Study for professional learning, the research questions, the purpose of the research, the research approach and its significance.

Chapter Two presents the review of literature about professional learning for teaching mathematics through problem solving and a theoretical framework for the study. With direction from the research question, this chapter discusses previous research projects that studied teachers' knowledge, professional learning, Lesson Study and problem solving. The theoretical framework was developed from the relevant research literature.

Chapter Three describes the methodology used in the study. The study, based on case studies, was developed using the quasi experiments in the Lesson Study Cycle process (plan the lesson, teach the lesson, reflect and evaluate and refine the lesson, and teach the refined lesson), classroom observation, Lesson Study meeting documentation, interviews pre and post Lesson Study and field notes. The study was conducted in three stages. The first and the second focused on observations from Lesson Study and interviews during and post Lesson Study. The third stage focused on interviews after one year to see the long-term effect of professional learning using the vehicle of Lesson Study.

Chapter Four presents the findings of the study. These findings describe changes in pedagogical content knowledge, mathematical content knowledge and teachers' beliefs about teaching and learning mathematics in primary school and also the impact of the Lesson Study Cycle on professional learning.

Chapter Five presents the discussion of the results presented in Chapter Four. It is based on findings described in the previous chapter and the implications according to the theoretical framework underpinning the study are discussed.

Chapter Six presents the conclusions of the study. It deals with the implications of the study for primary teachers and professional learning at the school level, as well as for policy makers. It also identifies suggested areas for further research. The last section presents the limitations of this study.

CHAPTER TWO

Learning for Teaching Mathematics through Problem Solving: A Review of the Research Literature

Introduction

This review focuses on several aspects of the topics of teacher knowledge, professional learning, and problem solving. Effective mathematics teaching requires a serious commitment to the development of students' understanding of mathematics. The development of students' understanding of mathematics may depend on how they learn and how the teacher teaches. Much of what the students learn about mathematics is learned through the activities that the teacher provides. To provide such activities, the teacher needs to understand what students know and need to learn and then support them to learn it well. For this to happen, the teacher should have good mathematical content knowledge, accurate knowledge of students, and knowledge of teaching strategies and methods. Different teachers have different styles and strategies for helping students learn and there is no best strategy to use in order to teach mathematics.

Many researchers have studied the relationship between mathematics teachers' knowledge and their practice (Shulman, 1986; Ball, 1990; Turner & Rowland, 2009). Shulman (1986) identified three forms of teachers' knowledge, *content knowledge, pedagogical knowledge*, and *curricular knowledge*. Later, Ball (1990) also focused on the subject matter knowledge of pre-service elementary and secondary mathematics teachers. Then, Rowland and Turner (2007) described the *Knowledge Quartet* as a framework to be used in practice for mathematics teaching development.

Specially designed professional learning often enhances the professional knowledge, skills and attitudes of teachers. One teacher professional learning model is Lesson Study. It originated in Japan and has been developed in many countries. Many studies about professional learning, including Lesson Study, have been conducted (Shulman, 1998; Saito, Harun, Kubokic, & Achbanad, 2006; Lieberman, 2009; McDonald, 2010). In the Indonesian context, professional learning is conducted in various aspects of teaching and at various levels. Teacher professional learning is provided by the government at the national level, the local level, and the school level. Besides that, professional learning is organised by professional organisations such as the Indonesian Teachers' Association (Persatuan Guru Republik Indonesia) (PGRI). In order to support the government policy National Ministry of Education (BSNP, 2007) for the academic qualification and teachers' competency standard, Lesson Study has been developed in Indonesia since 2001 to aid with professional learning. A number of studies using Lesson Study have been conducted in Indonesia (Karim, 2006; Marsigit, 2006).

Research in mathematics problem solving has been conducted in primary schools and secondary schools in some countries, for example in Australia and the United States (LeBlanc, 1982; Nisbet & Putt, 2000; Lesh & Zawojewski, 2007; Edens & Potter, 2007). Some studies (Nisbet & Putt, 2000; Wiest, 2001) of school mathematics at all levels have proposed to explain the appropriateness of problem solving, the interpretation of problem solving as part of a mathematics curriculum, and the way to use problem solving in mathematics. It is critical to support teachers to develop their content knowledge and pedagogical knowledge in their teaching of mathematics through problem solving. This study is about professional learning for teaching mathematics through problem solving in Indonesian primary schools through a Lesson Study approach. Therefore, this review focuses on teachers' professional knowledge, problem solving, professional learning, and Lesson Study.

Models of Teacher Professional Knowledge

Understanding teacher professional knowledge has been the focus of recent discussion by researchers and teacher educators. Shulman (1986) described teachers' knowledge in three categories. Ball, Thames, and Phelps (2008) introduced a framework of mathematical knowledge for teaching. Turner and Rowland (2009) identified aspects of teachers' actions in the classroom as the *Knowledge Quartet*.

Shulman's model of teachers' knowledge

Shulman, an educational psychologist, has made contributions to the study of teacher education. Through longitudinal studies, Shulman worked on teacher based knowledge including the constructs of pedagogical content knowledge, subject matter content knowledge, and curricular content knowledge (Shulman, 1986).

Subject matter content knowledge

Subject matter content knowledge (SMCK) is "knowledge of the content of the amount and organisation per se in the mind of the teacher" (Shulman, 1986). Knowledge of the subject requires an understanding of the structures of subject matter that includes the substantive and the syntactic knowledge. In mathematics, substantive knowledge refers to the variety of ways in which the basic concepts and principles of the discipline are organised to incorporate its facts. It includes the understanding of particular topics, procedures, and concepts and the relationships among these topics, procedures and concepts such as knowing the properties of a rectangle, knowing how to measure an angle and knowing how to multiply by a fraction. Teachers need to understand the procedure and concepts in ways to set the activities or tasks that help students do mathematics. Another aspect of subject matter knowledge that teachers need to understand is syntactic knowledge. Syntactic knowledge refers to the sets of ways in which truth or falsehood, validity or invalidity is established. It focuses more on the mathematical process than the product of such activity. It includes knowing how to prove a theorem, such as being able to demonstrate why an odd number multiplied by an odd number must result in an odd number (Shulman, 1986).

Pedagogical content knowledge

Pedagogical content knowledge (PCK) goes beyond subject matter knowledge for teaching. Shulman (1986, p. 6) identified pedagogical content knowledge as, "the most regularly taught topic in one's subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstration."

Pedagogical content knowledge includes the ways of representing and formulating the subject to make it comprehensible to learners. It involves "the knowledge and understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and then presented for instruction" (Shulman, 1987, p. 8). It also includes an understanding of what makes the learning of specific topics easy or difficult, for example, a number line model is often used by the teacher to demonstrate a way of representing calculations, including addition and subtraction. Pedagogical content knowledge includes the conceptions, preconceptions and misconceptions that students of different ages and backgrounds bring with them to the learning of particular concepts and skills (Shulman, 1986).

Curricular content knowledge

Curricular content knowledge (CCK) consists of the scope of material, subject matter and the sequence of teaching programs. Shulman (1986) divided curricular content knowledge into two aspects, lateral and vertical curricular knowledge. He stated that lateral curriculum knowledge includes the teacher's ability to relate the content of a given lesson being discussed with other classes (p. 10). Vertical curricular knowledge is "familiarity with the topics and organising them in the same subject area during the preceding and later years in school, and the materials that embody them" (Shulman, 1986, p. 10). Curricular knowledge refers to what students are expected to learn and knowledge of related resources such as textbooks.

Ball's model of teachers' knowledge

Ball is a researcher who focused on mathematical instruction and on the improvement of teacher professional learning. She is an expert on teacher education, with a particular interest in how professional learning and experience combine to equip teachers with the skills and knowledge needed for practice. She studied the mathematical knowledge needed for teaching and analysed teachers' mathematical knowledge.

Subject matter knowledge

In the area of mathematics, Ball (1990) defined subject matter knowledge for teaching as knowledge about mathematics that includes understanding the nature of mathematical knowledge and the mathematics field. Knowledge of mathematics includes what mathematics is, where it comes from, what it is good for, and how a correct answer is established. Knowledge of mathematics means an understanding of the substance the topics, concepts, procedures of the subject (Ball, 1988). Ball's model of the domain of mathematical knowledge for teaching is shown in Figure 2.1.

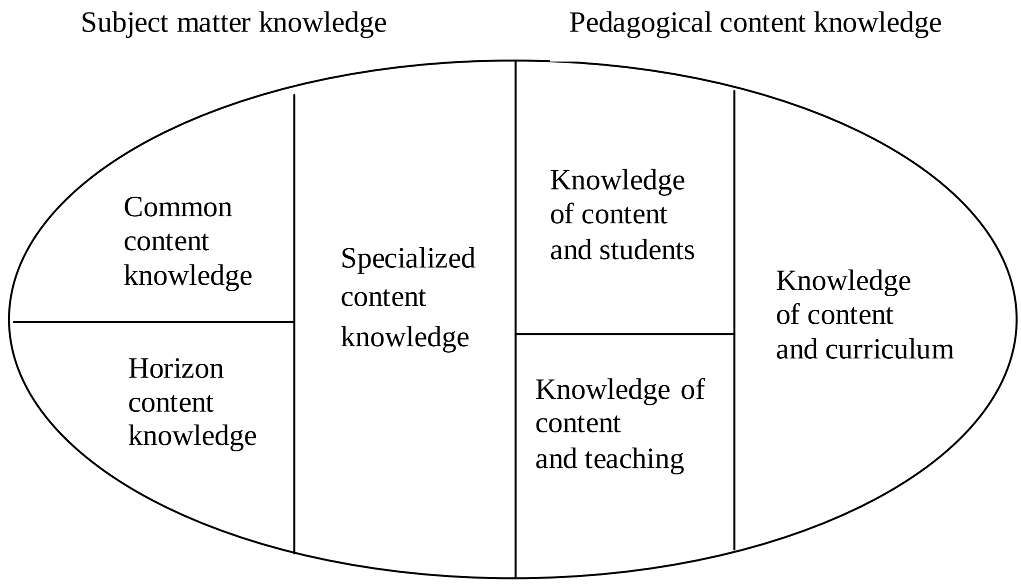


Figure 2.1: Ball's model of the domains of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008).

Subject matter knowledge is focused on substantive knowledge (e.g. facts, concepts and how they relate) and syntactic knowledge (e.g. how to prove theorem). It is described that 'substantive knowledge focuses on the organisation of key facts, theory and concepts and syntactic knowledge on the processes by which theories and models are generated and established' (Petrou, 2009, p. 2020). In Ball's model, subject matter knowledge is divided into common content knowledge, horizon content knowledge and specialised content knowledge.

Common content knowledge

Common content knowledge is the form of knowledge needed in common situations, which is different from special content knowledge used for teaching. For example, bank tellers may be highly skilled in arithmetic and be able to quickly find the difference between two numbers in their head, but if asked how the method works (e.g. Why do you borrow a digit from the next column?) may not to be able to give an explanation. Knowledge of an explanation is a form of specialised content knowledge.

Horizon content knowledge

Horizon content knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. For example, a teacher in Year Three may need to know how the mathematics he/she teaches is related to the mathematics students will learn as a student in Year Five, to be able to set mathematical foundations for what will come later.

Specialized content knowledge

This is different from common content knowledge in that knowledge is not mixed with knowledge of students. Specialized content knowledge that is used by teachers requires a specialised form of common subject matter knowledge. For example, if a student makes common errors with subtraction such as 307-168 = 169, a teacher must be able to identify this error.

Pedagogical content knowledge

Ball (1990) hypothesised that Shulman's pedagogical content knowledge was divided into three: knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum.

Knowledge of content and students

Knowledge of content and students is knowledge that combines knowing about mathematics and how students learn that mathematics. Knowledge about students as learners of mathematics should be understood by teachers. Teachers must anticipate what students are likely to think and what they will find confusing. Mathematical pedagogy includes involving students in mathematical activity and a view that all students are able to understand and do mathematics.

The way in which mathematics is represented should be appropriate for students' ages and students' understanding (Ball, 1988). It is helpful for teachers to consider what students of particular ages find interesting or difficult, for example how to measure the volume of a bottle of drink may be suitable for Year Five. In addition, the teacher's judgment of the level of what extension or emphasis is required depends on his/her knowledge of each of the student's knowledge and understanding. For example, teachers need to know that while most Year Five students struggle with the way to measure the volume of a bottle of drink, they probably understand how to work out the volume of any 3D object.

Knowledge of content and teaching

Ball (1988) noted four important ideas about mathematics pedagogy related to teaching and learning. First it is a core goal to help students develop their capacities to use, engage in, and appreciate mathematics with competence and confidence. Second, mathematical pedagogy is predicated on the concept of learning through making sense of mathematical problems. Third, the teacher's role as guide follows logically from this view of learning, as the teacher is a central source of mathematical experiences for students. Fourth, teaching entails representing the discipline in ways that engage and help students learn and enjoy mathematics.

Ball (1990) stated that the goal of mathematics teaching is for students to develop mathematical understanding. Mathematical understanding refers to learning about mathematical procedures and ways of thinking, as well as about mathematical facts, and skills, all in meaningful ways. It follows that a teacher should understand mathematics deeply to facilitate this kind of understanding. Ball, Thames, Bass, Sleep, Lewis, and Phelps (2009) argued that mathematical knowledge for teaching means the mathematical knowledge needed to carry out the work of teaching mathematics. Teaching means everything that teachers do to support the learning of the students. Teachers need to understand the ways in which mathematics is "useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students" (Ball et al., 2008, p. 404). Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways with story problems, pictures, situations, and concrete materials (Ball, 1990). Teachers must be able to generate explanations or other representations, often on the spot in response to students' questions. This includes the ability to talk about and model concepts and procedures.

Rowland's model of teachers' knowledge

In another model of teachers' knowledge, Turner and Rowland (2009) identified aspects of teachers' actions in the classroom that seemed to be significant and could be informed by their mathematics subject, mathematical content knowledge and pedagogical content knowledge. They used the concept of the *Knowledge Quartet* (Turner & Rowland, 2009) consisting of:

- Foundation;
- Transformation;
- Connection; and
- Contingency.

The distinctive features of each of the four dimensions of the Knowledge Quartet are as follows:

Foundation

The conceptualisation of '*foundation*' refers to teachers' beliefs, knowledge and understanding gained both from their personal education and from their learning in teacher education in preparation for their role in the classroom. Such knowledge and beliefs inform pedagogical choice and strategies in a fundamental way. Such beliefs typically include knowledge of the purposes of mathematics education, and the conditions under which students will best learn. The foundation dimension focuses on the knowledge that teachers bring to the teaching situation (Turner & Rowland, 2009).

Transformation

Transformation concerns knowledge in-action as demonstrated in both planning to teach and in the act of teaching itself. This includes the use of examples to assist concept formation, to demonstrate procedures, and the selection of exercise examples for student activity. In order to present the idea to the students, the teacher must find ways of representing what they themselves already know (Turner & Rowland, 2009).

Connection

Connection binds together certain choices and decisions that are made for the less prominent parts of the mathematical content. It focuses on the coherence of the planning sequence in a series of lessons. The coherence in mathematical content includes the sequencing of topics of instruction within and between lessons including ordering tasks and exercises (Turner & Rowland, 2009).

Contingency

Contingency can be witnessed in classroom events that have not been planned. This contingency concerns the teachers' response to these unplanned, unexpected classroom events. In some cases it is difficult to see how they could have been planned for, although that is a matter for debate. The key components of contingency are the readiness to respond to children's ideas and a consequent preparedness, when appropriate, to divert from a set agenda when the lesson was prepared. For example, in a scenario described by Rowland, Turner, Thwaites, and Huckstep (2009) the students were asked to give a fraction between ½ and ¾. One student answered 2/3 because the number 2 is between 1 and 3 and on the bottom the 3 lies between the 2 and the 4. Although this was a correct answer, the student's reasoning was incorrect. The way in which a teacher is able to respond to this situation would depend on the depth of knowledge of the teacher (Rowland et al., 2009).

Rowland et al., (2009) claimed that the *Knowledge Quartet* can be used as a framework or a tool for lesson observation and discussion for mathematics teaching development. Rowland et al., (2009) used 18 categories as a way for looking at and discussing teaching practices for primary mathematics. A summary of each of the dimensions of the Knowledge Quartet is shown in Table 2.1.

Table 2.1: The Categories of the Knowledge Quartet.

Foundation	Adheres to text book, awareness of purpose, concentration on procedures, identifying errors, overt subject knowledge, theoretical underpinning, and use of terminology.
Transformation	Choice of examples, choice of representation, demonstration.
Connection	Anticipation of complexity, decisions about sequencing, making connections between concepts, recognition of conceptual appropriateness.
Contingency	Deviation from agenda, responding to children's ideas, use of opportunities.

The four dimensions of the *Knowledge Quartet* can be generally related to the Shulman (1986) model of teachers' knowledge. The *foundation* dimension involves the subject matter knowledge classification and the knowledge that refers to teachers' beliefs about mathematics. The *transformation* dimension could relate to the Shulman model to describe the meaning of pedagogical content knowledge. The *connection* dimension is not directly linked to any single classification. The connectedness of teachers' own mathematical knowledge is an aspect of their subject matter knowledge, as is their understanding of the conceptual appropriateness of what is being taught. The *contingency* dimension involves all of Shulman's classifications: content knowledge, pedagogical knowledge and curricular knowledge.

Other models of teachers' knowledge

Other commentaries on models of teachers' knowledge are, for example Silverman and Thompson (2008), who argued that pedagogical knowledge is knowledge that lies at the confluence of content knowledge of students' thinking (the understanding they bring to a particular class or lesson and how it can be capitalised upon), and knowledge of mathematics education and pedagogy (e.g. curriculum, particularly difficult concepts, and effective images and instructional aids). However, An, Kulm, and Wu (2004) defined pedagogical knowledge as the knowledge of effective teaching and consisting of three components: knowledge of content, knowledge of curriculum and knowledge of teaching. Knowledge of content consisted of broad mathematics knowledge, as well as specific mathematics content knowledge at the grade level being taught. Knowledge of curriculum included selecting and using suitable curriculum materials and fully understanding the goals and key ideas of textbooks and curricula. Knowledge of teaching included knowing students' thinking, preparing instructions and mastery of modes of delivering instructions.

In other aspects An, Kulm, and Wu (2004) stated that teaching could be seen as either a divergent or convergent process. In this regard they noted:

> A divergent process of teaching is one that is based on content and curriculum knowledge, but is without focus and ignores students' mathematical thinking. A convergent process of teaching is one that focuses on knowing students' thinking, and consists of four aspects: building on students' mathematical ideas, addressing students' misconceptions, engaging students in mathematical learning, and promoting students' thinking mathematically. (p. 148)

In terms of students learning mathematics, An, Kulm, and Wu (2004) divided instructional beliefs into learning as knowing and learning as understanding. Learning as knowing assumes that mathematics is learned and understood if a concept or skill is taught. The teachers who are satisfied with students knowing or remembering facts and skills may not be aware of students' thinking. Learning as understanding recognises that learning as knowing is not sufficient and that understanding is achieved when the teacher does not only focus on conceptual understanding and procedure development, but makes sure that students comprehend and are able to apply the concepts and skills, and also constantly enquires about students' thinking.

Research on comparing the mathematical knowledge of prospective elementary and secondary teachers in the United States shows that “secondary teachers’ conceptual knowledge of elementary mathematics is not significantly stronger than that of their elementary counterparts” (Mewborn, 2001, p. 31). Mewborn also stated that many elementary teachers do in fact lack a conceptual understanding of the mathematics they are expected to teach. The teacher should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways. Then the teacher must appreciate and understand the connections among mathematical ideas.

Mathematics teachers not only learn mathematical knowledge but also learn about the practice of teaching: how to communicate to students and how students learn mathematics. Martin (2007) stated that knowledge of mathematics teaching and students learning are essential aspects of what a teacher needs to know to be successful. Teaching for understanding requires special mathematical knowledge for teaching.

For pedagogical content knowledge, Blanco (2004) noted two different aspects related to mathematics teaching and learning: a static component and a dynamic component. The static component included aspects of interest that are independent of the specific person who is teaching, along with the specific context in which the teaching activity is being performed. For example, specific knowledge about mathematics teaching and general psycho-pedagogical knowledge needs to be learned by pre-service teachers. Pedagogical knowledge is not only focused on what the teacher must know, but also how the teacher must know. It includes knowledge about how mathematics should be learned, knowing student learning outcomes, and knowing possible student difficulties. Therefore, it requires knowledge of how students think before and during learning the subject matter, and how students think about learning while they are learning.

The dynamic component of pedagogical knowledge involves personal knowledge, beliefs, and attitudes that require a personal involvement. It develops by means of a dialectic process between theory and actual experience. For example, the pre-service teacher may learn about problem solving strategies, but when she or he is in front of a Year Four primary class, she/he needs to readapt that scheme to the students' level and capabilities. This process of adaptation will be influenced by the concepts and beliefs held about mathematics, mathematics teaching and previous teaching experiences.

Research on pedagogical knowledge for problem solving has documented that teachers' knowledge of whether or not their own students could solve different problems was significantly correlated with student achievement (Carpenter, Fennema, Peterson, & Carrey, 1988). Problem solving involves the process of coordinating previous experience, knowledge, and intuition in an attempt to determine a method for resolving a situation, the outcome of which is not known.

Research on the knowledge of pre-service secondary school mathematics found that the participants were able to construct a deep understanding of problem solving (Chapman, 2005). Chapman suggested the need for teachers to reflect on learning experiences not only from the perspective as the learner, but also as a teacher to construct meaningful pedagogical knowledge. In the context of teachers' mathematical knowledge for teaching through problem solving, the ideas of teacher professional knowledge can be summarised in Table 2.2.

Table 2.2: Professional Knowledge of Teachers.

Subject Matter Content Knowledge (SMCK)	**Pedagogical Content Knowledge (PCK)**	**Curricular Content Knowledge (CCK)**
• Understanding SMCK • Mathematical content knowledge (MCK) • Common subject knowledge • Special content knowledge • Knowledge of mathematical problem solving - Understanding mathematical problem solving - Models of problem solving	• Understanding PCK • Mathematical knowledge for teaching • Knowledge of student learning • PCK for mathematical problem solving - Knowledge of student learning of problem solving - Knowledge of teaching mathematics through problem solving	• Understanding CCK • Knowledge level of student ability • Knowledge of student and their range of abilities • How students learn mathematics

The following Table 2.3 presents a summary of key aspects of beliefs and models about teacher knowledge.

Table 2.3: The Summary of Key Ideas about Teacher Knowledge.

Researcher	Subject Matter Content Knowledge	Pedagogical Content Knowledge	Curricular Content Knowledge
Shulman	It includes substantive (facts, concepts, principles and relationships between the concepts) and syntactic knowledge.	It includes the ways of representing and formulating the subject that make it comprehensible.	Sequential teaching program, scope of material, lateral curriculum, vertical curriculum.
Ball	It includes knowledge of mathematics and knowledge about the nature of mathematics.	It divides common content knowledge and specific content knowledge.	
Rowland	It is a foundation that consists of adhering to a textbook, awareness of purpose, concentration on procedures, identifying errors, overt subject knowledge, theoretical underpinning, and use of terminology.	It is a transformation that consists of choice of examples, choice of representation, and demonstration.	
Thompson		It includes content knowledge of students' thinking, mathematics education and pedagogy, assessing students' understanding, instructional strategies.	
An, Kulm, & Wu		It is the knowledge of effective teaching: knowledge of content, knowledge of curriculum and knowledge of teaching.	

Problem Solving

Introduction

Problem solving mostly is defined as figuring out what to do when you do not know what to do (Johnson, Herr, & Kysh, 2004). In school mathematics, problem solving could be a part of the curriculum content or a teaching approach. This section will discuss the nature of the problem, problem solving as a process, thinking mathematically, students constructing knowledge, problem solving as a mathematical process, and teaching approaches.

The nature of problem solving

Problem solving has been given a great deal of attention in the mathematics curriculum for schools in many countries. Much of this emphasis has developed from the pioneering work of George Polya, who characterised that solving a problem means finding a way out of a difficulty, a way around an obstacle and attaining an aim which was not immediately attainable (Polya, 1962). He believed that to have a problem meant "to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim" (Polya, 1962, p. 117).

The general definition of a problem is anything that is not working as well as it can work. It can be a difficulty of some sort, a state of relations that needs to be changed, and also it can be anything that people find is not in order and needs to be fixed (Fraenkel & Wallen, 2003). From a psychological perspective, Kilpatrick (1985) said that a problem was defined generally as a situation in which a goal is to be attained and a direct route to the goal was blocked. However, Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, and Human (1997) (cited in Van de Walle, Karp, and Bay-Williams, 2010, p. 33) defined "a problem in a school setting, as any task or activity for which the students have no prescribed or memorised rules or methods, nor is there a perception by students that there is a specific correct solution or method". Also, Johnson, Herr, and Kysh (2004) defined a problem as a task that may not have a clear path to the solution. From the perspective of a mathematics textbook, a problem is a situation that is in need of an answer, even if there is no available potential problem solver. For example: *How would you determine the number of hairs on your head*?

Siemon and Booker (1990) said that having a problem is a very personal phenomenon. It depends on who you are, what you know and how you feel about the task in question. For example: *Find the sum of 1 + 2 + 3 + … 100.* This is unlikely to be a problem for anyone who has noticed the pattern, but it may well be a problem for a large number of students in a lower grade. Problem solving occurs when an individual or group engages in a process, and directs and monitors what is known and how it is applied, in order to achieve a solution to a problem (Siemon & Booker, 1990).

A problem is different from an exercise. Johnson, Herr, and Kysh (2004) noted that an exercise asks the student to repeat a method that has been learned from a similar example, whereas a problem is usually more complex than this. For example, if students were asked to: C*alculate the area of a rectangle with a 6 centimetres length and 3centimetres width,* and the teacher had previously shown the students the method used to calculate the area, then this would be an exercise. However, if the students were asked to: *Find all possible rectangles that have an area of 18 cm*^{2} and the teacher had not demonstrated any procedures to follow, then this would be a problem to solve, as the students would need to find their own approach to discover the answers.

In a school mathematics context there are many kinds of problems. The research of Yan and Lianghuo (2006) on the comparison of mathematical textbooks, classified problems into the following seven types:

- **Routine problems versus non-routine problems.** The following two examples demonstrate routine problems and non-routine problems: (1) *Find the sum of 123 and 345.* This problem can be solved using the operation of addition, and is therefore a routine problem. (2) *Ali stands in line to buy a ticket. There are 4 people in front of Ali and 3 people behind Ali. How many* *people are in the line?* To solve this problem, the students will need to find a new strategy, which could be drawing or acting, in order to find the answer, and it is therefore a non-routine problem.

- **Closed problems versus open-ended problems.** The following two examples demonstrate closed problems and open-ended problems: (1) *Find the sum of 16 + 4,* is a closed problem because it has a single solution, whereas (2) *Find the positive integers with the sum of 20,* is an open-ended problem as it has more than one solution.

- **Traditional problems versus non-traditional problems.** The following two examples demonstrate traditional problems and non-traditional problems: *11 + 12 + 13 = …* is a traditional problem in that it can be found in a mathematics textbook, whereas: *Determine A from the sentence 1+2+3+... = AA*, requires not only skill, but also a strategy to solve it and is therefore a non-traditional problem.

- **Application versus non-application problems.** The following two examples demonstrate application and non-application problems: *Find the area of a rectangle with a length of 3 cm and a width of 2 cm,* can be solved using the formula to calculate the area of a rectangle, as an application to determine the area, whereas: *Write all the numbers between 1 and 50 that are multiples of 6* requires children to understand the concept of multiples of numbers and is therefore a non-application problem.

- **Single-step problems versus multiple-step problems.** The following two examples demonstrate single-step versus multiple-step problems: *Find the positive integers the sum of which is 20* - a single-step problem. Whereas: *Find the positive integers the sum of which is 20 and the product of which is the maximum possible* needs more steps to solve it and is therefore a multi-step problem.

- **Sufficient data problems, extraneous data problems, and insufficient data problems.** Here is an example of an extraneous data problem: *A boy* *scout in a jungle is heading south. He takes a right turn and walks for 40m. Then, he takes a left turn and walks again for a further 50m. He then takes a left turn and walks for another 45m. Finally, he takes a right turn. In which direction is he heading now? (Yan & Lianghuo, 2006, p. 615).* In this problem the distance that the boy walked is not needed to solve the problem. This problem compared to an example of an insufficient data problem: *How much will it cost to buy a 10 kg bag of rice today if it cost 500 rupiahs less last month?*

- **Problems in a purely mathematical form, problems in a verbal form, problems in a visual form, and problems in a combined form.** This classification depends on the form of representation of a problem.

  If the stem of a problem includes mathematical expressions, then the problem is classified into the category of problems (presented) in purely mathematical forms. If the stem is entirely verbal, namely, in written word only, then the problem is coded into the category of problems in a verbal form. If the stem simply consists of figures, pictures, graphs, charts, tables, diagrams, maps, etc., then such a problem is classified into problems in a visual form. The rest are problems in a combined form, presented in a combination of two or three of the above forms (Yan & Lianghuo, 2006, p. 615).

In word problems, Lesh and Akerstrom (1982) gave examples to illustrate some of the differences between typical word problems and real-world problems. They described that each problem was given in three distinct forms: as a “word” problem, as a “concrete” problem, and as a “real” problem. Also, Wiest (2001) divided word problem contexts into four categories: low fantasy contexts, high fantasy contexts, children’s real world contexts, and adult’s real world contexts. The following are examples of each category:

Low Fantasy Contexts

One day in December 1313 people shopping in a toy store were stunned when 157 toys on the shelves came to life. 49 of the toys danced around the store, 46 of them chatted with each other, and the others sang the song “Toyland”. How many toys in the store sang when they came to life? (Wiest, 2001, p. 78).

High Fantasy Contexts

The Secret Forest has 159 redwood trees and is the home of 134 animals that like to keep to themselves. Of the animals that live there, 37 are unicorns, 54 are fire-breathing dragons, and the others are horses with wings. How many animals living in The Secret Forest are horses with wings? (Wiest, 2001, p. 78).

Children's Real World Contexts

139 children rushed to their favourite ride at the yearly carnival held by the 137 business owners in their town. 34 children hurried to the Ferris well, 38 went to the merry-go-round, and the rest chose the roller coaster. How many children chose the roller coaster as their favourite ride? (Wiest, 2001, p. 78).

Adult's Real World Contexts

159 people visited the Westfield Art Show, held one weekend in August, to see the 136 works of art for sale. 58 works of art were sold on Saturday, 33 on Sunday, and the others were not sold, so they were stored for another art show. How many works of art were not sold at the Westfield Art Show? (Wiest, 2001, p. 78).

Generally, problems in a text book for school mathematics are mainly closed problems with only a few of the problems being open problems. However, more specifically in a school context, Jurdak (2006, p. 288) stated that "problem solving in the school context refers to a situation where the student is engaged in solving school-like problem tasks as part of an instructional sequence and as applications of taught mathematical concepts, principles, and algorithms". For example:

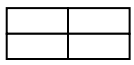
Rasamny Youniss Company is making a special offer on Nissan-Almera cars, model 1999, and automatics/full option for $13950. Now, you have two options for payment in instalments, either through the bank or through the company itself. Through the bank and with a down payment of $5,000 you can pay with a 12% annual interest on balance, $305 at the end of each month. However, the second option, and with a down payment of $5,000 you can repay, in equal monthly instalments for 36 months at an annual interest rate of 7.5% on total. (1) Suppose you wanted to pay the whole remaining amount after 6 months. In each option, how much do you have to pay to close your account?(2) Which is the most convenient option for paying for the car? (Jurdak, 2006, p. 299).

Stanic and Kilpatrick (1989) in Schoenfeld (1992) identified problem solving as three themes: problem solving as context, problem solving as a skill, and problem solving as an art. First, in regard to problem solving as context, problems are employed as vehicles in the service of other curricular goals that are identified into as one of five themes:

- As a justification for teaching mathematics. Problem solving becomes a part of the mathematics curriculum as justification for teaching all mathematics topics.

- Providing specific motivation for subject topics. A problem sometimes is used to introduce the topic as an indicator of understanding that when the students have completed the lesson that follows, they will be able to solve a problem of this sort. For example, a teacher may use the problem: *Find a number greater than* $\frac{1}{5}$, *but less than* $\frac{1}{4}$ to introduce the concept of numerator and denominator for fractions.

- As recreation. Recreational mathematics can demonstrate that mathematics can be fun and motivate students. For example: *How many rectangles do you find on the picture?*



- As a means of developing new skills. Problems can be used to introduce the new skills, such as skill of order of operation. For example: *Calculate* $1+2-3\times 4\div 5+6-7\times 8\div 9$.

- As practice. The problem mostly is used as a task, or exercise in a mathematics lesson until the students have knowledge of a specific technique or skill. For example: *Find the positive integers whose sum is 20 and whose product is the maximum possible?*

Second, problem solving is considered a skill, where a skill is "being able to obtain solutions to the problems given by other people, but worthy of instruction in its own right” (Schoenfeld, 1992, p. 14). The students need to learn the basic mathematical concepts and skills to solve a problem. For example: *Andy had saved Rp.2000. The next day he received his allowance. Now he has Rp. 12000. How much allowance did he get?* To solve the problem the students need the basic skill of addition and subtraction.

Third, problem solving is seen as an art, which takes the view that real problem solving involves problems of a “perplexing” kind, which is at the heart of mathematics. This view comes from mathematicians and philosophers. For example, a famous mathematical problem was named, “the four colour problem” which, when solved, became the four colour theorem (Schoenfeld, 1992, p. 16).

The Australian Curriculum Assessment and Reporting Authorities (ACARA, 2010) described the Australian Curriculum: mathematics as consisting of three content strands and four proficiency strands: understanding, fluency, problem solving, and reasoning. The problem solving strand “includes the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively” (p.3). Similarly, The National Council Teachers Mathematics Standards, (NCTM, 2000) described several standards, one of which was problem solving. Students should be able to build new mathematical knowledge through problem solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems and monitor and reflect on the process of mathematical problem solving. Lesh and Zawojewski (2007, p. 782) defined problem solving as:

> the process of interpreting a situation mathematically, which usually involves several interactive cycles of expressing, testing and revising, mathematical interpretations and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics.

In this, problem solving is seen as being related to an art in which there is interpretation of a situation. They also state that mathematical problem solving is about seeing (interpreting, describing, explaining) situations mathematically, and not simply about executing rules, procedures, or skills expertly.

The following Table 2.4 contains different views of what is meant by a problem.

Table 2.4: The Summary of Key Ideas about Problems.

Heibert	A problem is any task or activity for which the students have no prescribed or memorised rules or methods, nor is there a perception by students that there is a specific correct solution method.
Fraenkel & Wallen	A problem is anything that is not working as well as it might, could be a difficulty of some sort, or anything that people find is not in order and needs to be solved.
Siemon & Booker	A problem is a very personal phenomenon; it depends on who you are, what you know and how you feel about the task in question.
Johnson, Herr, & Kysh	A problem is a task that may not have a clear path to the solution.

With reference to the definitions above, the problems in this study can be defined as tasks that need to be solved, but where students do not know the way to solve them immediately and have no clear specific solution methods. To solve the problem depends on the teacher knowing what the students need to do when the students themselves do not know what to do.

Problem solving as a process

One way to approach the process of teaching mathematics is to view it as analogous to the process of doing mathematics. In teaching mathematics, a teacher must first understand the mathematics to be taught and the students who are learning the mathematics. The next step is to develop a plan that reflects that understanding. The teacher provides classroom instruction to implement the plan, and then the teacher evaluates the students' success, reflects and possibly revises the approach. This is somewhat analogous to the Lesson Study approach that is described later.

To be successful problem solvers, students need to understand for themselves the problem or question, understand what they should answer, and understand steps to solve the problem. Hence, students need to learn some aspects of the problem solving processes such as: understanding the problem, using mathematical skills and strategies, and reasoning related to the solution of the problem. The students trial their idea for a possible solution strategy and in this process they make a decision to try a possible solution and to check each step. Polya (1957) offered a four-step process for problem solving: understand the problem, devise a plan, carry out the plan, and look back. The first step, *understanding the problem,* is to determine what the problem is about and identify what the problem requires the solver to find out. The second step, *devise a plan,* is to think about how to solve the problem and what strategies will be used. The third step is the *implementation of the plan*, and the fourth step, *looking back*, is the moment when students determine if the answer from step three answers the problem as originally understood in step one. In other words, they check to see whether or not the answer makes sense.

Polya (1957) also suggested that strategies for problem solving include using diagrams, looking for patterns, listing all possibilities, trying special values or cases, working backwards, guessing and checking, creating an equivalent problem, and creating a simpler problem. However, Charles and Lester (1982) said that there are three sets of interacting factors that influence the problem solving process:

- Experience factors – environmental and personal;
- Affective factors – interest, motivation, and pressure; and
- Cognitive factors – reading ability, reasoning ability, and computational skill.

Problem solving emphasises the process rather than getting the answer. Hence the students need to have an understanding of the problem and mathematical skills and strategies to solve the problem.

Thinking mathematically

Many contemporary mathematics curricula contain a strand based on the process of mathematical thinking. For example, ACARA (2009, p. 7) said that "the content strands describe the 'what' that is to be taught and learnt, while the proficiency strand describe the 'how' or the way content is explored or developed i.e. the thinking and doing of mathematics." How students interact with the content was described with proficiency strands that included understanding, fluency, problem solving or reasoning. Mason and Burton (2003) mentioned that thinking mathematically was about mathematical processes and not about any particular branch of mathematics. To think mathematically students begin with intuitive thinking and end with conceptual ideas. So thinking mathematically can be improved by practice through developing problem solving skills, problem based learning, and a critical thinking approach. Mathematical thinking can be supported by an atmosphere of questioning, challenging and reflecting. A mathematical problem requires children to use their imagination to solve it and requires teachers to use their imagination to teach and to have a creative mind to challenge children in depth (Chiu, 2007). However, Booker, Bond, Sparrow, and Swan (2010) suggested that the process highlights the cyclical nature of problem solving and brings to the fore the importance of understanding a problem and its structure before proceeding. The process can be summarised in Figure 2.2.

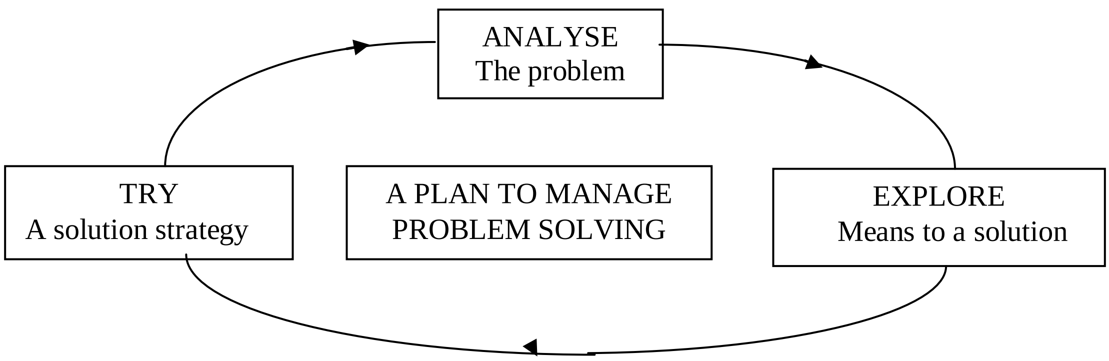


Figure 2.2: The cyclical nature of problem solving process (Booker, Bond, Sparrow, & Swan (2010, p. 56).

In solving any problems, the cyclical nature of the problem solving process helps to establish a working procedure. This begins with an analysis of the problem to see what the problem is really asking, followed by exploring strategies for possible solutions to try. Finally, possible solutions obtained are analysed to see if the solution is reasonable. These steps are explained in more detail.

Analyse

The students are encouraged to think and discuss what the problem is asking. During this step students may need to:

- Read the problem aloud;
- Think of previous problems and other similar problems;
- Select important information that may be useful from the problem;
- Discuss what the problem is asking.

Explore

The students see ways in which the problem can be solved. Using a range of strategies is an important aspect of the problem solving process. Students need to know a range of strategies such as:

- Drawing a diagram or graph;
- Using materials;
- Making a table or list;
- Working backwards;
- Working forwards;
- Looking for a pattern;
- Thinking of a similar problem;
- Try and adjust;

- Acting out the problem;
- Using smaller numbers.

Try

The students trial their idea of a possible solution strategy. In this process they make a decision to try a possible solution and to check each step.

Analyse

After getting the answer the students need to compare their answer with the original analysis of the problem to determine whether the solution obtained is a reasonable answer. This process is cyclic and should the answer be unreasonable, then the process would need to begin again.

The cyclical nature of the problem solving process is similar to the Polya problem solving process in general. For example at the *analyse* stage, students need to read the problem aloud, which is part of Polya's *understanding* stage.

Students constructing knowledge

Rowan and Bourne (1994) said that a child's ability to learn abstract processes and concepts integral to mathematics benefits from the opportunity to interact directly with the environment. As children manipulate objects, observe changes, develop trial and error methods of interaction, and reflect upon their experiences, they gradually construct their own understanding of the relationship between object and concepts. By placing children in controlled problem solving situations and providing a supportive environment to communicate ideas, teachers can build effective learning opportunities.

Students can construct conceptual knowledge and procedural knowledge through solving realistic problems in context. This is supported by social constructivist theory that suggests knowledge is constructed by the students and not by the teacher. Realistic problems and contextual problems support students in the construction of their understanding. Gravemeijer (1994) identified three principles of realistic mathematics: guided reinvention and progressive mathematisation, didactical phenomenology, and self-developed models. These all play a role in bridging the gap between informal knowledge and formal mathematics. In realistic mathematics the model is presented and developed by the students with the familiar problem. Through the process of formalising and generalising the model is developed until the students acknowledge formal understanding of the mathematics. The three principles can be looked at as a teaching design with heuristics. The aim of heuristics is to study the methods and rules of discovery and invention (Polya, 1957, p. 112). A heuristic approach to problem solving can help students improve mathematical thinking, reasoning, communication, and creative thinking (Susanta, 2006). A guided reinvention principle means the students are given opportunities to investigate mathematical concepts within various contextual problems. Contextual problems guide the students to construct mathematical concepts, model, apply concepts, and solve problems. In this way problem solving supports students to learn mathematics in a meaningful way and provides an opportunity for them to value the power and limitations of applying mathematics in the real world.

Table 2.5: The Summary of Key Ideas about Problem Solving as a Process.

Polya	Four steps problem solving: understand the problem, devise a plan, carry out the plan, and look back.
Booker, Bond, Sparrow, & Swan	The cyclic nature of problem solving: analyse the problem, explore means to solution, and try a solution strategy.
Gravemeijer	Three principles of realistic mathematics: guided reinvention and progressive mathematization, didactical phenomenology, and self-developed models.

Problem solving as a mathematical thinking process

Research suggests that the development of mathematical problem solving ability is influenced by five elements that are inter-related: concepts, skills, processes, attitudes and metacognition (Swee, 2002).

- Concepts – For example, mathematical concepts involving numerical and geometrical ideas, should be developed as integrated wholes, not as isolated pieces of knowledge;
- Skills – Mathematical skills such as procedural skills for numerical calculation and the use of mathematical tools are important in the learning and application of mathematics;
- Processes – Mathematical processes refer to the processes of applying mathematical knowledge that include thinking skills, communication and reasoning;
- Attitudes – Students' attitudes include beliefs and their interest in learning mathematics, and these are constructed from their learning experiences;
- Metacognition – Metacognition is thinking about thinking, and refers to the ability to control one's thinking process and use problem solving strategies. Similarly, students need to learn mathematical concepts and to see relationships between these concepts through problem solving. To learn mathematics, students must construct these concepts and relationships in their own minds. For example, the students need to describe their thinking in verbal ways and also by drawing in solving problems (Edens & Potter, 2007).

In addition, the process of mathematical abstraction needs time to develop. The students need to investigate, create, and solve problems. For example, the students may use systematic guess-and-check strategies in solving the problem or they might try to identify the structure of the conditions by taking into account the unknown quantities and the totality.

Problem solving as a teaching approach

In traditional teaching of mathematics teachers spend a small portion of the lesson explaining or reviewing an idea and then go into practice mode where students work through a set of exercises. Then students copy the problem written on the white board into their notebooks. The students would solve the problem following the teachers' example. In contrast, effective teaching has to relate to the nature of the learning process. Polya (1965) noted three principles of learning: active learning, best motivation, and consecutive phases. Active learning means changing from teacher-centred to student-centred learning. It claims that the teacher is not the source of information, but rather that the teacher supports students in their learning. The students are viewed as people who have the ability to construct knowledge for themselves. Best motivation refers to students' interest in the task. The consecutive phase means that the learning begins with the intuitive, proceeds to conceptions, and ends with ideas.

There are two important elements of teaching and learning mathematics: helping students to develop relational understanding, and helping students to construct mathematical concepts. Relational understanding includes a cognitive map of relevant topics and the interrelated network of concepts to build mathematics. Suggate, Davis, and Goulding (2010) said that "relational understanding helps to enable the effective teacher to frame explanations in a variety of language, to suggest a range of representations and models, and to inform appropriate questions in the classroom (p.12)". Teaching and learning with problem solving provides the students with an opportunity to use and apply their mathematical skills and knowledge. The focus is on teaching mathematical topics through problem solving contexts and inquiry oriented environments which are characterised by the teacher 'helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying' (Lester, Masingila, Mau, Lambdin, dos Santos, & Raymond, 1994, p.154). For example, Jones (2003) argued that a mathematics curriculum without problem solving can be likened to a diet of physical education in which children practice football or netball skills but never get to play the game.

Van de Walle et al. (2010) noted there are three ways in which problem solving might be incorporated into teaching mathematics: teaching for problem solving, teaching about problem solving, and teaching through problem solving.

Teaching for problem solving

Teaching for problem solving is teaching a specific skill first so that students can later solve the problems with the skills that have been taught formally. Most mathematics textbooks follow this format where the skill is taught first, and then this skill is used to solve the problem. In teaching for problem solving students begin with an abstract concept, then move on to solve the problem in a way that applies their learnt skills. For example, students learn the algorithm for the division of decimal numbers, and once that is mastered, solve story problems that involve dividing decimal numbers (Van de Walle et al., 2010).

Teaching about problem solving

Teaching about problem solving is teaching students how to solve a problem. It includes teaching a process of how to solve the problem using the steps outlined by Polya: understanding the problem, devising a plan, carrying out the plan and looking back. Also it involves general strategies for solving problems, such as look for a pattern, draw a picture, make a table, make an organised list, and write an equation (Van de Walle et al., 2010).

Teaching through problem solving

Teaching through problem solving means teaching where the students learn mathematics through real contexts, problems, situations, and models. It can be described as opposite to teaching for problem solving. In the beginning of a lesson, the problems are presented and skills develop from working with the problem. For example, when exploring the problem of drawing all rectangles with an area of 8 cm^2, in order to figure out how many rectangles there are of different sizes, the students would be led to use the procedure for calculating the area of a rectangle (Van de Walle et al., 2010). The problem tends to be more of an open problem and students learn by exploring the problem situation.

In teaching mathematics through problem solving the students play an active role in their learning to explore the problem situation with teacher guidance to develop their solution. Many researchers (Carpenter, Franke, Jacobs, Fennema, &

Empson, 1998) have investigated students' mathematical thinking and indicated that they can explore problem situations and develop ways to solve the problems. For example, to find traditionally the answer to the subtraction 65 - 19, students were expected to borrow one ten and add it to the 5, then by subtracting 9 from 15, write down 6 for the unit place. It might be a problem if they were asked to describe as many ways as possible for finding the answer. Carpenter et al. (1998) found that the students subtracted 65 - 19 by subtracting 20 from 65 and then adding 1 back. In this research, Carpenter et al. (1998) found that students who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than the students who initially learnt standard algorithms.

Hiebert (2003) suggested three signposts to guide the teacher in teaching mathematics through problem solving:

- Allow mathematics to be problematic for students;
- Focus on the methods used to solve problems; and
- Tell the right thing at the right time.

In addition, Van de Walle et al. (2010) noted that there are good reasons to go to the effort involved in teaching through problem solving. They state that teaching through problem solving:

- Focuses students' attention on ideas and sense making;
- Develops students' confidence that they are capable of doing mathematics and that mathematics makes sense;
- Provides a context to help students build meaning for the concept;
- Allows an entry point for a wide range of students;
- Provides ongoing assessment data useful for making instructional decisions, helping students succeed and informing parents;

- Allows for extensions and elaborations;
- Engages students so that there are fewer discipline problems;
- Develops mathematical power; and
- Is a lot of fun.

Similarly, Siemon and Booker (1990) incorporated three ways of problem solving in teaching and learning mathematics schematically as shown in Figure 2.3.

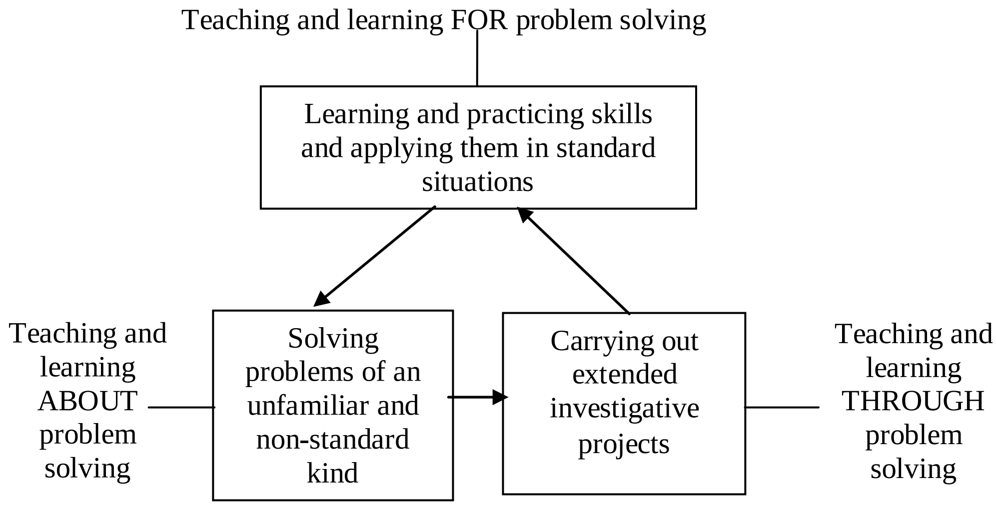


Figure 2.3: Incorporating the three ways of problem solving in teaching mathematics (Siemon, & Booker, 1990).

Teaching and learning FOR problem solving is needed to ensure the availability of appropriate knowledge, skills, and strategies that are built on understanding and exercised with confidence. Teaching and learning ABOUT problem solving is needed to provide the means to access, monitor and direct what is known and what can be done. Teaching and learning THROUGH problem solving is needed to provide a context for further learning and to exercise the application of the knowledge, skills and processes acquired as a result of the first two approaches.

Table 2.6: The Summary of Key Ideas about Problem Solving as a Teaching Method.

Polya	The teacher is not the source of information, but the teacher supports students in their learning.
Van de Walle et al.	Teaching for problem solving refers to teaching a skill so that students can later solve problems with skills taught first in formal ways. Teaching about problem solving involves teaching students how to solve problems and the processes or strategies for solving problems. Teaching through problem solving refers to students learning mathematics through real contexts, problems, situations, and models.
Siemon & Booker	Teaching for problem solving refers to learning and practicing skills and applying them in standard situations. Teaching about problem solving refers to solving problems of an unfamiliar and non-standard kind. Teaching through problem solving refers to carrying out extended investigative projects.

Professional Learning

Professional learning theory

In recent years, the professional learning of teachers has been considered a long-term process that includes regular opportunities and experiences planned systematically to promote growth and development in the profession (Reimers, 2003). Improving the quality of teachers and teaching methods through ongoing professional training has been a priority for many countries. Professional learning includes the processes referred to in the literature as professional learning, professional development, staff development, teacher development or in-service education (Muir & Beswick, 2007).

Reimers (2003) noted the characteristics of professional learning as follows:

- Professional learning is based on constructivism rather than on a ‘transmission-oriented model’;

- Professional learning is perceived as a long-term process as it acknowledges the fact that teachers learn over time;
- Professional learning is perceived as a process that takes place within a particular context;
- Many identify this process as one that is intimately linked to school reform, as professional learning is a process of culture building and not of mere skill training which is affected by the coherence of the school program;
- A teacher is conceived as being a reflective practitioner, that is someone who enters the profession with a certain knowledge base, and who will acquire knowledge and experiences based on that prior knowledge;
- Professional learning is conceived as a collaborative process;
- Professional learning may look and be very different in diverse settings, and even within a single setting, it can have a variety of dimensions.

Professional learning for teachers is a systematic attempt to bring about change in the classroom practices of teachers, change in their beliefs and attitudes, and change in the learning outcomes of students (Guskey, 1986). An alternative perspective on the "teacher change" process is illustrated in Figure 2.4. According to the Guskey model of professional learning teacher beliefs and attitudes change after changes in student learning outcomes are evident. The change in students' learning outcomes happens after changes in teachers' classroom practice.

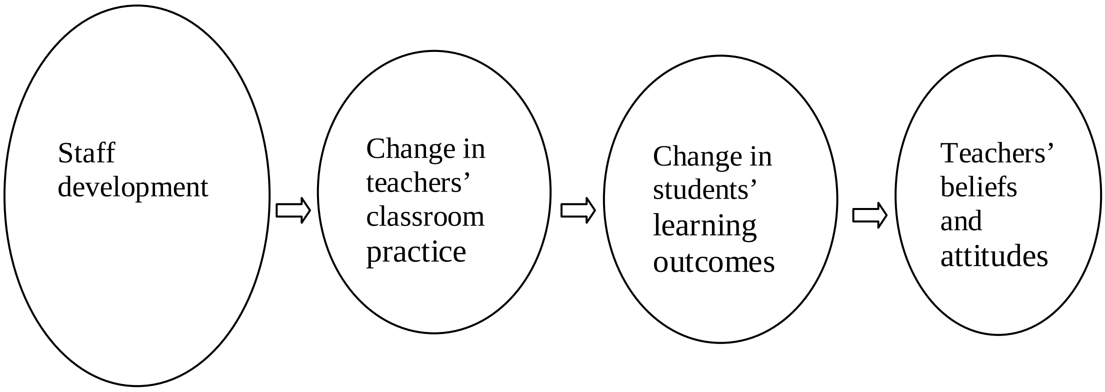


Figure 2.4: A model of the process of teacher change (Guskey, 1986).

According to various researchers (Gordon & Tyson, 1995; Shulman, 1998; Sparrow, 2000) there are three models of professional learning: transmission, partnership, and empowerment models. First, in the transmission model of professional learning teachers learn by demonstration. This approach tends to maintain the status quo rather than developing and supporting change in practice. For long-term change in teacher practice the transmission approach appears to be ineffective in most cases. Second, the partnership model of professional learning involves the partnership between researchers and teachers through collaboration such as action research. This approach may lead to teachers becoming disempowered because changes are seen as externally legitimised. In the empowerment model of professional learning, the teachers need to identify and meet their own needs. This model is determined by teachers' concerns, interests, and the realities of their classroom (Sparrow, 2000).

Teachers' reflection is an important characteristic of all empowerment models such as action research and case method meetings. Reflection is viewed as a process of becoming aware of one's context, of the constraints imposed by society and of the influence of ideology within practices previously taken for granted. Reflection is a way to gain control and direction over these influences.

In addition, Muir and Beswick (2007) noted the principles of effective professional learning as follows:

- Professional learning is more likely to be effective if it address teachers' pre-existing knowledge and beliefs about teaching, learning, learners and subject matter;

- Professional learning is more likely to be effective if it provides teachers with sustained opportunities to deepen and expand their content and pedagogical knowledge;

- Effective professional learning is grounded in teachers' learning and reflection on classroom practice.

Professional learning that focuses on developing the main attributes of an effective teacher can enhance teachers' understanding of the content, and teachers' teaching strategies that will enable their students to learn that content. Effective professional learning is directed towards providing teachers with the skills to teach and assess for deep understanding and to develop students' metacognitive skills.

There are many lists of characteristics of effective professional learning activities. The Victorian Department of Education and Training (2005) described the seven principles of effective professional learning as the following:

- Professional learning is focused on student outcomes (not just individual teacher's needs). It includes a support source from the school to provide a time frame to sustain teacher learning, and to encourage teachers to engage in professional learning;

- Professional learning is focused on and embedded in teacher practice (not disconnected from the school). It is built into the day-to-day work of teaching that involves teachers in analysing, identifying, and designing professional learning experiences;

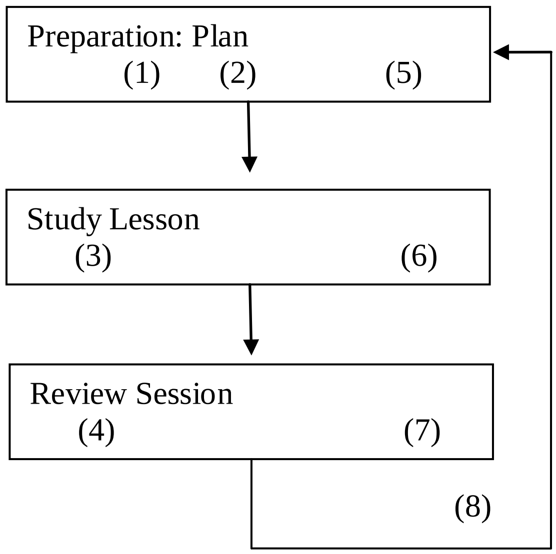
- Professional learning is informed by the best available research on effective learning and teaching (not just limited to what the teachers currently know). It develops an understanding of knowledge, uses multiple sources, and builds on the knowledge;

- Professional learning is collaborative, involving reflection and feedback (not just individual inquiry). It builds a school culture, and professional learning teams;
- Professional leaning is evidence based and data driven (not anecdotal) to guide improvement and measure impact;
- Professional learning is ongoing, supported and fully integrated into the culture and operations of the system - schools, networks, regions and the centre (not episodic and fragmented);
- Professional learning is an individual and collective responsibility at all levels of the system (not just the school level) and it is not optional.

Professional learning should be effective if it provides teachers with opportunities to deepen and expand their content and pedagogical knowledge and the schools provide a time frame to sustain teacher learning. The process of Lesson Study meets most of these seven criteria listed above. It is now described in detail.

Lesson Study

Lesson Study originated in Japan as a teacher professional learning model. Originally Japanese teachers used Lesson Study only to improve their instruction and develop their students' learning. Baba (2007) said that Lesson Study first developed in the Meiji period of Japan as an educational practice that enabled teachers to develop their teaching practice. In addition, Baba (2007) noted that Lesson Study involved preparation, actual class teaching, and a class review session. First teachers find and select teaching materials that are relevant to the purpose of the class. Second, teachers then refine the design based on the actual needs of the children, and design a new lesson plan. Third, the class is taught, based on the revised lesson plan, and observed by many teachers, who are sometimes joined by university instructors and supervisors from the Board of Education. Finally, a review session is held for all observers after the class. This process is shown in Figure 2.5. Steps (1) to (4) comprise the first stage, and the results of the evaluation in step (4) are utilised in the second stage, step (5) to (7), to refine the plan.



(1) Problem identification, (2) Class planning, (3) Class implementation, (4) Class evaluation, (5) Reconsideration of class, (6) Implementations of class plan based on reconsiderations, (7) Evaluation and review, (8) Sharing of result

Figure 2.5: Flowchart of pedagogical training (Stigler & Hiebert, 1999, in Baba, 2007)

Similarly, Leung (2006) noted that Lesson Study is considered a kind of action research, consisting of a spiral of steps involving planning, fact-finding and evaluation.

The successful Japanese Lesson Study has become a popular model for professional learning. Triwaranyu (2007) noted that Lesson Study as a professional learning method helped teachers develop ways of thinking about teaching and learning in the classroom; planning lessons; observing how students are thinking, learning, and taking appropriate actions; reflecting and discussing about teaching; identifying and recognising knowledge and skills necessary to improve their practice and see new solutions. In addition, Murata, Lewis, and Perry (2004) and Murata

(2011) noted that support for teacher learning through Lesson Study was developed and interacted in three areas: teachers' knowledge, teachers' commitment and community, and learning resources, as shown in Figure 2.6. Teachers' knowledge improvement and commitment to professional community growth interacts with the development of learning resources. Through Lesson Study, the learning resources are refined and improved in a meaningful context by the teachers' discussion that focuses on the lesson.

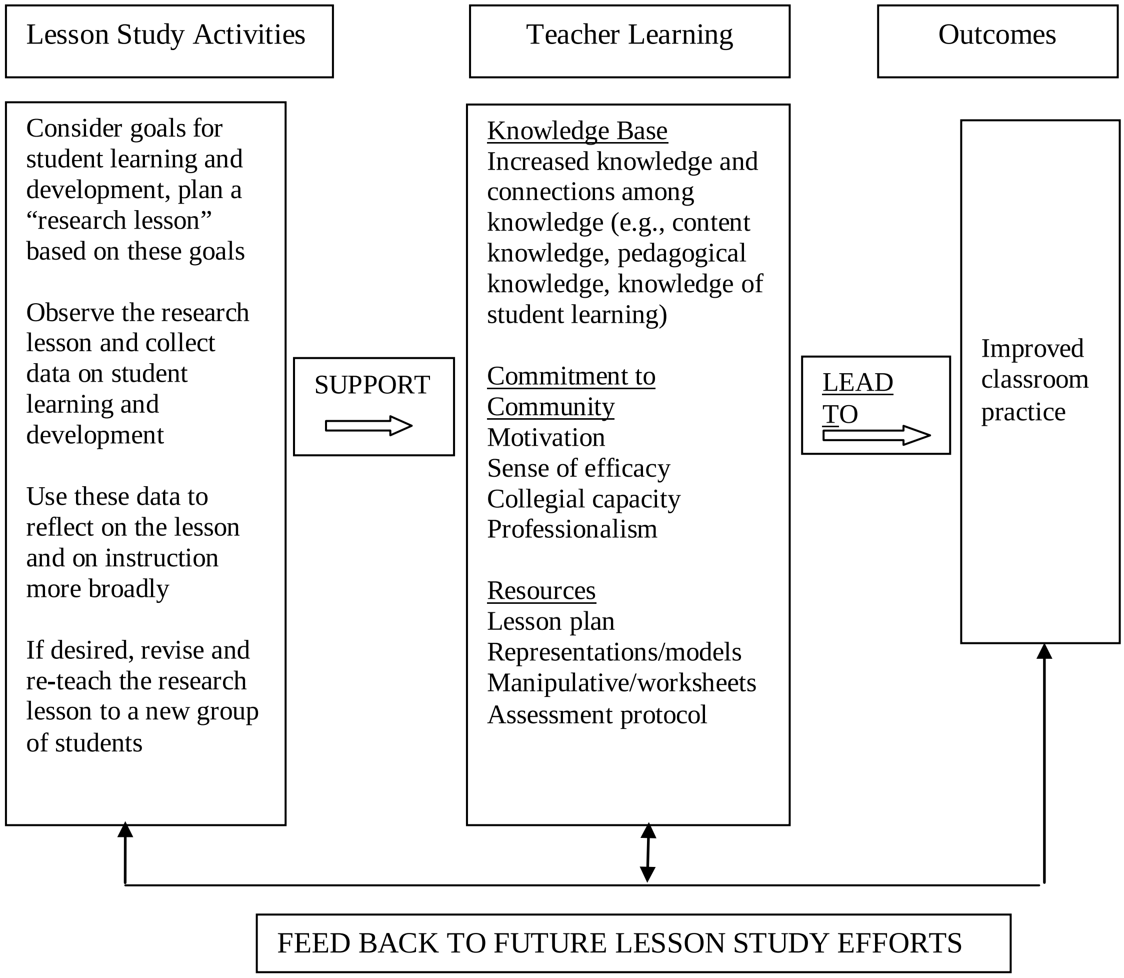


Figure 2.6: Teacher learning and outcomes (modified by Murata et al., 2011)

Lewis (2000) cited in Sowder (2007) listed eight ways that Lesson Study contributes to Japanese instruction.

- Lesson Study provides professional learning;
- Lesson Study helps teachers understand student thinking;
- Lesson Study spreads knowledge of new content and new approaches;
- Lesson Study helps individuals connect their practices to school goals and broader goals;
- Lesson Study allows competing views to be heard;

- Lesson Study creates a demand for improved instruction;
- Lesson Study shapes national educational policy; and
- Lesson Study honours the central role of teachers in the development of instruction.

Lesson Study is consistent with the parameters of effective professional learning and participants report that the process helps them identify and investigate challenging areas of instruction, and through collaborative inquiry, develop and test potential solutions (Audette, 2004). The Lesson Study model can serve as a means of teacher professional learning with a positive impact on teacher instructional practice (Rock & Wilson, 2005). This approach permits teachers involved in professional learning to become active in their learning. Rock and Wilson (2005) argued that as teachers work through the Lesson Study process there are multiple opportunities for them to reflect, analyse, create action steps, evaluate, and share understandings with other teachers. They believed that the Lesson Study process effectively assisted the teachers in improving their teaching practice. In addition, Audette (2004) said that Lesson Study will only work if the individual is open to learning with and from others. With regard to learning together, Murata (2011) modified the ideas about Lesson Study in many different aspects based on the localised knowledge.

- Lesson Study is centred on teachers' interests – Lesson Study about teachers' professional learning is focused on teachers' interest and relevant to their classroom practice;
- Lesson Study is student focused – Part of the activities are focused on teacher attention to student learning and its relationship to the teaching;
- Lesson Study has a research focus – The teachers are provided with opportunities to become researchers. They have shared data from observations and experience to improve the lesson;

- Lesson Study is a reflective process – Lesson Study gives opportunity for the teachers to reflect on their teaching practice and students' learning. Reflective process is very important and might lead to improvement in the next lesson cycle;

- Lesson Study is collaborative – In Lesson Study teachers work interdependently and collaboratively within a group of teachers (Murata, 2011).

During the Lesson Study process professional collaboration occurs as teachers of various levels of experience work together in groups to study their practice through the implementation of a research lesson. Research lessons are authentic classroom lessons that are: planned collaboratively, focused upon a particular pre-determined goal, observed by colleagues, recorded, reflected upon, and discussed (McDonald, 2010 p. 15). Also Cerbin and Kopp (2006) noted that the Lesson Study is a meaningful and manageable level of analysis for investigating teaching and learning.

There are various understandings of Lesson Study. Isoda (2010, p. 18) identified Lesson Study in Japan as having the following features:

- Process/Lesson Study Cycles – Plan (preparation), do (observation), and see (discussion and reflection) activities involving other teachers;

- Various dimensions of an open classroom – Personal (by master teachers), whole school, regional and national Lesson Study, but systematic;

- Theme of Lesson Study – Study topic and objective are different. An example of a study topic is developing mathematical thinking. The objective is specified at each class related with curriculum. The objective is often described by the sentence 'Through A, students learn/are able to do B'.

- Lesson plan – Format is developed depending on a study topic of Lesson Study;

- Teachers’ mind – Lesson Study is conducted by teachers for improving student learning in the classroom and helping each student develop him/herself, not for researchers who just observe a classroom from their perspectives. In this case, Lesson Study recommends that researchers are teachers who seek the improvement of their class, as well as teachers being researchers who analyse children’s understanding;

- Results – Usually considers achievement in relation to the study topic and objective (Isoda, 2010, p. 18).

Lesson Study in Japan has been conducted in many different formats: In-school training at the school level, with groups of teachers on a voluntary basis hosted by teachers’ union and academic societies, and Lesson Study provided by public funding (Baba, 2007). In addition, Taylor, Anderson, Meyer, Wagner, and West (2005) found that Lesson Study seems very suitable for rural settings because it does not require a complex or expensive infrastructure in terms of resources. Table 2.7 contains a summary of key ideas about professional learning.

Table 2.7: The Summary of Key Ideas about Professional Learning/Lesson Study.

Muir and Beswick	Professional learning includes the processes referred to in the literature as professional learning, staff development, teacher development or in-service education.
Guskey	Professional learning for teachers is a systematic attempt to bring about change in the classroom practices of teachers, in their beliefs and attitudes, and in the learning outcomes of students.
Baba	Lesson Study involves preparation, actual class teaching, and class review sessions.
Leung	Lesson Study is considered a kind of action research consisting of a spiral of steps involving planning, fact-finding and evaluation.
Murata	Lesson Study supports teacher learning and covers three areas: teachers’ knowledge, teachers’ commitment and community, and learning resources.

Theoretical Framework

Introduction

This project aims to study the effectiveness of the Lesson Study Cycle for professional learning of Indonesian primary school teachers. It is hoped that results might be used to inform the development of a professional learning package for primary school teachers in Indonesia. Recent documents from the Ministry of Education, BSNP (2006) about mathematics content standard and BSNP (2007) about teachers' competency standard, required teachers in the Indonesian province of Bengkulu to plan, teach, and evaluate using problem solving as a basis for their teaching.

The search of relevant research literature (see earlier in this chapter) suggested that the design of such a project be underpinned and informed by the following features:

- The experiences of professional learning in Indonesian contexts;
- The criteria for effective professional learning;
- The structure of Lesson Study; and
- The features of problem solving and the teaching of problem solving.

This next section will briefly consider and discuss each of these features and then outline the time frame for action.

The experience of professional learning in Indonesian contexts

The content standard of Indonesian Primary School Mathematics (BSNP, 2006) stated that problem solving was to be the focus in learning mathematics, so the use of a problem solving approach has become a challenge for primary teachers. Little is known about it as a teaching style by most primary school teachers in Indonesia.

Teacher professional learning has been conducted continuously by the Indonesian government through the Teacher Peer Group (Kelompok Kerja Guru) (KKG) for primary school teachers, and Subject Matter Teacher Peer Group (Musyawarah Kerja Guru Mata Pelajaran) (MGMP) for secondary school teachers. The KKG is a group of primary school teachers who work together to solve the problems of teaching. They try to think out, and develop new ideas for improving the quality of teaching and learning activities. In KKG, there is one specialist teacher (Guru Inti) who has been trained to help the other teachers. This person is proficient in the management of teaching, and has the knowledge and skills that can be developed with other teachers in the group. As a specialist teacher, the person becomes a facilitator, motivator, resource person, and innovator. The activities of teachers in the KKG are not only to resolve problems of teaching among the teachers, but also to develop academic contacts and self-reflection. Besides that, some teacher professional learning comes from professional organisations such the Teachers' Association. Generally, primary school teachers are familiar with working in a group with an experienced leader as a format for their professional learning. The project will continue with this format in the guide of the Lesson Study Cycle and use the researcher as the specialist teacher (Guru Inti).

Criteria for effective professional learning

Professional learning for teachers can improve the classroom practices of teachers, change their beliefs and attitudes, and change the learning outcomes of students (Guskey, 1986). Importantly, Reimers (2003) noted that the design of teachers' professional learning programs and activities should respond to teachers' professional needs, their personal and professional interests, the stage of professional learning attained at that particular time, and the level of the education system at which they work. An important aspect of professional learning is teachers' experience in teaching practice. Desimone, Porter, Garet, Yoon, and Birman (2002) found that professional learning which focused on specific instructional practices increased teachers' use of those practices in the classroom. However, Reimers (2003, p. 13) said that "the most effective form of professional learning is that which is based in school and is related to the daily activities of teachers and learners". The most successful teacher development opportunities are on-the-job learning activities such as study groups, action research, and portfolios (Wood & McQuarrie, 1999) and Lesson Study groups.

Desimone (2009) also identified some of the critical features of professional learning that increased teachers' knowledge and skills and improved their practice and which hold promise for increasing student achievement. Other researchers noted many of these earlier. These critical features of professional learning are:

- Content focus;
- Active learning;
- Coherence;
- Duration; and
- Collective participation. (Desimone, 2009)

The content focus of the teacher is the most influential feature in professional learning. Activities that focus on subject matter content knowledge and how students learn that knowledge influence and increase teacher knowledge, skills and improve practice, which increases student achievement. Another feature is active learning. The opportunities for teachers to engage in active learning are related to the effectiveness of professional learning. The coherent feature of professional learning depends on consistency of the school and policy with what is taught in professional learning. Duration is another critical feature of professional learning. Professional learning activities should be of sufficient duration of time, more often over a year rather than as a single presentation. The last feature is collective participation. This feature can be accomplished through participation of teachers from the same school, grade, or area. A model of the effect of professional learning on teachers and then on students is shown in Figure 2.7.

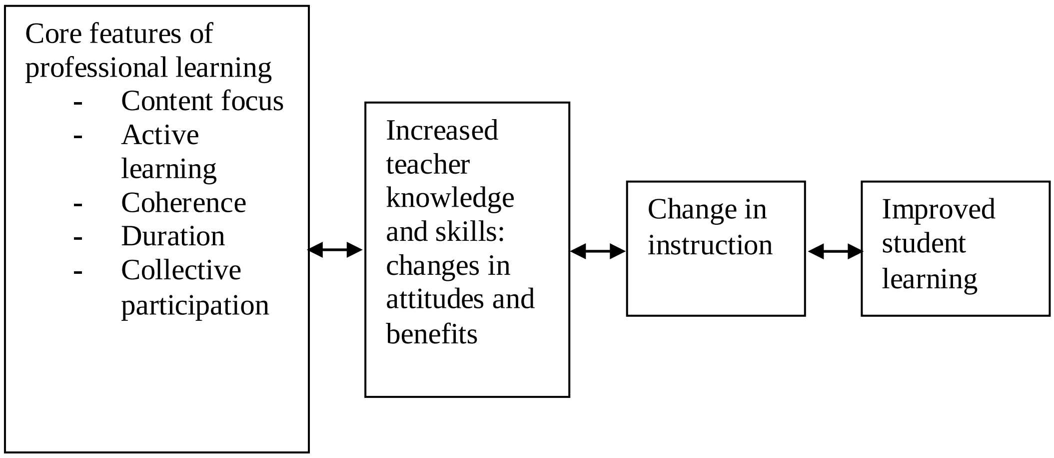


Figure 2.7: Proposed core conceptual framework for studying the effects of professional learning on teachers and students (Desimone, 2009, p. 185).

As reflected in the diagram, teachers first experience effective professional learning. Then the second step, the professional learning increases teachers' knowledge and skills and possibly forms a change in their attitudes and beliefs. The next step is for teachers to use their new knowledge and skills, attitudes, and beliefs to improve the content of their instruction or their approach to pedagogy or both. Finally, the instructional changes foster increased student learning. The present study will be designed to allow teachers to follow this development.

The structure of Lesson Study

Lesson Study is a form of professional learning that enables teachers to conduct personal learning in their classrooms. It includes discussion of subject matter, why they teach, how they teach and what students can learn. In Lesson Study teachers collaboratively plan the lesson, observe the lesson, improve the lesson, make changes to the lesson, and collect data to understand the effect of teacher action on student learning. In collaboration the teachers learn from and with one another, whether they are from the same or different school, or teach at the same or different year levels. Building communities of learning and teaching practice can occur as a result of Lesson Study. Sanders (2009) found that "the potential of Lesson Study as an effective model of school-based professional learning for teachers of mathematics became apparent" (p. 475). Successful Lesson Study was carried out by *Pursuing Good Practice of Secondary Mathematics Education* in Indonesia IMSTEP-JICA in collaboration with three universities from 2001 to 2003 (Marsigit, 2006). The success of Lesson Study in secondary school as an effective professional learning tool leads to the succes of Lesson Study in primary school. It will be the main vehicle for the project with primary teachers described here.

Lewis, Perry, and Murata (2006) described how Lesson Study can improve teaching practice in two ways. First, as Conjecture One, Lesson Study improves instruction through the refinement of lesson plans. Second, as Conjecture Two, Lesson Study strengthens three pathways to instructional improvement: teachers' knowledge, teachers' commitment and community, and learning resources. Models that specify the connection between Lesson Study's observable features and instructional improvement, even in a tentative, emerging fashion, can be useful in several ways shown in Figure 2.8 (Lewis et al., 2006).

IMPROVEMENT OF INSTRUCTION

↑

INTERVENING CHANGES

Lesson Study improves instruction through the refinement of lesson plans.

Lesson Study strengthens three pathways to instructional improvement: teachers' knowledge, teachers' commitment and community, and learning resources.

Examples of the three pathways:

Teachers' knowledge:

- Knowledge of subject matter;
- Knowledge of teaching;
- Capacity to observe students;
- Connection of daily practice to long term goals.

Teachers' commitment and community:

- Motivation to improve;
- Connection to colleagues who can provide help.

Learning resources:

- Lesson plans that reveal and promote student thinking;
- Tools that support collegial learning during Lesson Study.

↑

OBSERVABLE FEATURES OF LESSON STUDY

- Study existing curricula and standards;
- Plan and conduct research lesson;
- Collect data during research lesson;
- Present and discuss data from research lesson, draw out implication for future instruction.

Figure 2.8: How Lesson Study results in instructional improvement: Two conjectures (Lewis et al., 2006).

The goal of the lesson in the Lesson Study is selected on the basis of the curriculum, and a topic of interest. Teachers prepare a lesson plan that includes long term goals, a data collection plan, a model of learning trajectory and chosen teaching approach. A volunteer in the group teaches the lesson and the others observe and collect data. In reflection and discussion observers share data from the lesson, and use data to illuminate student learning and the broader issue of teaching and learning. This also raises new questions for the next cycle of the Lesson Study. This study followed the Lewis Lesson Study model.

The features of problem solving and the teaching of problem solving

The teachers in the project sought to develop their knowledge of teaching mathematics using a problem solving approach. Specifically, they had to know, understand, and apply the process of mathematical problem solving in their primary classrooms. They needed to understand how to apply and adapt a variety of appropriate ways to solve problems, to build new mathematical knowledge, and to monitor the process of mathematical problem solving, at both their personal levels and within their teaching.

Building new mathematical knowledge about problem solving is important for teachers. Hyde and Hyde (1989, p. 5) noted that problem solving activities can introduce students to new mathematical ideas, provide exciting experiences that develop a deeper understanding, and also help students apply what they understand to their lives. The teachers have to present problems and provide situations that enable students to construct their own understanding of mathematics. They need to develop the problem to establish a productive dynamic relationship between students' knowledge and mathematical ideas. Teachers will need to comprehend criteria for good problems. Hyde and Hyde (1989) noted some as the following criteria for good problem:

- There is no obvious way to work on the problem;
- The problem is set in a meaningful context for the students;
- The problem provokes students' interest in pursuing it;
- Working on a problem should use mathematical thinking and knowledge that is developmentally appropriate;

- Discussion of a solution should allow the teacher to build on the problem to explore mathematical ideas.

Monitoring and reflecting on the process of mathematical problem solving is part of the assessment of student learning. Also, Hyde and Hyde (1989) suggested that teachers assess students' work in problem solving more analytically than the traditional simplistic right or wrong. Many of these features were new to the participating teachers and formed a core of the teaching and discussion during the professional learning based on the Lesson Study Cycle.

Professional learning program

Based on the theoretical framework, to enhance primary teachers' understanding and their ability to implement teaching mathematics using a problem solving approach, the researcher used Lesson Study as a vehicle for professional learning. The professional learning program was followed by 12 participants and was conducted between September 2010 and December 2010.

The professional learning, in-service program consisted of components of workshops and Lesson Study. The purposes were:

- To enhance teacher's mathematical knowledge of problem solving;
- To enhance teacher's pedagogical knowledge of problem solving;
- To design lesson plans using a problem solving approach;
- To understand Lesson Study as related to professional learning;
- To know how to do Lesson Study themselves;
- To understand the role data plays in Lesson Study and how to collect data during Lesson Study.

The material for the workshops to support those purposes consisted of teaching mathematics using a problem solving approach (e.g. understanding the problem, strategies for teaching problem solving, helping students with problem solving, assessing teaching with problem solving), lesson plan design, introducing Lesson Study (e.g. understanding Lesson Study, the benefits of Lesson Study, doing Lesson Study), and the Lesson Study process (e.g. planning, teaching, observing, debriefing, reteaching, reflecting and sharing).

Lesson Study as a professional learning strategy required teachers and other educators to work collaboratively to strengthen a given lesson until it had been refined as much as possible and then teach it to gather powerful data about how well the lesson worked. The participants were able:

- To prepare, implement, and revise teaching material using a problem solving approach;
- To apply their mathematical content knowledge of problem solving in teaching practice;
- To apply pedagogical content knowledge of problem solving in teaching practice;
- To decide whether to revise the tested lesson and re teach or apply what they have learned to another lesson.

Based on their understanding of the basics of Lesson Study and engaging in teaching practice, participants would learn about Lesson Study variations. They would plan the implementation of Lesson Study in schools, and they would work together to solve problems related to its implementation. The professional program was aimed at helping teachers know, understand, and develop their mathematical knowledge and pedagogical knowledge in implementing a problem solving approach in their mathematics for certain topics (see Table 2.8).

Table 2.8: Timetable of Professional Learning.

Date	Meeting	Content	Purpose	Involved
20 Sept. – 25 Sept. Week 1	1	Teaching with problem solving - Type of problem, strategies for teaching, helping students, and assessing teaching	Enhance teachers' mathematical & pedagogical knowledge of problem solving	12 teachers
	2	Lesson plan and teaching mathematics with problem solving	Develop a lesson plans using problem solving approach	12 teachers
27 Sept. – 2 Oct. Week 2	3	Introduction to Lesson Study - What is Lesson Study? Benefits? How does it work?	Improve basic familiarity with the Lesson Study Cycle	12 teachers
	4	The process of Lesson Study - Plan, teach, observe, debrief, revise, reteach, reflect and share	Recognise the process of Lesson Study	12 teachers
Cycle for working on Lesson Study for each group: Group A and Group B				
4 Oct. – 24 Dec. Weeks 3 – 13	5 – 25	Group Meeting - Research & prepare study lesson	Prepare teaching material and a lesson plan	6 teachers from each of Group I and Group II
		First Study Lesson - Teaching practice	Implement lesson plan and classroom observation	
		Group Meeting - Reflection and revising	Improve lesson plan and teaching practice	
		Second Study Lesson (optional) - Reteaching practice	Implement lesson plan revision	
		Group Meeting - Reflection and revising	Improve lesson plan and teaching practice	
Final review of Lesson Study cycle				
27 Dec. – 31 Dec. Week 14	26	Lesson Study open house - Review the Lesson Study - Feedback	Obtain feedback about Lesson Study and assess teachers' professional learning about Lesson Study and problem solving	6 teachers 3 principals (Group I) 6 teachers 3 principals (Group II) 1 mentor

Chapter Summary

Research described here shows that there may be inadequacies in teachers' mathematical and pedagogical content knowledge and in their practices for teaching problem solving. It also suggests that Indonesian primary teachers may benefit from professional learning based on experiences in other countries. The research literature relating to effective teacher professional learning suggests that Lesson Study as a professional learning approach can improve teacher knowledge of practices based on a problem solving approach.

This chapter has described teacher professional knowledge, professional learning, Lesson Study, and problem solving in order to establish the case for study in this area. Previous research related to aspects of the research question has been reviewed and a review of community of practice theory, which is the underpinning philosophy for this research has been provided.

CHAPTER THREE

Methodology

Introduction

This chapter describes the methodology used in this study, beginning with an overview of the approach followed by a rationale for the study approach and design. The technique used to select the participants and the developments of the instruments in this study are also described. The processes of collecting data and analysing data in this study are then presented.

Research Design

A qualitative research design was used in this study. Some researchers (Merriam, 1998; Creswell, 2005) believe that qualitative research is best used to discover themes and relationships at the case level. Qualitative research is an umbrella concept covering several forms of inquiry that help to understand and explain the meaning of social phenomena with as little disruption of the natural setting as possible (Merriam, 1998, p. 5). Also, Creswell (2005) stated that qualitative research is a model of research in education in which the researcher identifies the views of participants, asks broad, general questions, collects data consisting largely of words (or text) from participants, describes and analyses these words for themes, and conducts the inquiry in a subjective manner.

This study used case study as an analytical method to provide an in-depth perspective based on extensive data collection. Gall, Borg, and Gall (1996, p. 545) defined case study research as the in-depth study of instances of a phenomenon in its natural context and from the perspective of the participants involved in the phenomenon. The purpose of doing case studies is to produce detailed descriptions of a phenomenon, to develop possible explanations of it, or to evaluate the phenomenon

(Gall et al., 1996). Qualitative case studies in education can be defined by arranging them into categories or type based on disciplinary orientation or by function, that is, whether the overall intent is to describe, interpret, or evaluate some phenomenon or to build theory (Merriam, 1998, p. 34). Also, Sowder (2007) noted case studies have become a useful way to understand and evaluate the effectiveness of professional development or intervention in the teacher preparation process.

This study used a case study approach and was concerned with the process and meaning rather than outcomes (Merriam, 1998). In case studies the process can be viewed in two ways, as monitoring and as causal explanation. Merriam (1998, p. 33) noted that the meaning of process as monitoring included describing the context and population of the study, discovering the extent to which the treatment or program has been implemented, providing immediately feedback of a formative type, and the like. Process as causal explanation included discovering or confirming the process by which the treatment had the effect that it did.

Quasi-experimental design is used in this research to reduce the threats to internal validity. Quasi-experiments are experimental situations in which the researcher assigns, but not randomly, participants to groups because the experimenter cannot artificially create groups for the experiment (Creswell, 2005). In this case, groups consisted of teachers who continued to teach in their regular class situations.

The study is about teacher professional learning that involved a small group of primary school teachers. The effect of a Lesson Study Cycle on the teaching practices of a small group of Indonesian primary school teachers will be described. Research questions were developed to address the main goal of this research. Throughout the study, data were collected using a range of methods: teacher interviews, in-class observations of teacher practice, small group discussion of Lesson Study meetings, and analysis of documents. These data allowed the researcher to focus on insight, discovery and interpretation.

Research questions

The purpose of this research was to describe and identify the effect of professional learning using a Lesson Study Cycle, on the teaching of a small group of Indonesian primary school teachers. The objectives of professional learning were to improve teachers' knowledge in teaching mathematics through problem solving in the primary school. In general, the study sought to evaluate the effect of Lesson Study in bringing about change in teachers' knowledge and practices in primary school mathematics.

The general research question in this study was: *To what extent does professional learning based on problem solving and a Lesson Study approach effect: Teachers' pedagogical content knowledge, teachers' mathematical content knowledge, and teachers' beliefs about teaching and learning mathematics?*

It was supported by a subsidiary question: *In what ways does Lesson Study bring about change in teachers' practice in teaching mathematics through problem solving in an Indonesian primary school context?*

The specific research questions, data needed, and data sources, are shown in Table 3.1.

Table 3.1: The Connection of Research Question to Data and Data Sources.

Research Question	Data Needed	Data Source
To what extent does professional learning based on problem solving and a Lesson Study approach effect teachers' pedagogical content knowledge about teaching and learning mathematics?	Teachers' pedagogical knowledge Classroom practice	Interviews Observation in class Lesson Study – Group Meetings – audio tapes Documents – lesson plans Video recording
To what extent does professional learning based on problem solving and a Lesson Study approach effect teachers' mathematical content knowledge?	Teachers' mathematical content knowledge	Interviews Observation in class Lesson Study – Group Meetings – audio tapes Documents – lesson plans Video recording
To what extent does professional learning based on problem solving and a Lesson Study approach effect teachers' beliefs about teaching and learning mathematics?	Teachers' beliefs	Interviews Lesson Study – Group Meetings
In what ways does Lesson Study bring about change in teachers' practice in teaching mathematics through problem solving in Indonesia primary school context?	Impact of Lesson Study Factors for change. Negative effects of Lesson Study.	Interviews Observation Lesson Study – Group Meeting

Participants

This study was conducted at primary school level in an inner city area and a suburban area of Bengkulu City, Bengkulu, Indonesia. Bengkulu City is a small regency of Bengkulu province of Indonesia with an area of about 150 square kilometres consisting of four district areas and about 107 primary schools.

The Indonesian Central Statistical Agency (Badan Pusat Statistik) (BPS) identified an urban area as a village level administrative area that meets certain requirements in terms of population density, percentage of agricultural households and a number of urban facilities, such as transportation, means of formal education, public health facilities and so on. In general, an urban area is close to the central government administration and the centre of economic activity. Physically, the urban area serves as the centre of economic, and social cultural activity. In contrast the suburban area is quiet, far from the central government administrative area, with less economic activity, less population, less transportation facilities and less traffic.

Both the schools in the urban and suburban areas use the same KTSP curriculum and national exam. However, the school in the urban area is easier to access than the school in the suburban area. It provides better opportunity for teachers' professional learning. The official language of teaching and assessment is the Indonesian language (Bahasa Indonesia). The Indonesian educational system is made up of six years for primary school, three years for secondary school, and three years for senior high school.

The study is based on case studies of primary teachers who participated in Lesson Study. Purposive sampling was used in this study for several reasons. First, it was decided to use upper primary teachers (Grade Four and Five) to ensure their students had at least basic mathematics knowledge and skills needed for problem solving. Thus, the participants were 12 primary school teachers of upper grade students divided in two groups. Second, it was planned to involve teachers from different locations. Hence, each group consisted of six primary teachers who came from two different school locations: inner city and suburban schools in Bengkulu city, Indonesia (see Table 3.1). Three primary schools in each area were selected based on proximity to each other and the level of accreditation. Schools in each group are less than 4 kilometres apart. Each group consisted of the schools, which have A and B accreditation score by National School Accreditation Agency (Badan Akreditasi Nasional Sekolah/Madrasah) (See http://www.ban-sm.or.id/). Third, the teacher participants were classroom teachers who teach mathematics and other subjects in their classes. All participants generally believed in teaching mathematics based on a text book format and lacked knowledge of problem solving as a teaching approach. Fourth, teachers were selected on the basis of their proposed learning experience. Prior to the preliminary phase of this study, these teachers had experienced limited participation in professional learning workshops. There were no participants with experience in Lesson Study. Due to the limited funding for professional learning, the teachers who attended the workshop depended on the nature of the available professional learning funding. Normally, the most frequent kind of professional learning is a one-day seminar such as an education policy program meeting funded by the government.

Generally, Bengkulu was selected as the site for the study due to its lack of involvement in Lesson Study, and the fact that the teachers were unfamiliar with problem solving in mathematics. Even though the teachers lacked knowledge in problem solving and professional learning, their willingness to develop their professional practice by participating in the study was enthusiastic. Another reason for selection is that Bengkulu is the home area of the researcher, which facilitates establishing the professional contacts needed to conduct research and collect data.

Table 3.1: The Participants of the Study.

No	Teacher	Age (Years)	Experience School (Years)	School	Grade	Urban/ Suburban	Volunteer (Vr) Observer (Ob)
1	Ha	54	32	SD 9P	V	Suburban	Vr & Ob
2	La	41	19	SD 9P	IV	Suburban	Ob
3	Wi	34	11	SD 1P	IV	Suburban	Vr & Ob
4	Yu	37	14	SD 1P	IV	Suburban	Ob
5	Mi	25	2	SD 8P	V	Suburban	Vr & Ob
6	Hen	37	11	SD 8P	IV	Suburban	Ob
7	Zu	37	10	SD 8M	IV	Urban	Vr & Ob
8	Ri	54	15	SD 8M	IV	Urban	Ob
9	Sa	34	10	SD 1M	IV	Urban	Vr & Ob
10	Ma	48	25	SD 1M	V	Urban	Ob
11	Si	56	33	SD 4M	IV	Urban	Ob
12	Yi	37	10	SD 4M	V	Urban	Ob

Data Collection

This study investigated the participation of 12 teachers in professional learning designed to enhance their teaching of mathematics through problem solving in primary schools. The project involved classroom observations, interviews with teachers, audio recording of Lesson Study Group Meetings, video recording, and analysis of documents. This reflects the three kinds of data collection as noted by Patton (1990) as relevant for qualitative research: in-depth interviews, direct observation, and written documents.

Observations

Observation is the process of gathering open-ended, firsthand information by observing people and places at a research site (Creswell, 2005). There were two observations in this research: classroom observation and Lesson Study meeting observation. Classroom observations were conducted to determine in detail how teachers translated the professional learning workshop information into their teaching practice. Lesson Study meeting observations were conducted to record the teachers' discussions after a research lesson. Observation methods are powerful tools for gaining insight into situations (Cohen, Manion, & Morrison, 2007). In qualitative research observations provide an alternative source of data for verifying the information obtained by other methods (Gall et al., 1996).

The proforma for classroom observation and Lesson Study meeting observation was developed after considering some aspects: what the research questions indicated about data needed; how this led to initial criteria categories; then discussion with other researchers to revise and refine categories. The major structural components of a category observation instrument are: (1) A set of operationally defined categories of behaviours; (2) A set of rules and priorities for observation and coding; (3) A standardised recording form in the matrix; and (4) A series of instructions for organising and analysing the observation data. The development of the category observation instrument followed the steps: (1) Descriptive observation about which behaviours were to be recorded; (2) Categorising observation into a number of generic blocks which are meaningful in relation to the purposes of the instrument developer; (3) Formal categorisation and prioritising the more important behaviours; (4) Standardising through discussing with supervisor and other researchers. The researcher/observer then recorded which behaviours occurred and how often they occurred during the period of observation.

The proforma classroom observation was discussed with the teacher participants prior to the commencement of the Lesson Study sessions. This approach was important in determining the quality of data and response rates. The teacher participants constructed the lesson plan included students worksheets. These constructions were considered as teachers contributed in their practice as a principle of Lesson Study. In October of 2010 as the first time on Lesson Study session, the classroom observation instrument was trialed with a teacher at one school, in order to ascertain: an appropritae the level of readability and potential ambiguity of any aspect of observation. Feedback from teachers led to modifications to some aspect observation and refined the format observation in various nature, ranging from focus on fluency with which teachers executed the stages of teaching practice, procedures and resulted in the creasion of concise instructions for teachers to ensure consitency for this study. To increase credibility the teacher participants were told that the observations were for research purposes in order to try and investigate ways on improving teaching learning mathematics with problem solving approach in a Lesson Study Cycle. The aspects of teacher participants in teaching and learning that were observed by the researcher included knowledge of teaching and students' learning: the use of resources and the representation of ideas, the demonstration of procedure, the classroom management, the use type of problems, the level of teacher's questioning, and the use of diagnosis and assessment.

Gold (1969) has identified four models of observation in qualitative research, shown in Table 3.2

Table 3.2: Possible Roles for Researchers in Qualitative Studies.

Complete participant	Identity not known to group. Researcher interacts naturally with group as a member.
Participant as observer	Participates fully with group, but identity as researcher known to group.
Observer as participant	Identity of researcher known, but no attempt made to participate as a member of the group.
Complete observer	Researcher observes without any involvement in group activities.

In the relationship between observers and their roles, Gall et al. (1996, p. 345) noted that the observers in qualitative research vary along a continuum from complete observer to complete participant. Between these two extremes is the 'observer as a participant' and 'participant observer roles'. This study involved two models of observation, a complete observer and observer as participant.

Classroom observation

For classroom observations, the researcher used the role of non-participant observer or complete observer. That is, the researcher was an observer who visited a class and recorded notes without becoming involved in the activities of the participants. Classroom observation was conducted by the researcher and the other teachers in the classroom, when the member of the Lesson Study Group carried out the lesson in his or her classroom. Based on the lesson plans and through the technique of non-participant classroom observation the teachers were observed for how they implemented content knowledge and pedagogical content knowledge for problem solving in their classroom teaching practice.

Observational data were gathered until the researcher began essentially replicating earlier data. Data from classroom observation related to teacher activity and student activity in teaching and learning mathematics with a problem solving approach. That is, they were quantified and described in relation to problem solving, what they said, did, or demonstrated, and what the students said, did, and showed. Observations focused on the teacher teaching mathematics using a problem solving approach in terms of using types of problems, using classroom organisation for students working, using classroom discourse, and using assessment (See Appendix A).

During the Lesson Study classroom observation the observers sat in the back of the class. Sometimes the observers looked closer at the student group discussion. The observers also brought the lesson plan that the teacher had developed as well as the observation instruments. The observers also made notes about extra activities that were not covered on the instruments. Each observation was used for reflection in the Group Meeting after the lesson in the Lesson Study Cycle.

Lesson Study meeting documentation

Lesson Study Group Meetings were audio and video recorded and conducted after the focus lesson to consolidate teachers' reflections and to revise the lesson. The Lesson Study Group Meeting was used to collect shared understandings from several teachers as well as to get views about specific topics. It provided interaction among teachers, collection of extensive data, and participation by all teachers in a group. Pierce and Stacey (2009) noted that focus group discussions gave sufficient information for analysis, reworking of lessons and evaluation of the revised lessons.

In the Lesson Study meeting, the researcher was an observer as participant. Sometimes the researcher first entered Lesson Study meetings and observed as non-participant, then became involved, as the participants needed it. The researcher offered comments, asked questions, and asked for explanation and clarification. Soon after each lesson the researcher conducted a Lesson Study meeting with each group of teachers. The researcher observed the participant activities during Lesson Study meetings, focusing on the content of the group's discussions and participants' field notes. Field notes were made by the researcher during each Group Meeting as they prepared and designed the lesson plan. This observation focused on some aspects: the ability of the participants to contribute to lesson plan design, the ability of the participants to work collaboratively in the team and the ability to be critically responsive when working with other participants. Documentation of the Lesson Study Meeting consisted of group discussion, reflection, feedback, and the revision of data from the research lesson (See Appendix B). Both classroom observations and Lesson Study meeting documentation was conducted from October 2010 to December 2010.

Interviews

Interviews with teachers were conducted to capture the varied perspectives of teachers in two different school areas. In this study, the one-to-one interviews were conducted in three ways, before Lesson Study, at the end of Lesson Study, and one year after the Lesson Study program. The interview before Lesson Study focused on each teacher's background of teaching mathematics using a problem solving approach, their views of professional learning through Lesson Study, and their beliefs (See Appendix C). The interview at the end of Lesson Study program focused on the effect of professional learning, and teachers' beliefs about teaching and learning mathematics (See Appendix D). The interview one year after the Lesson Study program focused on the impact of professional learning and teacher beliefs about teaching and learning mathematics. The interview used predetermined questions to encourage teacher participant reflection on professional learning interventions. The interviews were semi-structured and designed to: assess teachers' new knowledge and views about problem solving, and explore teachers' beliefs and knowledge related to the professional learning workshops (See Appendix E). Bibier and Leavy (2006) argued that semi-structured interviews rely on a certain set of questions and try to guide the conversation to remain, more loosely, on those questions. The researcher also allowed respondents to talk about what was of interest or relevant to them. Informal notes were taken during the interviews, which were audio recorded.

Video recordings

The Lesson Study meetings were video recorded. As distinct groups, the 6 inner city teachers and the 6 suburban teachers participated in separate Lesson Study Cycles. The analysis focused on both the students and teachers - what the students and teachers were doing and thinking, and what the teachers were learning about student problem solving. The video recording may not adequately capture much of what happened in the Lesson Study meeting, but it did provide a helpful additional source of data. As a supplement, the video recording supported the observation data to ensure key happenings and things said or discussed were not missed. Also, the videos provided primary data of what people actually said and did.

Documents

A document can have different meanings at different levels of analysis. For example, the content of a lesson plan can be analysed to determine what concept it covers (See Appendix F). In qualitative research, the same document can be analysed at different points in the study with each analysis yielding new constructs, hypothesis and insights (Gall et al., 1996, p. 636). Document analysis of lesson plans, and teacher reflections on the lessons were conducted. The documents were examined for information about teachers' learning about problem solving and related content knowledge and pedagogical knowledge that might not be directly observable in the lessons. These data were developed and analysed.

Reliability, validity, and triangulation

In qualitative research reliability can be regarded as a fit between the collected data and what actually occurs in the natural setting that is being researched (Cohen, Manion, & Morrison, 2007). Also, in qualitative data validity might be addressed through the honesty, depth, richness and scope of achieved data, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher (Cohen et al., 2007). Triangulation can be defined as the use of more than one method of data collection in the study of some aspects of human behaviour. The Triangulation process is the process of using multiple data-collection methods, data sources, analysis, or theories to check the validity of case study finding (Gall et al., 1996). In qualitative research, triangulation can be achieved through the use of a range of data collection techniques, which enhances the validity of data generated. To minimise observer bias, the researcher used standard procedures between the two groups of teachers in the inner area and the suburban area. The same format was used during the classroom observation and Lesson Study meeting observation. The observation began with the classroom observation where the volunteer carried out the lesson. Soon after this the Lesson Study meeting observation was conducted. The validity of the study has been enhanced through the researcher's choice for standardisation of the conduct of interviews. The pre-interview and post-interview were semi-structured and included open-ended questions. The researcher distributed the open-ended question to the teachers for guidance during interview. Interview bias was minimised through audio recording transcription to ensure accuracy of the data transcribed. One potential issue in interviewing is that an interviewer might give some explanations and examples that could subtly influence the subject into giving answers skewed towards the interviewer's own opinions, prejudices and values. These could be checked by reading the interview transcription. In this study the researcher carefully avoided the use of leading questions or examples. Merriam (1998, p. 204) noted in qualitative research the researcher can use six basic strategies to enhance internal validity:

- Triangulation – using multiple investigators, multiple sources of data, or multiple methods to confirm the emerging findings. For example, the process of corroborating evidence from methods of data collection using document and interview;

- Member check – taking data and tentative interpretations back to the people from whom they were derived and asking them if the results are plausible. For example, asking the participant in writing or interview about the accuracy of the report, such as whether the description is complete and realistic;

- Long-term observation at the research site or repeated observation of the same phenomenon – gathering data over a period of time in order to increase the validity of the findings. For example, data collected over the long term, after one year the situation specific influences are canceled out;

- Peer examination – asking colleagues to comment on the findings as they emerge. For example, asking colleagues about findings as they emerge;

- Participatory or collaborative modes of research – involving participants in all phases of research from conceptualising the study to writing up the findings. For example, the participants reflected on what has happened and decided whether they can practically apply the collected information and whether more research is required;

- Researcher's biases – clarifying the researcher's assumptions, worldview, and theoretical orientation at the outset of the study. For example the researcher used double observation from real classroom and video recording to eliminate the posibility of reseracher bias.

Each of the points noted above by Merriam are reflected in what was done in this study.

Ethical considerations

In this study teachers and students were involved in providing information. Informed consent was obtained from the teachers, the principal of schools and Head of Department of Education as appropriate. The teachers were informed of all aspects of the study that might be expected to influence their willingness to participate. The participants were informed of procedures for contacting the researcher within a reasonable time period and consent form as depicted in Appendix G. The researcher agreed to sign a confidentiality agreement. All interviews were transcribed, duplicated and names of participants were changed to pseudonyms. Also all classroom observations and Lesson Study Group Meeting observations were audio and video recorded. Only the researcher knew the real names of the people involved in the study. All data collected within the study has been kept secure and will be maintained for five years by the researcher. During the research program all data was stored on a password-protected computer at the School of Education, Curtin University.

Data Analysis

Data were analysed using inductive and interpretive analysis, where the researcher looked for natural variation in the data. According to Patton (1990, p. 423) the interpretation process means attaching significance to what was found, offering explanations, drawing conclusions, extrapolating lessons, making inferences, building linkages, attaching meanings, and dealing with rival explanations, disconfirming cases, and data irregularities as part of testing the viability of an interpretation.

Interview transcripts, classroom observations, Lesson Study meeting documentation and field notes, were used to portray each teacher participant in the Lesson Study program. Transcripts were coded and analysed to better understand the participants' professional learning through the process of Lesson Study. The interviews revealed the participants' views of their professional learning that cannot be directly observed. The data from interviews were analysed by inductive analysis, going from the particular data to the detailed data (e.g., transcriptions or typed notes from interviews), to the general data and then to codes and themes.

The data from the interview immediately after Lesson Study were analysed immediately after transcription, translation and then compared to the transcript of the interview before the following professional learning workshop. The post interviews were analysed to determine the effect of Lesson Study on teachers' knowledge while determining what changes had occurred after the Lesson Study program.

The data from interviews one year after Lesson Study program were analysed after transcription, translation and then compared to the interview immediately after Lesson Study. The data from the interviews one year after Lesson Study were analysed to discover the impact on teachers' knowledge after they had participated in the Lesson Study program. The findings from the pre-interview, interview immediately after Lesson Study, and the interview after one year were used together with the other methods to describe the effect on each teacher's knowledge of the professional learning.

Observation and notes made by the researcher during the Group Meetings were analysed to determine the impact of professional learning on teachers' learning and their classroom practice.

Timetable of Data Collection

Data collection was conducted in Bengkulu Indonesia with the time line shown in Table 3.3.

Table 3.3: Timetable Data Collection.

Date	Instrument	Purpose	Recording and Analysis
Sept. 2010	Pre-interview (one-one) with each teacher	To record the background of teachers knowledge (pedagogical content knowledge, mathematical content knowledge)	Transcript Type noted/field noted
Oct. 2010	Classroom observation	To record the teacher's teaching practice (content, questioning, resources, classroom organisation, discourse, and assessment) and students response	Type noted/field noted
	Lesson Study Group Meeting documentation	To record the group discussion (reflection, comment, suggestion, revision) and handling the communication	Type noted/field noted Video recording
Nov. 2010	Classroom observation	To record the teacher's teaching practice in pedagogical content knowledge, mathematical content knowledge, and students response	Type noted/field noted Video recording
	Lesson Study Group Meeting documentation	To record the group discussion and handling the communication	Type noted/field noted Video recording
Dec. 2010	Classroom observation	To record the teacher's teaching practice in pedagogical content knowledge, mathematical content knowledge, and students response	Type noted /field noted
	Group Meeting documentation	To record the group discussion (reflection, comment, suggestion, revision) and handling the communication	Type noted/field noted Video recording
	The first interview (one-one) with each teacher	To record the teachers' knowledge (pedagogical knowledge, mathematical content knowledge),teaching practice, and teachers' beliefs after Lesson Study	Transcript Type noted/field noted
Dec. 2011	The second interview (one-one) with each teacher	To record the impact of professional learning in teachers knowledge (pedagogical knowledge, mathematical content knowledge) and teachers' beliefs	Transcript Type noted/field noted

Summary

This chapter has described the methodology used in this study. This study was designed to study changes in teacher knowledge about teaching mathematics using a problem solving approach in an Indonesian primary school context. It focused on teacher professional learning to evaluate the changes in teachers' knowledge and their practices following participation in Lesson Study based on using a problem solving approach. Qualitative case studies were used as the basis for this study. Data collection instruments were interviews, field notes and observations, and data generated from them were analysed using inductive and interpretative analysis. Triangulation was achieved by the use of a range of instruments and data collection methods and measures to ensure reliability and validity were implemented.

CHAPTER FOUR

Results

Introduction

This research project aims to identify and describe the effect of professional learning through the use of a Lesson Study Cycle in an Indonesian primary school context. The study attempted to answer the following research question: *To what extent does professional learning based on problem solving and a Lesson Study Cycle approach affect teachers' pedagogical content knowledge, teachers' mathematical content knowledge, and teachers' beliefs about teaching and learning mathematics?*

It is supported by a subsidiary question: *In what ways does Lesson Study bring about change in teachers' practice in teaching mathematics through problem solving in an Indonesian primary school context?*

Presented in this chapter are data sets from interviews, observations, video recordings, fields notes and documentation. The interview was conducted for 12 participants in three phases. In the early stage of professional learning the interview was conducted to identify the teachers' background on professional learning and teachers' knowledge. At the end of professional learning the participants were interviewed to see the teachers' change and enhancement in teachers' knowledge. Then, one year after the Lesson Study program the participants were interviewed to see the impact of professional learning in their school classroom teaching. Observation was conducted during the Lesson Study Cycle from October 2010 to December 2010 as a part of the principles of the Lesson Study process. Video recording was conducting during the Lesson Study process. Field notes and documents were collected during the process of professional learning. A detailed description of the instruments that were used to analyse the data has been given in Chapter 3.

This chapter describes briefly the experience of twelve primary school teachers who participated in the Lesson Study Cycle as a vehicle for professional learning. Focusing on the Lesson Study Cycle, using open direct observations, video recordings of lessons and group discussions, and post teaching interviews, the researcher summarised the teaching practice of each participant, described their changing teaching practice (through demonstrated change in mathematical content knowledge and pedagogical content knowledge), and described changes in the teachers' beliefs about teaching and learning mathematics.

Project Participant Cases

At the beginning of the project a group of twelve teachers participated in the professional learning program that was conducted by the researcher. The program content was split into two sections: teaching mathematics using a problem solving approach, and the Lesson Study model as a vehicle for professional learning. First, the professional learning focused on using problem solving approaches for teaching mathematics. In this section the participants discussed the characteristics of mathematical problem solving and mathematics problem examples, such as routine and non-routine mathematics problems. They also discussed issues with teaching mathematics using a problem solving approach, such as teaching based on problem solving, heuristic strategies to solve problems, types of problems, and teaching about problems that provided for individual student differences. Secondly, the professional learning also focused on the details and format of Lesson Study as a professional learning approach. In this section, the participants discussed the Lesson Study Cycle, the model of Lesson Study and lesson planning. As the next step, the participants were divided into two groups for Lesson Study, with six teachers from a suburban or rural area in Group I, and six teachers from an urban area in Group II.

During the Lesson Study Cycle, five of the original twelve teachers had an opportunity to become a volunteer teacher and to be observed by other group members. The five teachers who acted as volunteer teachers included three teachers from Group I and two teachers from Group II. The following lessons produced by Group I and Group II are shown in Table 4.1.

Table 4.1: Lessons /Volunteer Teachers for Group I and Group II.

Lesson	Teacher	Basic Competence	Grade	School	Group
1A	Ha	Determining the area of a trapezium	VA	SD 9P	I
1 B	Ha	Determining the area of a trapezium	VB	SD 9P	I
2 A	Wi	Measuring angle with non-standard units and degrees	IVA	SD 1P	I
2 B	Wi	Measuring angle with non-standard units and degrees	IVA	SD 1P	I
3 A	Mi	Determining the volume of a cube and a rectangular prism	VA	SD 8P	I
3 B	Mi	Determining the volume of a cube and a rectangular prism	VB	SD 8P	I
4 A	Zu	Determining the volume of a cube and a rectangular prism	VA	SD 8M	II
4 B	Zu	Determining the volume of a cube and a rectangular prism	VA	SD 8M	II
5 A	Sa	Determining the perimeter and area of a parallelogram	IVA	SD 1M	II
5 B	Sa	Determining the perimeter and area of a parallelogram	IVB	SD 1M	II

In each instance, the initial lesson (Lesson A) was planned by the teachers in the Lesson Study Group I or II. It was taught by one teacher, observed by members of the group, and then analysed by the group. The lesson was then replanned (Lesson B), based on the group's analysis and advice, and retaught, sometimes by the same teacher, sometimes by a different teacher. The five lessons were taught by five teachers in their own classrooms. The duration of each lesson was 70 minutes.

Documentations of Lesson Study meetings were focused on reflections, feedback, revision of plans from previous lessons, and the involvement of participants in the group discussion. In the classroom observations, the teachers were observed as to how they implemented their mathematical content knowledge, their pedagogical content knowledge, and how their stated beliefs about teaching and learning mathematics through problem solving were reflected. The observations focused on the implementation of their mathematical knowledge through a problem solving approach, in terms of the types of mathematical problems used, the classroom organisation, the use of classroom discourse, and the use of assessment.

Pedagogical Content Knowledge

Many studies on pedagogical content knowledge have been conducted (e.g., Shulman, 1986; Ball, 1990; Turner & Rowland, 2009). Shulman (1986) noted that pedagogical content knowledge included the ways of representing and formulating the subject that made it comprehensible. Pedagogical content knowledge has been defined as the knowledge of effective teaching which includes three components: knowledge of content, knowledge of curriculum, and knowledge of teaching (An, Kulm, & Wu, 2004).

In this project, teachers' pedagogical content knowledge (PCK) was considered to include aspects of their knowledge about teaching, knowledge of students' learning, and knowledge of curriculum. Throughout the Lesson Study Cycle, the researcher attempted to obtain a more representative picture of the primary teachers' pedagogical content knowledge. An understanding of the teachers' pedagogical content knowledge was obtained from the observations of teaching practice, Group Meetings, and individual interviews. Knowledge of teaching was considered to include how the teachers represent ideas, demonstrate procedures, use examples, explain topics, manage the classroom, engage and acknowledges students, and connect new concepts with previous knowledge. Knowledge of students' learning was shown by how teachers diagnose and deal with student learning difficulties, how they anticipate students' thinking, how they involve the students, and how they assess the students. Knowledge of curriculum was shown by how teachers sequence their teaching programs, choose teaching objectives, select suitable teaching materials and textbooks, and design lesson plans. This study looked at teachers' pedagogical content knowledge about teaching mathematics through a problem solving approach. This was examined through two groups of Lesson Study participants teaching the topics as shown earlier in Table 4.1 earlier.

Mathematical Content Knowledge

Teachers' mathematical content knowledge has been studied by a range of researchers (e.g., Shulman, 1986; Ball, 1990; Turner & Rowland, 2009). During the Lesson Study program, the researcher attempted to obtain a more representative picture of the primary teachers' mathematical content knowledge (MCK). This is considered to include common subject knowledge, special content knowledge, and knowledge of mathematical problem solving. Through observation of teaching practice the teachers' mathematical content knowledge (MCK) was described as :

- Their understanding of the topics, procedures, concepts, and relations among the topics;
- Their understanding of mathematical processes;
- How they demonstrated their knowledge;
- How they identified students' errors.

This study also looked at teachers' mathematical content knowledge about teaching mathematics using a problem solving approach. This was examined through looking at the Lesson Study participants' teaching of the topics previously mentioned. The teachers' mathematical content knowledge was demonstrated by their involvement in discussions about preparing materials, designing lesson plans, and reflections during the Group Meetings.

Teachers' Beliefs about Teaching and Learning Mathematics

In this project, the teachers' beliefs about teaching and learning mathematics were mainly determined from their statements during interviews and Group Meetings during the Lesson Study Cycle. Among other things, the teachers' actions during the lessons, the ways of presenting the problems, the management of the classroom, and approaches to managing student work, while teaching using a problem solving approach, were discussed. The interviews provided information about how the teachers viewed the nature of problem solving, and their mathematical and pedagogical knowledge of problem solving.

The results from the lesson observations, video recordings, Group Meetings, and post teaching interviews are grouped here according to five cases, with each case being one of the five volunteer teachers.

Case 1: Lesson 1A and 1B by Teacher Ha

Teacher Ha had 32 years teaching experience in primary schools. He was involved in some professional learning in mathematics as a participant and facilitator ten years ago. He teaches upper level Year Five and Six (students aged 11-12) at SD 9P Bin located in a suburban area about 3 kilometres away from SD 1P Bin. His class has 35 students who live in the neighbourhood of the school. He stated his belief that teaching problem solving is very important because this approach allows students to be creative and develop logical thinking. About ten years ago he undertook some professional development in teaching methods, student-centred learning, and active learning. He felt confident about using a problem solving approach because he felt that he had good knowledge of mathematical content and teaching methods. Even though he had undertaken some professional development the Lesson Study process was new to him. In the Lesson Study group he became a volunteer teacher. He taught the same topic twice at Year Five with different classes during the study (Lessons 1A and 1B). Lesson A was taught to the original lesson plan designed by the group, and Lesson 1B was taught to a lesson plan based on Lesson 1A and revised following feedback from the group. The lessons were based on determining the perimeter and area of a trapezium. Both Ha's lessons were video and audio recorded, and his classroom was observed by his Lesson Study group. He was also interviewed after the Lesson Study intervention.

Description of Lesson 1 A

In Lesson 1A on 14 October 2010 Ha segmented the lesson into three distinctive phases: the initial activity, the main activity, and the end activity. His class consisted of about 35 students, with the students sitting in rows facing the white board. The objective of the lesson was for students to determine the area of a trapezium. Teacher Ha mostly used direct teaching, where he worked through the topic in a step-by-step manner.

He spent about five minutes at the start of the lesson introducing the topic. First, he asked students to show some examples of two-dimensional shapes: triangle, rectangle, and parallelogram. Then he asked the class for the formula for calculating the area of a rectangle, and the properties of a rectangle. Students answered orally in a whole class setting. He explained also the learning objective for the lesson.

The main activity took about 50 minutes. On the white board, he drew a rectangle. He divided the rectangle into two equal parts, each being a trapezium as shown in Figure 4.1.

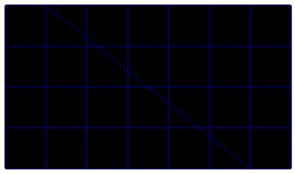
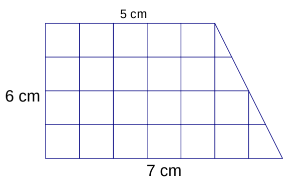


Figure 4.1: Rectangle divided into two equal parts.

Using a piece of grid paper, he counted the unit squares on this figure. He explained to the students how to determine the formula for the area of a trapezium step by step directly by first finding the area of a rectangle.

The students sat in their rows, listening, answering teacher questions, and following teacher explanations until the teacher reached the formula for the area of a trapezium $A = \frac{1}{2}(a + b) \times h$, (where A = area, a = base side, b = top side, and h = height of a trapezium). After the teacher found the formula for the area of a trapezium, he gave the class an example problem: *Calculate the area of a trapezium with the size: base side 10 cm long, top side 8 cm, and the height 9 cm long.* By drawing a trapezium on the white board with this size, the teacher demonstrated answering the example by applying the formula to determine the area of trapezium. After that, the students worked in groups, and he gave applied problem exercises, where the questions were structured in the same way as in the teacher’s example, such as: *If a square with sides of 8cm is drawn on grid paper, measure each side of a trapezium on grid paper. Calculate the area of trapezium.* The teacher gave each group of students a trapezium on grid paper. Each group worked with different problems in size of the trapezium to calculate the area of the trapezium. The teacher observed the group and asked some questions to students in their groups and helped students in difficulty.

At the end of the lesson, the teacher asked a student from each group to present the group's working. He wrote it on the white board, and it was discussed in the whole class. For the last 15 minutes the teacher gave individual applied problems such as: *Calculating the area of trapezium on the figure.*



The problems were presented on student worksheets and student answers were checked in a whole class setting.

Lesson Study Group Meeting after Lesson 1A

A Lesson Study Group Meeting was held after Lesson 1A to provide feedback on the lesson. The Lesson Study Group commented in the following ways:

- Classroom management – A few students did not involve themselves in the group activity. The number of students was different for each group;

- Demonstration of procedures – Steps presented in the classroom were different from the lesson plan. The teacher spent more time on explanation than suggested in the plan and the students listened and answered some of the teacher's questions;

- Representation of ideas – The teacher cut the rectangles on grid paper for each group. Cutting the rectangles could be done by the students, not the teacher;

- Types of problems – The students did not solve any challenging problems. They could answer all the problem examples by using exactly the same procedure as in the teacher's example.

Based on their discussions, the group made some suggestions for changes to the next lesson. These are summarised in Table 4.2.

Table 4.2: Suggested Changes for Lesson 1B.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Classroom management	Ensure all students are involved in the group discussion. Consider the number of students in each group, not just based on the closeness of student seats to form the group.
• Demonstration of procedures	Follow the steps in the lesson plan. Write the teaching learning activity in more detail on lesson plan, not brief notes.
• Representation of ideas	Allow students to cut their own rectangles from grid paper.
• Types of problems	Develop more challenging problems for students to determine the area of a trapezium.

Description of Lesson 1B

Ha taught Lesson 1B to a different class six days later on 20 October 2010. Using the revised lesson plan, Ha again segmented the lesson into three distinctive phases: the initial activity, the main activity, and the end activity.

In the first five minutes he asked students for the dimensions of a rectangle and the formula for finding its area, and explained the learning objective of the lesson. In the main activity of about 50 minutes he gave this task to the groups*: Cut rectangles on the grid paper into two parts of equal size and shape.* The students produced two congruent shapes in a range of ways, for example, rectangles, triangles, and trapeziums. The teacher asked the students to cut the rectangle on grid paper again into two trapeziums of equal areas. Students then counted the number of squares within the trapezium and size of the trapezium they had cut and compared it to the original rectangle. Using this task, the teacher guided students to find the area of a trapezium by counting the total number unit squares within the trapezium, and the size of trapezium. Then the teacher gave out a worksheet and each group worked on a different problem from the worksheet such as:

a) *Calculate the area of a rectangle on Figure A. Measure the size of the rectangle.*
b) *Calculate the area of trapezium on Figure B. Measure the size of the trapezium.*
c) *Compare the results of the area of a rectangle on Figure A and a trapezium on Figure B.*

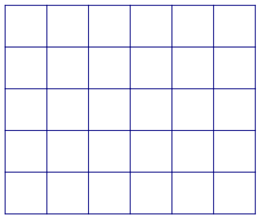


Figure: A

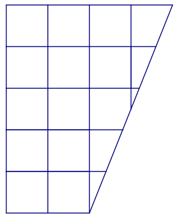


Figure: B

Students compared the area of a trapezium with the area of a corresponding rectangle. All groups presented their solutions by filling out a table on the white board. By looking at the patterns in the table, the teacher guided students to find the formula for the area of a trapezium. After the students and the teacher found the area of a trapezium the teacher gave students practice in solving the problems using a closed problem and an applied problem such as in Lesson 1A. Students answered individually for about ten minutes at the end of the lesson.

Lesson Study Group Meeting after Lesson 1B

The Group Meeting was held soon after Lesson 1B and all members attended the meeting. In general comments all participants noted that the teaching and learning in Lesson 1B were better than in Lesson 1A. A few comments were noted:

- Demonstration of procedures – Teaching and learning were better than in Lesson 1A. The participants stated that they had seen more students working to solve the problems in their groups, more discussion between students, and more students explaining the process to their fellow group members. They solved the problem and discussed in a group. There was more discussion and more explanation of the solution to their classmates in groups;

- Classroom management – Getting students to sit in groups at the beginning of the class made teaching easier to organise. The students looked like they were enjoying learning, and enjoying solving the problems successfully;

- Representation of ideas – Using a chart with an ink pen and paper in the same colour for writing and drawing made it difficult to read on the white board.

Based on the discussion, the group made suggestions for changes for the next lesson, as summarised in Table 4.3.

Table 4.3: Suggested Changes for Lesson 1B the Next Lesson.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Demonstration of procedures	Encourage students to communicate their solution process to check student understanding and to encourage them to use their own words.
• Representation of ideas	Use diagram or chart more clearly for the class.

At the end of the group discussions the group members decided that they were satisfied with the second lesson and that they should move on to a new topic in a different school for the next lesson. The second Group Meeting involved less discussion than the first Group Meeting.

A summary of the changes in pedagogical content knowledge based on the researcher's observation of the teacher in action in his classroom are shown in Table 4.4. For this table and those that follow a check (√) symbol indicates one observed incidence of the particular criterion for every five minute period of teacher activity and students working.

Table 4.4: Pedagogical Content Knowledge as Demonstrated in Teaching Practice (Lessons 1A and 1B).

Teacher Using Activity	**Student Working**							
	Individual		**Pair**		**Group**		**Class**	
	1A	**1B**	**1A**	**1B**	**1A**	**1B**	**1A**	**1B**
KNOWLEDGE OF TEACHING								
Representation of Ideas and Use of Resources								
Concrete	√	√			√	√√√√	√√	√√
Pictorial	√	√			√	√√	√√	√
Real world					√	√	√	√
Symbolic/abstract							√	√
Demonstration of Procedures								
Tasks/problems					√	√√√√		
Discussions					√√√	√√√√		
Student presentations					√	√	√√	√√√√
Teacher presentations					√	√	√√√√√	√√
Classroom Management								
Positive teacher statement						√	√√√	√√√
Negative teacher statement								
Encouraging students in difficulty					√√	√√√√	√	
Encouraging students to have different points of view							√	√
Encouraging students to participate					√√	√√		
Encouraging students to explain their reasoning								
Type of Problem								
Open						√√		
Closed	√√√	√√√			√√	√√	√√√√	√√√√
Drill	√	√			√	√	√	√
Applied	√	√			√	√	√	√
Puzzle problem								
KNOWLEDGE OF STUDENTS' LEARNING								
Level of Teacher Questioning								
Knowledge					√√√	√√√√	√√√	√√√√
Comprehension					√√√	√√√	√√√√√√	√√√√√√√√
Application								√√
Analysis								
Synthesis								
Evaluation								
Diagnosing Learning Difficulties								
Oral question					√	√√√√	√√√√√	√√√√√√
Written question					√	√		√
Written test	√√√	√√√					√√√	√√√

Change in Pedagogical Content Knowledge

Observations of Ha, the first volunteer teacher in the Lesson Study Cycle, show changes in some aspects of his teaching practice between Lesson 1A and 1B. With regard to pedagogical content knowledge in teaching mathematics using a problem solving approach, Ha changed over the two lessons. Changes were noted in the following:

- Representation of ideas – In the first lesson Ha used more pictorial representation (e.g. drawing a rectangle shape on the white board). However, in the second lesson he used more concrete representation (e.g. cutting rectangle paper model);

- Demonstration of procedures – He moved from a teacher-centred instructional approach in the first lesson to a more student-centred, active learning approach in the second lesson;

- Type of problems – In the first lesson Ha used primarily closed problems. However, in the second lesson he moved to using open problems;

- Level of teacher questioning – In the first lesson Ha used knowledge level questions. However, in the second lesson he used more comprehension and application level questions;

- Diagnosing learning difficulties – In the first lesson Ha mostly used oral questions to identify students' level of understanding. However, in the second lesson he used oral and more written questions.

The changes demonstrated in Ha's pedagogical content knowledge have been classified into knowledge of teaching, and knowledge of students' learning.

Knowledge of teaching

Ha demonstrated changes in his teaching over the two lessons – in the way he represented ideas, demonstrated procedures, and in the types of problems he chose to use.

1. Representation of ideas

Representation of the calculation of the area for a trapezium involved physical, pictorial and symbolic models in both the lessons. However, in Ha's second lesson there was more emphasis on concrete representation (through the use of hands-on paper models). In Lesson 1A he provided a figure of a rectangle and asked some questions about the rectangle and area. For example, he showed a piece of white paper as a model of a rectangle in the whole class. He asked: *What is the formula for area of a rectangle?* Together students answered: length multiplied by width. The students already knew how to calculate the area of a rectangle. Then he asked about: *What is the length of a rectangle? What is the width of a rectangle?* Next, he introduced the trapezium shape that came from dividing a rectangle on grid paper into equal parts. Through a step by step approach and teacher questions, he determined the formula for area of the trapezium in the symbolic form $A= \frac{1}{2} \times (a + b) \times h$. Apart from the initial paper model, the majority of the class used drawings on the whiteboard and symbolic representation through mathematical formulas.

He used more concrete representations in Lesson 1B where the students actually physically handled the paper. The students, in groups, cut some rectangles on grid paper, re arranged the shapes, and counted the squares. He used more grid paper to describe the area of a 2-D shape. He used a concrete model of a rectangle in grid paper and helped students to visualise and explore and to connect learning with their own experience. The students calculated the areas of trapeziums by counting the total of square units on the surface of 2-D shapes. Also he emphasised how to measure the side of a shape by counting the square units. Then he brought the students into the symbolic representation to determine the formula for calculating the area of a trapezium. Part of the activities went as follows:

Teacher : What part of the area of a rectangle is the area of trapezium?
Students: No answer.
Teacher : Look at the table, and fill in the table.

Each group filled in the table in white board similar to this:

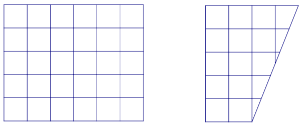
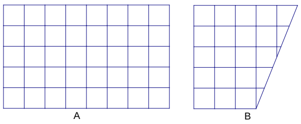
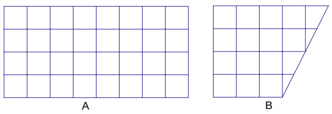
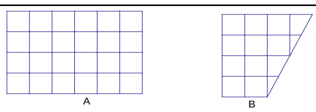
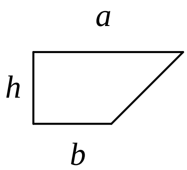
The area of a rectangle divided into two equal of the trapezium	*Top side of trapezium A*	*Base side of trapezium b*	*Height of trapezium h*	*The area of the trapezium*
 *area= 30 area=15*	*4*	*2*	*5*	*15 = ½ ×30 =½ ×(4+2)×5*
 *area = 40 area= 20*	*5*	*3*	*5*	*20= ½ × 40 =½ ×(5+3)×5*
 *area =32 area = 16*	*5*	*3*	*4*	*16 = ½ ×32 =½ ×(5+3)×4*
 *area =24 area= 12*	*4*	*2*	*4*	*12 = ½ × 24 = ½ ×(4+2)×4*

Figure 4.2: Results of group calculations of area.

The teacher drew a trapezium on the whiteboard and marked its sides.



Then, on the whiteboard the teacher wrote the area of trapezium = ½ × l × w
= ½ × (……..) × h
= ½ × (……..) × 5 = 20.

Teacher : What is a?
Students : 5, then the teacher wrote "5" on the whiteboard.
Teacher : What is b?
Students : 3, then the teacher wrote "3" on the whiteboard.
Teacher : What is the operation so the mathematical statement is correct?
Students : Addition.
Teacher: The area of trapezium = ½ × (5 + 3) × 5 = 20 so in general the area of trapezium is ½ × (a + b) × h.

He commented that before he participated in Lesson Study he only used teaching aids or manipulative materials available at the school, but he seldom used the manipulatives as teaching aids because he needed extra time to prepare these materials. He used manipulatives that were easily available in the class room situation only. He explained:

> *Manipulative and teaching materials are available at school, but I need extra time to use them. I tend to use manipulatives that are available in the class. For example in teaching geometry, I show 3-D objects in the class, and tell the students. After Lesson Study with problem solving, now it is not just the teacher who shows 3-D objects, but students must also show 3-D objects and describe the components of the 3-D object. Students measure, compare, and calculate the volume of 3-D object.* (Interview, Ha, 23 December 2011)

2. Demonstration of procedures

Ha moved from a teacher-centred presentation approach in the first lesson to a more student-centred active learning approach in the second lesson. In Lesson 1A he tended to dominate classroom communication by explaining procedures, demonstrating examples, and guiding questions and answers in a whole class setting. The students listened and answered the teacher questions orally. He presented and explained step by step how to find the formula for the area of a trapezium. While he was writing on the white board he asked questions to the whole class. At the end the students worked on the teacher's method to get the formula for calculating the area of a trapezium.

Group discussion was emphasised more in Lesson 1B. The groups were set at the beginning of the lesson. He gave each group a problem to solve. While students were working in groups, the teacher observed, walked around the groups, and helped students having difficulty. Students in the groups were instructed to cut the rectangle to create two congruent shapes. He did less presentation in Lesson 1B and he paid more attention to student presentations of their group's work.

In Lesson 1A Ha tended to demonstrate the process or procedure and the students listened and answered. Students solved the problems based on an example of one way to find the solution. They tended to look for a formula and looked at the text book when doing the exercises. He explained:

> *I demonstrate the concepts and the students listen. They solve the problem following the teacher's example and the students have a lack of skill such as solving the problem.* (Interview, Ha, 23 December 2011)

After the Lesson Study Cycle with a problem solving approach, he explained:

> *Students demonstrate solving the problem in their own way and I pay attention on what students are doing with manipulatives and they improve their skills in solving on their own.* (Interview, Ha, 23 December 2011)

This is a clear example of how Ha's teaching approach became more student centred after Lesson Study meetings.

3. Types of problems

Closed problems were given by the teacher in both lessons as the problem example for students' tasks, and exercises. Ha moved from using mostly closed problems in Lesson 1A to also using open problems in Lesson 1B.

In Lesson 1A he drew a rectangle that could be divided into two equal sized trapeziums and he gave a closed problem: *Find the area of the trapezium.* Through a step by step instruction the teacher used the problem to show a procedure to determine the formula for calculating the area of a trapezium. The students followed the teacher's explanation and the new procedure for calculating the area of a trapezium was found. Then he gave the students problems in which to apply the formula for calculating the area of trapezium. He gave closed problems as exercises in the same format as the problem examples.

In Lesson 1B Ha started by giving an open problem in groups: *Divide a rectangle into two equal parts in more than one way.* He emphasised student exploration and that they could reach the solution in more than one way. The students found solutions to the problem by using grid paper models and discussion. Then he gave closed problems to groups (with different problems to each group) to calculate the areas of rectangles and trapeziums. The teacher gave the students a table to complete and the students compared the area of a trapezium and a rectangle for each problem. By looking at the pattern in the table, the formula for finding the area of a trapezium was found by the students. After the formula was found, he gave the students exercise problems to solve by applying the same formula. This supports the comment made in the previous section that Ha's teaching became less teacher directed and that he allowed students to take more control of their learning, following the Lesson Study meetings.

Before Lesson Study in his teaching practice generally he taught mathematics using the problems based on the textbook, where most problems were closed problems and drill exercises. Some changes happened after the Lesson Study meeting. For example, he modified one original task: *Determine the perimeter of a rectangle on the diagram* to become: *Using a string, calculate the perimeter of a table*. While this change may not involve using a completely open problem, it does represent a change in Ha's thinking and teaching.

Knowledge of students' learning

In his knowledge of students' learning Ha demonstrated changes in his teaching over the two lessons with regard to using different levels of teacher questioning, and diagnosing learning difficulties.

1. Level of teacher questioning

In Ha's second lesson he used more questions, more questions at different levels, and more comprehension and application level questions. In Lesson 1A he mainly asked knowledge questions about what the students had already learned: *Have you remembered the formula for area of a rectangle?* The students answered together: *Yes the area of a rectangle was length multiplied by width.* In Lesson 1B he asked more questions to guide students in comprehending the process of calculating a trapezium's area, such as: *How long is the length of the two parallel sides of the trapezium?* He directed more questions in groups in Lesson 1B compared to 1A - 4 knowledge and 3 comprehension questions compared to 3 knowledge-recall questions and 3 comprehension questions. Also he directed more questions to the whole class in Lesson 1B compared to 1A - 4 knowledge and 8 comprehension questions compared to 3 and 6. Also he asked 2 application questions. His questioning tended to use lower level questions to check for students' recall and descriptions of correct facts or procedures such as students remembering the formula for area of a rectangle. He used application questions to see whether or not students could think more deeply and show understanding of the procedures. It could be said that this changes in his questioning were in response to discussion during Lesson Study meetings.

2. Diagnosing learning difficulties

In Lesson 1A Ha used only oral questions to identify students' difficulties, whereas in the second lesson he used written questions as well. In both classes the teacher asked questions to check students' understanding. For example, the teacher showed a rectangle and asked the students: *What is the difference between the length and the width of a rectangle?* The students did not respond. They understood the length and the width of a rectangle, but they did not understand the difference between them. The teacher asked: *Why is it called length? Can you distinguish between length and width?* No students responded. Ha responded by showing the whole class a rectangle model. By rotating the rectangle model with the length in a horizontal position, vertical position, and any position, he showed the students the meaning of "length". In the end they understood the length was longer than width.

In Lesson 1B Ha identified students having difficulties, not only using oral questions, but also in written questions, through group discussion and by observing the students in groupwork. For example, he observed that one group had difficulty in working out the area of a shape on their worksheet by counting the grid squares. Then he helped students in their groups to calculate the total number of unit squares to determine the area of the trapezium. When students were having difficulty in transferring the dimensions of a trapezium from a pictorial model to symbolic level to fill in the table he showed them how to count the squares along the side of the trapezium figure.

In the first lesson Ha had paid little attention to students who had difficulties. If any student made a mistake he said that it would lead to failure and he did not follow up the difficulty. He paid no attention to students' misconceptions. But after the Lesson Study program he paid more attention to students who had difficulties. Through observing students talking and working, he could identify their misconceptions. When he saw students making mistakes he asked other students to provide guidance rather than directly explaining the solution himself. As he explained:

> *I ask students why something is incorrect, and then I come back with the question to give the question to the other students in the class or in groups.* (Interview, Ha, 23 December 2011)

Initially, assessment of student learning was based on the final score on a test from the student worksheet. It was different after the Lesson Study program, as he explained:

> *Even though the students' final scores have not reached the target that I had set, I know their thinking processes have improved. This is because the lesson is improved to have different ways of students showing their thinking. We are not only looking at the final scores.* (Interview, Ha, 23 December 2011)

These changes occurred following the Lesson Study meeting and reflect changes in how Ha assessed student learning.

Change in Mathematical Content Knowledge

The mathematical content knowledge shown by Ha changed somewhat over the two lessons. Changes were noted in the following:

- Understanding of procedures – In the first lesson Ha used more algorithms to introduce the new concept or procedure. However, in the second lesson he moved to begin with problems;
- Understanding of mathematical relationships – In the first lesson Ha divided a rectangle into two congruent trapeziums to introduce the area of

trapeziums. However, in the second lesson he moved to divide a rectangle into two congruent shapes and required students to use more than one shape.

1. Understanding procedures

Ha moved from demonstrating an algorithmic understanding of the process for finding the area of a trapezium to creating a setting for students to find the process through their own exploration. In Lesson 1A with Ha guiding the class step by step, the area of a trapezium was derived from the formula for calculating the area of a rectangle: length × width. He demonstrated that the width of a rectangle is the same as the height of a trapezium, and that the length of a rectangle is the sum of horizontal sides of a trapezium (the trapezium is a half part of the rectangle). He demonstrated competence in explaining the procedures. At the end of the lesson he proposed the new formula for calculating the area of a trapezium:

Area = ½ × (a + b) × h,

where a = length of the top side of the trapezium, b = length of the base side of the trapezium, and h = height of the trapezium.

In Lesson 1B Ha began with giving the class an open problem: *Divide a rectangle into two equal parts.* This activity was set up to introduce the concept of the area of a trapezium. Through the principle that the area of a trapezium is a half the area of an original rectangle, he set problems comparing the area of a trapezium and the area of a rectangle, to lead up to development of the formula for finding the area of a trapezium. He knew that the formula for the area of a trapezium can be illustrated by dividing a rectangle into two equal right angled trapeziums. In addition, he understood that the area of a right-angled trapezium is calculated in the same way as the area of any other trapezium. Through exploration and discussion he conjectured that the area of the trapezium was the average length of the parallel sides times the perpendicular distance between them, is ½ × (a + b) × h. In doing so, Ha demonstrated development of his mathematical content knowledge. While he originally knew the formula or procedure for finding the area of a trapezium, he now knew how to demonstrate a way of deriving the formula. He applied this to his teaching in order to better engage the students.

2. Understanding of mathematical relationships

Ha’s teaching moved from dividing a rectangle into equal parts with one given shape to using more shapes. He knew the definition of a trapezium was a quadrilateral that has a pair of parallel lines, and a rectangle is a quadrilateral with all right angles. He described this by representing the relationship between concrete representations (dividing a shape) and the properties of rectangle and a right-angled trapezium. In Lesson 1A he divided the rectangle into two equal parts, each a right-angled trapezium. However, he used a rectangle shape of grid paper that was divided into two equal parts to see the relationship between, and properties of a rectangle and a right angled trapezium.

In Lesson 1B he offered an open problem to the students to divide a rectangle into equal parts that became two right angled triangles, two rectangles, or two right-angled trapeziums. He knew any rectangle can be divided into two equal parts with right angled triangles, rectangles, or right angled trapeziums. This meant the area of a trapezium is a half of the area of an original rectangle. He showed students that he understood the properties of a right-angled triangle and a right angled trapezium are related to properties of a rectangle.

In the interview Ha explained about his mathematical content knowledge before and after the Lesson Study program. He explained:

> *I am confident about mathematics content for teaching and more confident after Lesson Study because of receiving input and sharing with other teachers.* (Interview, Ha, 23 December 2011)

His ability to demonstrate his own mathematical content knowledge was shown by the way he changed from a procedural approach in Lesson 1A to a more conceptual approach in Lesson 1B. He also gave an alternative solution and suggestions to the other colleagues who had difficulty with teaching in a group after the Lesson Study meeting. Again, there is a clear indication that Ha's teaching, this time with respect to his mathematical content knowledge, changed after the Lesson Study meetings.

Beliefs about Teaching and Learning Mathematics

According to what Ha said in his interview before the Lesson Study program, he already believed that mathematics should be taught using multiple methods, and that students should be active in their learning. Before Lesson Study began, he explained:

> *Problem solving is very important because it improves students' logical thinking processes so the students have a more valuable learning experience.* (Interview, Ha, 20 September 2010)

Classroom observations of Ha's teaching practice during the Lesson Study program showed that he taught the concepts differently in the two lessons. Exercise problems were presented after the teacher demonstrated finding the formula in Lesson 1A, but open problems were presented to help the students discover the new formula in Lesson 1B.

In Lesson 1A after explaining how to find the formula he gave the children an applied problem where they applied the formula to solve the problem. His lesson was mostly based on teacher explanation, and teacher-centred question and answer sessions with the students.

At the beginning of Lesson 1B the students were already in groups working on the open problem. Based on students' answers and contributions, he guided the students to construct the new formula. Ha presented different problems and different solutions for each group to introduce the new concept. Students were involved with paper models. He was interested in offering individual and group feedback. He was managing the class activity rather than explaining. This is shown in Table 4.4 where there is less time spent on teacher presentation in Lesson 1B than Lesson 1A. He allowed student doing activities in Lesson 1B compared to Lesson 1A – 4 group tasks to 1 group task, 4 class students' presentations to 2 class students' presentations, and 4 discussions in groups to 2 discussions in groups.

In Lesson 1A the students looked at the teacher using a hands-on teaching aid and pictures to explain the solution because of the limited number of hands-on teaching aids. In Lesson 1B the students used paper models to solve the problems. They counted square units and were cutting shapes. Students were more active and Ha had more time to observe, but more time was also needed for the activity.

At the conclusion of the Lesson Study program Ha stated the belief:

> *To get success in teaching and learning, the teacher should not use just a single method but various methods, and this should involve students being creative.* (Interview, Ha, 10 December 2010)

One year after the Lesson Study program Ha was again asked about his beliefs about teaching mathematics through problem solving. He described them as being the same as at the end of the Lesson Study program. He explained:

> *With a problem solving approach, students are doing a mathematics activity do not simply memorise a formula. With the old approach, if a student forgot the formula they were stuck - they would not go on. With a problem solving approach, if students forget the formula, they recall what they have done, and imagine what they did with the hands-on model. They can find the solution of the perimeter of a rectangle without memorizing the formula*. (Interview, Ha, 23 December 2011)

While Ha had already realised the value of problem solving, his comments after the Lesson Study program show that he then also believed how a problem solving approach could replace the need for learning procedures as students could work them out. It could be said that this change resulted from his reflections for following Lesson Study.

Summary

Ha changed his methods of teaching mathematics using a problem solving approach from the first lesson to the second lesson. The way he taught was possibly based on increased pedagogical content knowledge.

- He changed the emphasis from pictorial to concrete representation in the second lesson;

- His teaching changed from teacher centred to student active learning;
- He used different types of problems, not just closed problem but also open problems;
- He used more questions and more questions of different levels and more comprehension and application level questions;
- His diagnosis of learning difficulties changed from not only using oral questions but also using written questions.

In his mathematical knowledge he displayed greater understanding of procedures, in understanding algorithms, and using exploration of mathematical procedures.

His beliefs about teaching and learning mathematics in his interviews before and after Lesson Study did not change, regarding the values of problem solving, but they became more expansive. He believed that mathematics should be taught using multiple methods and those students should learn actively. He explained that with a problem solving approach students learned mathematics with an emphasis on meaning and the process, not only on memorising. In a problem solving approach if students forgot the formula, they recalled what they had done, or imagined and looked at what they had done with manipulatives. Following Lesson Study his beliefs expanded and his teaching, as described earlier, changed to reflect those beliefs.

Case 2: Lesson 2A and 2B by Teacher Wi

Wi is a female teacher who teaches Year Four (students aged 10) at SD 1P Bin located in a suburban area. Her class has 40 students who come from the neighbourhood of the school. At the time of the study she had 11 years teaching experience in primary schools as a classroom teacher and she held a bachelor's degree in primary school teaching.

At first, when teaching mathematics, she did not use a problem solving approach because she lacked knowledge about this method. She taught mathematics according to the textbook with a traditional approach of teacher explanation, giving examples, and then having the students doing exercises. She did not have any experience in personal professional learning in mathematics, and Lesson Study was new for her. During the Lesson Study program she became a volunteer teacher, teaching the same topic twice with classes at Year Four level. Both the lessons were video-recorded, audio recorded and observed by fellow participants. The first lesson with the original lesson plan is called here Lesson 2A, and Lesson 2B is the revised version of the first lesson. The lessons focussed on measuring an angle using non-standard units with a fixed angle as a unit, and using standard units in degrees.

Description of Lesson 2A

On 28 October 2010 Wi taught the lesson for the first time (Lesson 2A). She segmented the lesson into three distinctive phases: the initial activity, the main activity, and the end activity. The students sat in groups of five. At the beginning of the lesson the teacher presented the objective of the lesson: S*tudents are able to measure an angle with non-standard units*. She asked the students to give examples of real-life angles by looking around the classroom. Students gave examples such as the corner of a table. Most students were able to give correct responses orally.

She drew a point on the white board with three rays originating from it as in the Figure 4.3 below. She asked the students: *How many angles do you see?*

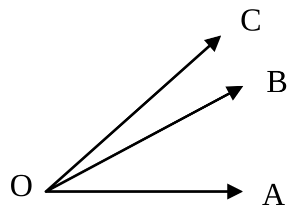


Figure 4.3: Two adjacent angles.

The students became confused about the diagram. She then asked the students: *What type of angle is this*? The students knew this as an acute angle. Then she labelled the diagram with capital letters and she asked students: *What is the name of this angle*? Students answered her questions orally but not all students could give the name of the angle. Then she explained how to describe the name of an angle by using points at the apex and at the ends of the rays. She asked the students to give all possible names of angles on the diagram.

Before she started the main activity she asked students if they could name the three types of angle: acute angles, right angles, and obtuse angles. After students understood the three types of angle she distributed two circles of coloured paper to each group. By using the rotation of the two papers model she compared the two angles.

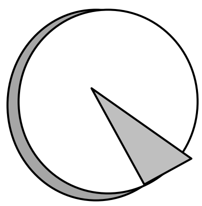


Figure 4.4: Two circles of paper model.

At the front of the class two students demonstrated how to construct an angle by rotating the paper model. The other students compared the two angles made by the two students. The students said whether it was bigger or it was smaller. This was then followed by all the students working in groups where they constructed two angles using the rotation of the paper model and compared them as shown in the teacher demonstration. All students participated actively.

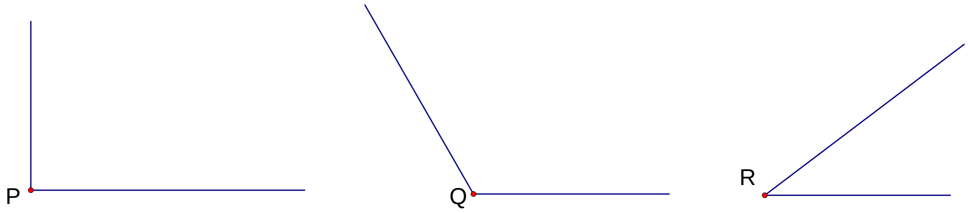
For the next step she explained measuring angles with non-standard units. Then she showed how to measure an angle with non-standard units. By using a fixed acute angle as a non-standard unit angle she demonstrated how to measure and compare the larger angle. A non-standard unit is a fixed small angle that was made by the teacher to measure the big angle. The students watched the teacher measure the angle using the non-standard unit measurement instrument.

She then distributed worksheets and asked students working in groups to measure the angle with non-standard units and give the name of the angle. During group discussion the teacher observed and helped students with difficulties. Some students were having difficulties in giving correct alternative name of angle, so that $\angle$JKL became $\angle$KJL not $\angle$LKJ. The right name should have the letter at the vertex placed in the middle of the sequence. Representative students from each group presented their answers at the front of the class by writing on the white board or reading out group results.

By opening the door in front of the class Wi went to show another real-life example of an angle. She asked students to find out where the angle was constructed and one of the students traced the angle that was constructed between the door panel and the wall. The other students looked at the angle drawn and said the type of angle. The students measured the door angle with a non-standard unit angle. She continued to ask students to find other angles around the classroom such as the corner of the white board, and the corner of a table, and measure them.

In the final activity she summarised the lesson to make sure students understood the concepts of the lesson. She drew adjacent angles and the students counted the total number of angles and gave the names of the angles. Then the teacher gave the students closed problems (task 1) and drill exercise (task 2) problems for individuals task on work sheets which were to be continued as homework.

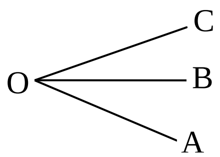
Task 1: Observe the angle on the picture below:

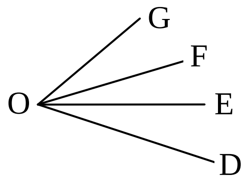


Based on the figures above, answer the following questions

a) *Which one is greater, angle P or angle Q?*
b) *Which one is greater angle Q or angle R?*
c) *Write all three angles in order from the largest*

Task 2: How many angles do you see? Give the name for each angle!





a) *Name of each angle.……* b) *Name of each angle.….*

Lesson Study Group Meeting after Lesson2A

The Group Meeting was held on 28 October 2010 soon after Lesson 2A and all group members attended the meeting. The group made the following comments:

- Demonstration of procedures – The students were confused when asked to count the number of angles on adjacent angles where two or more angles have the same vertex of angles;

- Representation of ideas – The use of the door and wall as a real-life example of an angle could not be seen by all students in the class, especially those at the back;

- Demonstration of knowledge – When demonstrating measuring angles using the non-standard unit the teacher confused the terms “unit angle” and “angle unit”. She was inconsistent in the way she wrote the names of the angles by using three capital letters (e.g. ∠ ABC) sometimes, and only one capital letter (e.g., ∠A);

Based on the group discussion and comments the suggested changes for the next lesson are summarised in Table 4.5.

Table 4.5: Suggested Changes for Lesson 2B.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Representation of ideas	Use concrete model of an angle, use something that can be seen by the whole class. Provide manipulatives and more protractors to measure angles for each group.
• Demonstration of procedures	Put a mark on an angle that has been counted on the white board so it is easy to keep track.
Mathematical content knowledge • Demonstrate knowledge	Explain that each angle has two alternative names, and that each name should have three capital letters. Use definition for understanding an angle.

Description of Lesson 2B

Lesson 2B was taught to a different class seven days after Lesson 2A. Wi segmented the lesson into three distinctive phases: the initial activity, the main activity, and the end activity. In the initial activity she asked students to give an example of a real-life angle in the classroom. The students were responsive right away with answers such as: the corner of the white board, and the corner of the table. The teacher asked the students to define an angle but no student was able to respond and there was silence. Then by using more specifically a piece of paper folded into a circle, she showed an example of an angle, then by drawing an angle on the white board she gave a definition of an angle as the intersection of two rays in one point and showed the elements of the angle, point and ray. The students followed the teacher's explanation. She asked students to identify the angle by names.

In the next activities she introduced students to the three different types of angles. By folding a paper circle she constructed a right angle that was followed by students constructing their own angles. The teacher distributed pictures of different angles on worksheets to the groups. Then, using a right angle constructed from folded paper, the students compared this right angle to the angles she gave them to find out whether the angles were acute, obtuse or right angles. The teacher guided the students to find the acute angles and the obtuse angles.

She introduced a protractor as a standard tool for measuring angles. She showed how to read the protractor and the meaning of a “degree” such as 1 degree, and 30 degrees. Then she used a hand clock model to show that the angle of a full rotation is 360 degrees. She guided the students to rotate the hands to show angles of 30°, 60°, and 90°. The students followed her explanation and they tried to move the hand of the clock so that every step was 30°.

Wi returned to use the protractor as a tool to measure an angle. She demonstrated how to measure an angle by drawing an angle on the white board and measuring it. The students followed her explanation. Then the students measured the angles on worksheets using a protractor. During group discussion the teacher observed the groups and helped students having difficulties. After finishing group work the students presented their results at the front of the class. One student from each group read the group’s results and filled in the results on the table on the white board. The teacher checked the students’ results. After collecting the students’ worksheets she reviewed the lesson with the class.

Lesson Study Group Meeting after Lesson 2B

The Group Meeting was held after Lesson 2B and all members attended the meeting. A few comments were made, as follows:

- In demonstration of procedure – The teacher taught many sub topics in a short time such as: introducing the concept of angles, comparing angles, measuring angles, but for each topic the procedure was not developed much;
- In classroom management – Students did not appear to be motivated and did not engage with the lesson. The teacher spent more time giving explanations and less time on the student activity.

Based on the group discussion and comments the suggestions for the next lesson are summarised in Table 4.6.

Table 4.6: Suggested Changes for the Next Lesson.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Demonstration of procedures	Develop problems that allow for more student activity for each topic.
• Classroom management	Improve students' motivation so that all students are motivated and engaged.

The suggestions for the next lesson were for the mathematical content to be taught with fewer sub topics and with more student activity for each topic. The Group Meeting decided however to change the topic and school for the next lesson.

A summary of changes in pedagogical content knowledge in practice based on researcher observation is shown in the Table 4.7. For this table and those that follow, a √ symbol indicates at least one observed incident of the particular criterion for every five minute period of teacher activity and student working.

Table 4.7: Pedagogical Content Knowledge as Demonstrate in Teaching Practice (Lesson 2A and 2B).

Teacher Using Activity	Student Working							
	Individual		Pair		Group		Class	
	2A	2B	2A	2B	2A	2B	2A	2B
KNOWLEDGE OF TEACHING								
Representation of Ideas and Use of Resource								
Concrete					√√√	√	√	√√√
Pictorial					√√	√	√√√	√√√
Real world							√	√
Symbolic/abstract							√	√
Demonstration of Procedure								
Tasks/problems					√√√√	√√	√	
Discussions					√√√√ √√	√√√		
Student presentations							√√√√√	√√√√
Teacher presentations					√√	√√√	√√√	√√√√√
Classroom Management								
Positive teacher statement							√√√√√	√√
Negative teacher statement								
Encouraging students in difficulty					√√√			
Encouraging students to have different points of view					√	√	√	√√
Encouraging students to participate					√		√	
Encouraging students to explain their reasoning								
Type of Problem								
Open	√	√			√		√√√	√
Closed	√	√			√√√	√√	√√	√√√√
Drill	√	√			√√	√√	√√	√√√√
Applied					√	√	√	√
Puzzle problem								
KNOWLEDGE OF STUDENTS' LEARNING								
Level of Teacher Questioning								
Knowledge					√√√	√√√	√√√	√√√√
Comprehension					√√√	√√√ √	√√√√√√	√√√√√√ √
Application								√
Analysis								
Synthesis								
Evaluation								
Diagnosing Learning Difficulties								
Oral question					√	√√	√	√√
Written question					√	√	√	√
Written test	√√	√√					√√	√√

Change in Pedagogical Content Knowledge

Based on the observation of Wi's teaching practice over two lessons, changes in pedagogical content knowledge were noted in the following:

- Representation of ideas – In the first lesson Wi used a simple concrete representation. However, in the second lesson she used a concrete representation by using a numerical scale for measuring angles;

- Demonstration of procedures – Wi changed from a student centred teaching approach in the first lesson to the use of more teacher explanation in the second lesson;

- Level of teacher questioning – In the first lesson Wi asked students knowledge level questions. However, she changed to asking students more comprehension and higher level questions in the second lesson;

- Diagnosing learning difficulties – In the first lesson Wi used written questions to diagnose learning difficulty. However, in the second lesson she used written questions and more oral questions at different levels.

These aspects of pedagogical content knowledge can be classified into knowledge of teaching and dealing with students' difficulties.

Knowledge of teaching

Over Wi's two lessons for Lesson Study she demonstrated changes in how she represented ideas, and demonstrated procedures.

1. Representation of ideas

In both the lessons Wi involved various forms of representation to build students' concepts of angles. To introduce the idea of an angle she used physical objects around the classroom and pictorial objects (drawing on the white board). She asked the students, *Which of the objects has an angle in it*? Then by using the symbol of an angle (∠) she introduced the method of naming angles. The students already knew the conventions of angles, and of naming an angle.

In Lesson 2A, Wi used a rotation model of two papers in different colours to introduce the concept of a smaller angle and a bigger angle. The rotation model was constructed from two circles of paper of the same size but in different colours with the same centre point. In pairs, students constructed models with different angles. The students compared the two angles as a dynamic model several times and they began to understand the concept of smaller angles and bigger angles. The teacher then introduced the terms acute and obtuse angles and drew some examples of each. The students showed examples of acute or obtuse using the circular paper model. Then the teacher explained to the class about the angle on the white board, and explained to the class what the terms “acute angle” and “obtuse angle” meant.

The concept of measuring angles was introduced by measuring an angle with non-standard units in which she folded a paper circle to construct a model of an angle. Then she demonstrated to the students how to measure another angle by placing the first angle model into it, and comparing the two. The students then measured some other angles by using a fixed angle model.

In Lesson 2B she used a more complex numerical representation for measuring angle. Wi began to introduce the concept of acute angles and obtuse angles with the use of a right angle made of paper. She demonstrated how to make a right angle by using paper folding and she explained that a right angle is an angle of 90^0. Then, this right angle construction was used by the students to compare with other angles. Finally, the students understood the concept: if the angle is smaller than a right angle, it is called an acute angle. If it is bigger, it is called an obtuse angle. The teacher used a right angle as a standard to compare with the other angles.

Next she showed the students a protractor and described it as a tool to measure an angle in standard units. Presenting to the whole class she introduced what a protractor was, what it was used for, how to read the scale, and how to use it. Then, she used a hand clock model to show that the angle of a full rotation is 360^0, and then she guided the students to rotate the hands to show angles of 30^0, 60^0, 90^0. The students followed her explanation, and then they tried to move the hands of the clock so that every step was 30^0. This was in direct response to suggestions made at the Group Meeting, which included the need to provide more manipulatives and protractors for groups to use.

In both lessons she demonstrated and explained to introduce the procedures with teaching aids. Then the students solved exercise problems following the same procedures. In Wi's interview before the Lesson Study program she stated that she only used teaching materials and hands-on teaching aids available at the school. It was different after the Lesson Study program - she not only used teaching aids and manipulatives available at the school, but she also created her own manipulatives such as, two circular papers model to show an angle rotation (Figure 4.4 on page 114). Her use of manipulatives was influenced by the Lesson Study meeting which stated that using manipulatives helped to make students active in their learning.

2. Demonstration of procedures

Demonstration of procedures changed from being student-centred to a greater use of teacher explanation. She gave tasks more in a group in Lesson 2A than Lesson 2B. In Lesson 2B she presented to the whole class setting. In the beginning of Lesson 2A, Wi introduced an angle by asking students and setting problems. For example, the students worked in groups to solve the open problem in Figure 4.3 above, *How many angles do you see?* Students had differing answers. She provided guidance (to students in their groups) to help them answer the problem. In the next activity, she had students working in pairs, comparing the size of two angles using a paper model. Two students in each group made angles and the other students compared them. She observed the group work and asked questions to students in their groups.

It was different in Lesson 2B where she introduced mathematical concepts in a traditional teacher explanation style. The students paid attention and answered teacher questions. In this class she introduced a protractor and a hand clock model and showed the students how to use them to measure angles.

Before the Lesson Study program she taught mathematics only by explanation methods with the concepts based on the text book. As she explained:

> *I explained the concepts, gave an example, then students would do exercises in the text books.* (Interview Wi, 21 December 2011)

Students wrote the results on the white board and then the teacher corrected the results. After the Lesson Study Cycle, she taught using a problem solving approach for very limited topics that she learned during her participation in the Lesson Study program. She explained that she taught the topic of measurement using a problem solving approach. She gave an example of how she modified a text book problem, which originally stated: *Find the result 8 kilometres – 4000 metres = ….* It was modified to become a story problem: *The length of a broken road, some of which has been repaired is 8 kilometres. So far 4000 metres has been repaired. How long is the road still under repair?* The teacher modified drill exercises which contained only numerical problems to be become story problems. She explained that when she used a problem solving approach in her teaching practices:

> *Students were actively learning doing mathematics or solving the problem in group then I guided the students on problem solving.* (Interview Wi, 21 December 2011)

These changes occurred following her reflection about teaching during Lesson Study meetings.

Knowledge of students' learning

Over Wi's two lessons for Lesson Study she demonstrated changes in the level of questions she asked, and how she diagnosed students' learning difficulties.

1. Level of teacher questioning

In the first lesson Wi asked students knowledge level questions and in the second lesson Wi used more questions, both knowledge level questions, and comprehension and application level questions. In both of the lessons the teacher used both lower-order knowledge recall questions as well as comprehension questions. In Lesson 2A the teacher asked the students questions in whole-class and group settings. Generally, she asked the students questions to guide their thinking, and to gain students' attention, both in the group and whole-class situations. But in Lesson 2B questions were mostly used by the teacher in the whole class setting. In both lessons she very seldom used higher order questions. Only in Lesson 2B was it noted that she used one higher order question in asking students: *Why is it called an obtuse angle?* A few students were able to offer the right answer. She used more questions in the whole class setting in Lesson 2B (4 knowledge and 7 comprehension questions) compared to Lesson 2A (3 knowledge and 5 comprehension questions). She asked slightly more questions to students working in groups in Lesson 2B (3 knowledge and 4 comprehension questions) compared to Lesson 2A (3 knowledge and 3 comprehension questions). Also she used 1 application question in Lesson 2B. So she started to use slightly more questions and higher order questions. An improved level of teacher questioning occurred in the second lesson following the Lesson Study Group Meeting.

2. Diagnosing learning difficulties

In the first lesson Wi mostly relied on written work to assess students' learning difficulties. By the second lesson she started to assess students' knowledge by the use of oral questions. She checked how well students understood the new concept. In Lesson 2A written tasks were given to the whole class with questions such as: *Give an example of things which have angles in them; Give the names of the angles in the figure as shown; and Count the total number of angles in the figure as shown.* She found that some students had difficulties in giving the name of an angle and in counting the number of angles. She also found that some students used three capital letters but not in the right order, such as when $\angle$AOB was written as $\angle$OBA. The teacher drew another angle ($\angle$ABC) on the white board and explained again how to give the name of the angle: ABC. The teacher emphasised that when giving the name of an angle the middle letter was the name of the vertex of the angle and the other letters were the names of the sides of the angle. This was in direct response to suggestions made in the Lesson Study meeting, which were that the explanation and demonstration of naming angles needed to be clearer.

Diagnosis of learning difficulties changed from using written questions to oral questions at different levels, such as asking students: W*hat is the name of an angle*? She not only identified students' difficulties by asking questions but also by defining the concept. She used tasks, questions, and problems. In Lesson 2A the tasks were given to the whole class with written tasks such as: *Show the things that have angles. Give the name of angle in the figure. Count the total angles in the figure.* She found that some students had difficulties in giving the name of an angle and counting the number of angles. The teacher re-explained by drawing and asking questions orally. She also found the students had difficulty in counting the total number of angles when they had the same vertex. The teacher demonstrated counting the different angles, naming each one as she did so.

Similarly, in Lesson 2B she used questions, tasks and problems to build student understanding of the concept. The students understood the idea of angles better then when they had studied the topic previously. When she asked students to define an angle at the start of the lesson no students answered. Then she drew an angle on the white board and she defined an angle as two rays that intersect in one point. She asked questions about the elements of an angle, such as: *What is the name of an angle drawn on the white board?* Most of the students answered correctly.

Wi said that before the Lesson Study program, if any students had difficulty with a concept, she would give an explanation again to try to overcome the students' difficulty. She assessed student learning by the final scores on their written work. But after Lesson Study program she identified student difficulties and tried to find a different teaching approach for that topic to help the students overcome their difficulties. As she explained:

> *Traditionally to diagnose student difficulty, I used the format of re-explanation of the concept, but after Lesson Study program to diagnose student difficulty I identify what students have difficulty with and I try to find the teaching methods to help the students understand.* (Interview, Wi, 21 December 2011)

Another change she made after the Lesson Study program was that she not only used written tests to assess students' level of understanding, but she also observed the process of students doing mathematics when students worked in groups. These changes could be said to be a direct consequence of Wi reflecting on her teaching following the Lesson Study meetings.

Change in Mathematical Content Knowledge

Wi demonstrated some changes in her teaching practice over the two lessons, which possibly reflected some changes in her mathematical content knowledge.

- Understanding mathematical relationships – Wi knew about sizes of angles and how to measure them and she demonstrated this in different ways during Lesson 2A and Lesson 2B;

- Demonstrating knowledge – Wi gave different definitions of angles in order to help children understand the concept and this reflected a growing level of mathematical content knowledge.

1. Understanding of mathematical relationships

Wi demonstrated knowledge of properties of angles. In Lesson 2A she described the properties of angles, such as comparing angles, and measuring angles. She demonstrated how to count the total angles that have the same vertex (adjacent angle). She described that the total angle in Figure 4.3 were 3 angles ($\angle$AOB, $\angle$AOC, and $\angle$BOC) and 3 names of angles ($\angle$AOB, $\angle$AOC, and $\angle$BOC).

In Lesson 2B Wi introduced a range of ideas to help students understand the concept of angle. For example, she introduced an acute angle and an obtuse angle by using a right angle paper throughout comparison. Next she introduced a protractor for measuring angle and showed how to use it. Then she used a dynamic hand clock model to show angles.

2. Demonstration of knowledge

Wi demonstrated her knowledge of angle by using different models in Lesson 2A and Lesson 2B. She introduced an angle by using examples of a real object and pictorial model of angles in both the lessons. Her examples could be classified into dynamic and static ideas of angles. The dynamic view is an angle as the amount of turning, such as using rotation in the two papers model and using a hand clock model. The static view is when an angle as is shown as two straight lines meeting each other. For example, the teacher drew an angle on the white board. Also, she introduced an "angle unit" (a paper model she had made) as non-standard unit for measuring other angle.

In Lesson 2B she introduced an angle not only using examples, but also giving a definition. She defined an angle *as an intersection of two rays that meet at one point.* Then she drew an angle to show the elements of an angle, side of angle, and vertex of an angle. In terms of giving examples she demonstrated an example of an angle by opening a book. This supported and reflected her understanding of an angle being *an amount of turning,* which is shown by her use of the two dynamic models (clock face and paper circles).

Wi's demonstrated mathematical content knowledge changed during and after the completion of the Lesson Study Cycle. She stated before the Lesson Study program she had limited understanding of the concept of angles. Following the Lesson Study Group Meeting she demonstrated her ability to present a range of views of angles in order to help students understand the concept. Before Lesson Study she taught mathematics based on the textbook only. She did not develop her own problems but she selected the problems from the textbook. The problems were solved using a single strategy, such as applying a formula to solve the problem. After Lesson Study she had a better understanding and insight into the concepts especially the topics taught during Lesson Study program. For example, she always remembered from group discussion about the difference between "unit angle" and "angle unit". Also she used problems not only from the textbook but also from discussion with her peers. She was able to try developing problems based on the textbook.

Beliefs about Teaching and Learning Mathematics

During the interview with Wi before the Lesson Study program started she believed that mathematics was taught by the teacher explaining the procedure and giving examples, and then students doing the relevant tasks or exercises. She thought that the students would become good at mathematics by doing the tasks and solving the problems. She viewed a problem as a story problem in which the problem translated from the words of a story problem into some kind of mathematical sentence or operation. However, when describing her beliefs about teaching with problem solving approach before Lesson Study she explained:

> *Teaching mathematics using a problem solving approach needs too much time. The students find it difficult to understand the story problem.* (Interview, Wi, 21 December 2011)

During the classroom observations of her teaching practice it was noted that her teaching practice was primarily based on teacher presentation, used a variety of technique to explain the concepts, including manipulatives as teaching aids, and students were given tasks to work in groups. However, this generally was only for short time periods of the lesson. Reflecting her stated beliefs about teaching, Wi first explained the procedure and gave examples, then presented exercise problems for students to solve. In explaining a concept, she sometimes used hands-on teaching aids to involve the students. She would demonstrate the use of the aid (e.g. folded paper, or a protractor) and the students would follow her example.

At the interview one year after the Lesson Study program was completed, she explained her beliefs about teaching mathematics:

> *I teach mathematics and begin with a question, giving a problem related to real life and then putting students in groups to solve the problem.* (Interview Wi, 21 December 2011)

The teacher's role during group work was as a motivator, prompting students to solve problems on their own. The teacher allowed students to work cooperatively, to solve problems in their groups. She believed that students would learn mathematics as they solved many problems and discussed their solutions.

Summary

Changes were noted in the following aspects of Wi's teaching, which may reflect changes in her pedagogical content knowledge.

- Representation of the idea to build on students' mathematics changed from simple representation to more specific representation involving numerical representation of scale to measure angles;
- Demonstration of procedures changed from student centred to teacher explanation;
- Questioning changed slightly to include more comprehension level questions and application level questions;
- Diagnosis of learning difficulties changed to using more oral questions in different levels.

In the mathematical content knowledge Wi changed as follows:

- She showed more understanding of properties of angles and demonstrated this by using a wide range of representations of angle;
- She gave different definitions of angles in order to help children understand the concept, reflecting a growing level of mathematical content knowledge.

In her stated beliefs before Lesson Study, mathematics was taught by the teacher explaining the concept and giving examples, followed by the students doing tasks or exercises. Students could learn mathematics well by doing the tasks and solving the set problems. Through the Lesson Study she changed her belief so that mathematics was based using a problem approach allowing students to work in group settings, and encouraging students to solve problems according to their own methods. However she did not seem to have the confidence to translate this belief into her own practice, as shown by her teaching in Lesson 2B.

Case 3: Lesson 3A and 3B by Teacher Mi

Teacher Mi has 2 years teaching experience in a primary school. She teaches Year Four at a suburban school, SD 8P Bin, about one kilometre away from SD 1P. Her class has 40 students who come from the neighbourhood of the school. She graduated as a primary school teacher in 2008, and has had no professional development in teaching mathematics since she graduated. The first time she heard about Lesson Study as a professional learning method was from the researcher when she was interviewed prior to the start of the program. She was a volunteer teacher for 2 lessons in the Lesson Study program. She taught the topic “Finding the volume of a cube and a cuboid” in Lessons 3A and 3B with two different classes. Lesson 3A was the original lesson designed by the group and Lesson 3B was a revised lesson plan version of Lesson 3A. Both the lessons were video recorded and audio recorded and her classroom was observed by the researcher and her fellow Lesson Study program participants.

Description of Lesson 3A

In Lesson 3A, taught on 15 November 2010, teacher Mi segmented the lesson into three distinctive phases: the initial activity, the main activity, and the end activity. The students were put into groups of five. At the beginning of the lesson she reviewed some of the differences between 2-D shapes and 3-D objects shapes with the class. Then she showed the students some unit cubes. She asked the students to identify the characteristics of a cube and a cuboid. The students compared the different characteristics of a cube and a cuboid.

She distributed a different amount of cubes for each group. Then she gave a task as an open problem*: Using the cube units, construct a cuboid with as many unit cubes as you have*. Each group worked together to construct a cuboid, while the teacher observed the groups. She reminded students that all the unit cubes had to be used to construct a cuboid. Students constructed their cuboid and determined the dimensions of the cuboid. For example, one group of students constructed a cuboid of dimensions 2 × 2 × 2, and another group constructed a cuboid of dimensions 4 × 2

$\times$ 1. The students determined the length, width, and height of the cuboid they had constructed. During the group work students discussed the task, and the teacher observed and helped students who had difficulty. The teacher prepared a table format on the white board with the headings: length, width, height, and volume of cuboid. Then one student read the group's results aloud while another student filled in the table on the whiteboard based on all the groups' results. After completing the table for all groups, the teacher asked students to discuss the students' answers from the table in the whole class.

In the discussion she demonstrated the cube construction for each dimension of the cuboid on the table. Students classified the three dimensional objects into a cube or a cuboid by looking at the pattern and the relationship between the volume, length, width, and height of the shape. Based on the data table, she guided the students to find the formula for calculating the volumes of a cube and a cuboid. After finding the formula for calculating the volume of a cuboid, she gave students an application problem to determine the volume of a cube and a cuboid. By drawing a cuboid and a cube with certain dimensions on the white board she asked students to calculate the volume of the shapes. The teacher asked who could answer the problem. Two students, who volunteered, then demonstrated solving the problem for the rest of the class.

Next, she gave students an evaluation task through a written individual exercise. While the students worked, the teacher walked around and observed them. Students' answer sheets were collected by the teacher. At the end the teacher and students reviewed and summarised the lesson.

Lesson Study Group Meeting after Lesson 3A

A Group Meeting of teachers was held after Lesson 3A. Based on the observations and general comments, all participants said that the lesson was a good teaching and learning activity, and students had learned actively. Some comments and recommendations from the group were as follows:

- Representation of ideas – Students were not familiar with volume units, such as cm^3, because the teacher did not introduce standard volume units in the learning process to determine the formula for the volume of a cube. The teacher introduced this convention later after some students made mistakes in writing the volume units in their results. One of the groups wrote the dimensions of a cube as being length 2, width 2 and height 2. In this case the teacher needed to emphasise the concept that the sides of a cube all have the same size. Also with another group she found that they constructed rectangular prisms and they wrote the dimensions as length 1, width 4 and height 2. This was not consistent with the convention that length is longer than width. The observer suggested that it was better if the students were shown the cube or cuboid with viewing from any side position;

- Demonstration of procedures – Mi did not construct the cube model well because some unit cubes moved. The observer suggested that constructing a cuboid would be better to be delegated to the students so that they could be more active. The teacher could use that time and opportunity to ask students questions and give attention to all students;

- Demonstration of knowledge – Using the data on the table was not enough to draw conclusions about the volume of a cuboid. The table only showed data for one cube so she needed more than one cube example. The students had a lack of understanding about the standard units for volume, for example, they said *centimetre to the power of three* not *centimetre cubed*, in using the units of standard volume. The observer suggested that during the process of finding the formula would be better to introduce the standard volume unit.

Based on the group discussion and comments the group's suggested changes for the next lesson are summarised in Table 4.8.

Table 4.8: Suggested Changes for Lesson 3B.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Representation of ideas	Show the cuboid and draw attention to the different sizes of its sides to help students understand the meaning of length, width and height.
• Demonstration of procedures	Give opportunity for the students to be involved in using manipulatives to construct a cuboid.
Mathematical content knowledge • Understanding mathematical processes	Introduce the standard volume unit during the process of finding the formula for calculation of the volume. Use enough data to be able to make a conjecture or a conclusion. Emphasise the meaning of length, width, and depth on a cuboid.

Description of Lesson 3B

Lesson 3B was taught to a different class nine days later on 23 November 2010. At the beginning of the lesson the teacher asked the class for the differences between 2-D shapes and 3-D objects. Then she asked them to give an example of a cuboid and a cube using objects around the classroom as examples. By showing a model of a cuboid the teacher asked students to compare the characteristics of a cuboid and a cube. The students identified the elements of a cube and a cuboid.

Next, she distributed a worksheet and a set of unit cubes to each group. She constructed a cuboid and showed it to the whole class. She reminded them of the dimensions of a cuboid using length, width, and height. Then she gave each group an open problem: *Construct a cuboid using all the unit cubes you have.* Each group then constructed a cuboid of different sizes depending on the number of cubes they had. They solved problems on a worksheet that asked them to find the dimensions and volume of the cuboid they had constructed. After all groups had finished she asked students to write the dimensions of their cuboid in the table on the white board. Two students from each group presented their group's results; one student read the results while the other wrote in the table.

Based on the table she asked the whole class to look at the relationship between the length, width, height, and volume of a cuboid. She reviewed the data in the table to check that the answers were correct, and students from each group demonstrated their group's model to the class. She used the data table to guide the students towards determining the formula for calculating the volume of a cuboid. For example, she gave 2...2...2 = 8 so for the correct answer the students put in the operation "×" (multiply). Then she made a generalisation that the volume of a cube is s × s × s with s as one side. In the same way, she identified a cuboid with its dimensions. Then she made a generalisation that the volume of a cuboid is l × w × h. The next step was that she wrote two questions to apply the formula for calculating the volume of a cuboid on the white board. The students solved the problems individually, and one student wrote the answers on the white board. She set a problem example on the white board, for students to calculate the volume of a cuboid. One of the students answered the question and explained his method to the teacher, so that the whole class could hear. After students understood the formula for calculating the volume of a cube and cuboid, she gave the students written individual exercises. In the whole class situation she reviewed the lesson by asking the students some questions.

Lesson Study Group Meeting after Lesson 3B

A Group Meeting involving all group members was held after the lesson. All participants said that the lesson was a good teaching and learning activity and used active learning. Some comments from group discussion were noted:

- Demonstration of procedures – Hands-on learning aids (unit cubes) were distributed to the students early in the lesson. Students tended to play with these instead of paying attention to the mathematics concepts. It was better that the teacher demonstrated the concept with the manipulatives and not give these to the students until later. The unit cubes would be distributed later, after the explanation by the teacher.

- Classroom management – When the students presented their results to the class the teacher forgot to praise them for correct answers. It would be better if the teacher acknowledged student answers as correct or incorrect and gave suitable feedback.

- Diagnosing learning difficulties – The teacher asked questions to the whole class. It would be better choosing individual students to answer in the whole class setting. Also, if a student answered incorrectly, the teacher should pass the question to other students to respond;

- Demonstration of knowledge – The formula for the volume of a cuboid – $V = L \times W \times H$ should be written as $V = l \times w \times h$. Standard convention is to use small letters for $l \times w \times h$, as well as for segments of a shape. Capital letters are conventionally used for points.

Based on the group discussion and comments, the suggested changes for the next lesson are summarised in Table 4.9.

Table 4.9: Suggested Changes for the Next Lesson.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Demonstration of procedures	Distribute manipulatives or learning aids after the teacher has demonstrated and explained the concept or procedure.
• Classroom management	Give positive feedback for student answering questions, whether they are right or wrong. Encourage students who are in difficulties by asking them questions to guide them.
• Diagnosing learning difficulties	When asking questions to the whole class choose individual students to respond.
Mathematical content knowledge • Demonstration of knowledge	Use writing mathematical expression in the right way, such as lines was represented using lower case letters.

A summary of changes in pedagogical content knowledge in practice based on researcher observation is shown in the Table 4.10. For this table and those that follow, a √ symbol indicates at least one observed instance of the particular criterion for every five minute period of teacher activity and student working.

Table 4.10: Pedagogical Content Knowledge as Demonstrated in Teaching Practice (Lessons 3A and 3B).

Teacher Using Activity	Student Working							
	Individual		Pair		Group		Class	
	3A	3B	3A	3B	3A	3B	3A	3B
KNOWLEDGE OF TEACHING								
Representation of Ideas and Use of Resource								
Concrete					√√	√√√	√√√	√√√
Pictorial					√	√√	√√	√√
Real world							√	√
Symbolic/abstract							√	√
Demonstration of Procedures								
Tasks/problems					√√	√√	√√	√√
Discussions					√		√	√√√
Student presentations					√√√		√√	√√
Teacher presentations					√	√	√	√
Classroom Management								
Positive teacher statement					√	√√	√√	√√√√
Negative teacher statement								
Encouraging students in difficulty					√	√	√	√
Encouraging students to have different points of view								
Encouraging students to participate					√√	√√√	√	√√√
Encouraging students to explain their reasoning								√√
Type of Problem								
Open					√			
Closed					√√			
Drill								
Applied					√	√	√	√√√
Puzzle problem								
KNOWLEDGE OF STUDENTS' LEARNING								
Level of Teacher Questioning								
Knowledge					√√	√√√	√√√	√√√√
Comprehension					√		√√√	√√
Application								
Analysis								
Synthesis								
Evaluation								
Diagnosing Learning Difficulties								
Oral question					√√	√√	√√√	√√√√
Written question					√	√	√	√
Written test	√√√√	√√ √√					√√√√	√√√√

Change in Pedagogical Content Knowledge

The pedagogical content knowledge shown by Mi changed over the two lessons. The changes were noted in the following:

- Using representation of ideas – In the first lesson Mi used a few unit cubes to demonstrate a concept. In the second lesson she incorporated various models with many unit cubes in various colours to demonstrate concepts;

- Demonstration of procedures – Mi changed to demonstrate an example from static viewing (via a cuboid in one dimension only) in the first lesson to dynamic viewing (view a cuboid in more than one dimension) in the second lesson;

- Classroom management – Mi used positive feedback for students' responses in the first lesson and changed to use more positive statements and encouragement of student thinking in the second lesson.

- Level of teacher's questioning – In the first lesson Mi asked students most questions at beginning of the lesson. However, she changed by asking more higher level questions in the second lesson.

Mi displayed changes in her knowledge of teaching as part of her pedagogical content knowledge over the two lessons.

Knowledge of teaching

Although Mi did not show a big change in her knowledge of teaching over the two lessons, there were some changes in the way she represented ideas, demonstrated procedures, and in her classroom management.

1. Representation of ideas

In building the volume of a cuboid the teacher used unit cubes to construct a cuboid model representation for each group in both lessons. In Lesson 3A she used only a few unit cubes to construct a rectangular prism because there was a limited quantity of unit cubes. Also because of this limitation the teacher demonstrated constructing the cuboid rather than giving this task to the students. Each group constructed a cuboid with different dimensions. Only one group constructed a cube the others constructed rectangular prisms.

In Lesson 3B the building of cuboids could be done in many ways by the students. Each group constructed a different size rectangular prism. The teacher obtained extra unit cubes from other schools and teachers. In this way she had enough cubes for students, in groups, to construct their own rectangular prisms. The students were more involved in the activity of constructing a cuboid than the first lesson. In both lessons, they counted the total number of cube units to measure the length, width and height of their construction. Also they counted the total number of cube units needed to construct the rectangular prism. The teacher asked students to look at the relationship between length, width, height, and the volume of the rectangular prism from the dimensions shown on the table on the white board. Then the teacher guided students to find the formula to calculate the volumes of a cuboid and a cube, using symbolic representation. She used modelling to help the students find the formula for calculating the volume of a cuboid. She helped students become aware of what they do know and how they use what they know to find the formula for calculating the volume of cuboid. The students developed better conceptual understanding of the volume of a cuboid from using physical representation.

Mi taught mathematics using the limited manipulatives or teaching aids available at the school and sometimes based on the textbook only, without additional teaching aids. But after the completion of the Lesson Study program she not only taught mathematics using the teaching aids at the school but she also modified or created the teaching aids. It showed the positive effect of Lesson Study in that, as a result of the discussions in the Group Meetings, Mi and other teachers became aware of the situation about the lack of resources. They also realised the importance of having more manipulatives available and shared their own resources with Mi.

2. Demonstration of procedures

Mi changed her presentation techniques so that her demonstrations became more dynamic. Students gave more presentations in their group settings in Lesson 3A than she did in the later lesson. When demonstrating the dimensions of a rectangular prism to show its length, width and height, she constructed a rectangular prism using cube units. In Lesson 3A she put the prism on the table in front of the class in a static position. But in Lesson 3B she asked the students to explore to find a cuboid of a different size with the same amount of unit cubes. She rotated the rectangular prism in any position to enable dynamic viewing to help students understand the concepts of length, width and height. She began with constructing a cuboid with an open problem: *Construct a cuboid using all the unit cubes you have* which was the same as she had in Lesson 3A, but this time the teacher emphasised that the length of the shape was longer than its width. A part of the demonstration went as follows:

Teacher : How long is the cuboid? (She showed the cuboid as below)

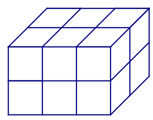


Figure 4.5: A cuboid 3 × 2 × 2 size.

Students : Three.
Teacher : If the position is like that, how long is the cuboid? (She showed the same cuboid, but rotated it so the short side was towards the class, as below)

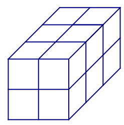


Figure 4.6: A cuboid 3 × 2 × 2 size.

Students : Three units.

When the students had found all they could, the teacher asked them to discuss what they had found. When each group had generated a number of possibilities, they wrote down answers on the board. She took care when giving the meaning of "length" to explain that it should be longer than the "width" of a rectangular prism. Here, she allowed students to construct the meaning of length and width based on their own understanding and experience.

Before the Lesson Study program Mi mostly taught mathematics in a routine way by explaining the concepts, giving examples, and then doing exercises with the students. She would demonstrate solving a problem and then the students would copy textbook examples. In this traditional way, she taught the example directly from the textbook or from other text books.

After the Lesson Study program she started to teach mathematics using a problem solving approach for certain topics. She stated that she felt more capable of reviewing the topic taught having participated in the Lesson Study program. When she taught mathematics using a problem solving approach she gave the students open problems or closed problems and the students solved them using various methods. She was more of a guide in helping students understand the concepts. She demonstrated the problem solving processes with examples relevant to the students and used simple manipulative materials or teaching aids to help students find solutions. For example, she gave the students an open task: *If the volume of a cuboid is 10 cm^3, show all possible cuboids in different size.* The students answering the sizes of cuboid are 5 cm × 2cm ×1cm; 10 cm × 1cm ×1cm. Next she used the same task with the volume of cube 12 cm^3. She allowed them to use manipulatives, draw the cuboid, and write the size of cuboid for all cuboids.

3. Classroom management

When the groups were able to report how they solved the problem and to answer the teacher’s question correctly, she gave more positive feedback statements in Lesson 3B, such as: *Good job, Wonderful*. The teacher successfully gained the students’ attention in both lessons. When she asked a question she gave time for students to answer the question. She observed students working in groups by walking around the students. She also gave an opportunity for all groups to present their results by writing on the white board.

Before the Lesson Study program she taught in a traditional teacher-centred way. She explained the concepts and asked questions to the whole class with students answering together or individually. But after the Lesson Study program she began to teach using both whole-class and group settings. She said that group settings made it easy to identify student understanding by asking students questions and observing. As she explained:

> *Using grouping makes it easier for me to classify weak students or strong students. By observing, I see that normally strong students do not like being in the same group with weak students.* (Interview, Mi, 22 December 2011)

Mi's practice changed in that the Lesson Study program had provided her with the experience of how to manage a classroom when using a problem solving approach.

Knowledge of students' learning

In her knowledge of students' learning Mi demonstrated change in her teaching over the two lessons with regard to using levels of teacher questioning. By the second lesson, Mi began asking more questions, making more positive teacher statements and offering more encouragement to students' thinking. To encourage students' thinking and participation she used questions, problems, and positive statements in both lessons. At first, she asked lower order questions, mostly knowledge questions with some comprehension questions. For example, in Lesson 3A by showing a 3-D construction she asked one group of students to identify a range of different facts:

> *Mi : What is the name of this shape?*
> *Students : Cuboid.* (Students answered together)
> *Mi : How long is it?*
> *Students : 4.* (Students answered together)
> *Mi : How wide is it?*
> *Students : 2.* (Students answered together)
> *Mi : How high is it?*
> *Students : 2.* (Students answered together)
> *Mi : What is its volume?*
> *Students : 16.* (Students answered together)

The way Mi asked questions changed from Lesson 3A to Lesson 3B. In the first lesson most of the teacher's questions happened at the beginning of the lesson but in Lesson 3B the teacher's questions occurred not only at the beginning of the lesson but also almost all the time from the beginning until the end of the lesson.

More importantly, the level and nature of her questions had changed such that she used more *why* questions or comprehension level question in Lesson 3B. For example, part of the discussion during Lesson 3B to find the dimension and the formula for calculating the volume of a cuboid was as follows:

> *Teacher : From the table which one is a cube?*
> *Students : Shape number 7.*
> *Teacher : Why is it called a cube?*
> *Students : Because all its sides are the same.*
> *Teacher : What are the others called?*
> *Students : A cuboid.*
> *Teacher : Why are they called a cuboid?*
> *Students: Because the sides are different lengths.*

Change in Mathematical Content Knowledge

Mi demonstrated some changes in her mathematical content knowledge over the two lessons. She paid more attention to accuracy of details in the second lesson. She clarified the meaning of height, width and length, and ensured that formulas were written with the correct mathematical conventions with lower-case letters in Lesson 3B. In both lessons, she reviewed some of the differences between 2-D shapes and 3-D objects with the class. She was able to demonstrate the properties of 2-D shapes and 3-D objects.

To introduce the process for calculating the volume of a cube and a rectangular prism, the teacher defined volume as being the number of unit cubes required to build a shape. She described the unit cube as being a unit of volume. The students constructed a large cube or cuboid shape using as many unit cubes as they could. Then, they determined the length, width and height dimensions of their shape by counting the unit cubes along each side. They determined the volumes of their shapes by counting the total number of unit cubes within the shapes. Her lesson was mostly based on students' sharing and discussing. She was able to demonstrate the volume of a cube or a cuboid for a real object by counting its constituent unit cubes. She was also able to demonstrate the formula for the volume of a cube or cuboid by looking at the relationship between the lengths of each side and the total volume of the cuboid. Also, she was able to select problems that have more than one solution.

In her interview one year after the Lesson Study program finished she stated that before the Lesson Study program she had lacked confidence and had limited understanding of some of the mathematics concepts she was teaching. As she explained:

> *I sometimes lacked understanding of the problems, but nobody challenged me, because no students objected to my explanation.* (Interview, Mi, 22 December 2011)

Hence, she did not develop her lessons beyond what was in the text book. The problems she set were limited to the textbook examples. Also the problems were all solved in the same ways in the textbook. After the Lesson Study program she said she had more understanding of the mathematical concepts and more confidence. As she explained:

> *I feel more confidence in understanding mathematical concepts. The problems are not just solved using a formula, and that is the end of it. But through teaching different ways to solve the problems, I am able to introduce the new concepts.* (Interview, Mi, 22 December 2011)

During the Lesson Study meeting her mathematical understanding was challenged. She was open-minded about learning mathematical content from other teachers. The Lesson Study Group Meetings contributed to changes in Mi's teaching in this way and also in the way in which she obtained more manipulatives resources so that her students were able to build more shapes and better explore the concept of 3-D objects.

Beliefs about Teaching and Learning Mathematics

Before the Lesson Study program Mi believed that the way to teach mathematics was for the teacher to explain the concept, demonstrate the formula, and demonstrate solving mathematics problems. The students solved the problem individually, and had opportunities to ask questions. As she explained:

> *I would explain the concepts or formulas to the students and they would solve the problems individually.* (Interview, Mi, 22 December 2011)

Before Lesson Study program Mi believed teaching and learning mathematics through a problem solving approach emphasised training students to solve problems individually or in a group. It appears that Mi interpreted problem solving as students applying a formula to solve a mathematics problem. She imagined that real problem solving was difficult to apply in teaching mathematics. As she explained:

> *Teaching using a problem solving approach emphasises students doing exercises to solve the math problems individually or in groups. I explain the problem example, and then discuss students' solutions.* (Interview, MI, 22 December 2011)

This changed after her participation in the Lesson Study. Classroom observation of her teaching practice during the Lesson Study program found that in Lesson 3A she distributed hands-on teaching aids and worksheets to the different groups. Students presented their group's results in front of the class. During the lesson she allowed students to work on their own while she asked questions. The students shared, discussed, and constructed a rectangular prism to solve the problem in both the lessons. In Lesson 3B the students constructed a cuboid using unit cubes in many different ways. At the end of the lesson the students could determine the volume of a cuboid by using the formula. Also by having students solve the problem using hands-on teaching aids, it gave them a deeper understanding. As well, she offered individual and group feedback and asked questions to assess student understanding.

After the Lesson Study Cycle she believed that by using the problem solving approach the students were better able to understand the new concept. She noted that the students enjoyed working in groups. She found that using a problem solving approach made her more enthusiastic about teaching. She felt that problem solving allowed the students to learn by doing and to explore to find different ways of solving problems.

Summary

Over the two lessons Mi demonstrated changes in her pedagogical content knowledge in the following ways:

- She used different models to represent ideas changing from a simple model constructed of a few unit cubes to various models constructed of many unit cubes;

- Her demonstration of concepts changed from static viewing to dynamic viewing to enable students to view objects from different positions;

- She changed her classroom management style to ask more questions, give more positive feedback to students, and give more encouragement to student thinking;

- Her level of teacher's questioning changed from knowledge level questions to more comprehension of higher-level questions.

Mi demonstrated some changes in her mathematical content knowledge over the two lessons. In particular, she was more careful to use mathematical terminology and conventions accurately.

As a result of her involvement in Lesson Study her beliefs changed. Previously she taught mathematics by explaining, giving students the formulas, and solving problems in the same way. After the Lesson Study program she believed that through problem solving students were able to solve in various ways, to learn by doing, and to understand the new concepts.

Case 4: Lessons 4A and 4B by Teacher Zu

Teacher Zu has 28 years teaching experience with more in science than in mathematics. She graduated with a Master of Educational Management. She teaches a Year Five class at SD 8M Bin where students come from the neighbourhood of the school. The school was located in the inner city close to public offices, a shopping centre, and busy traffic. She has had no previous professional development in teaching mathematics but quite a lot in science teaching. She was a volunteer teacher in the Lesson Study and she taught twice with the same topic for Lessons 4A and 4B. Lesson 4A was the original lesson that was designed by the group and Lesson 4B was the revised lesson from Lesson 4A. Both the lessons were video recorded, audio recorded, and observed by another teacher. The lessons taught were determining the volume of a cube and a cuboid.

Description of Lesson 4A

Lesson 4A was taught on 15 November 2010. Teacher Zu segmented the lesson into three distinctive phases: initial activity, main activity, and end activity. At the beginning of the lesson she showed models of two dimensional shapes and three dimensional objects. In showing the shape selected she asked students to identify the name of the shape in two dimensions then she continued to identify the name of the object in three dimensions, such as cylinder and cuboid. Frequently she used questions at the knowledge level with short answers, such as, *What is the shape's name?* Almost all students were able to answer orally with the right answer. Then she explained the objective of the lesson was to determine the volume of a cube.

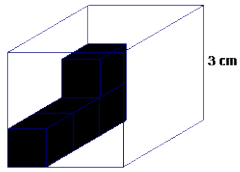
In the main activity she began to divide the class into groups and distributed unit cubes and a large transparent cube model to each group. The large cube was open and could be filled with the smaller unit cubes. She gave tasks in a closed problem for each group: *How many unit cubes can be placed in the large transparent cube?* All groups were successful and filled in the large cube with unit cubes. Using the large cube as their construction they answered the questions on the worksheet and explained the solution to their classmates. When students were working and discussing in groups she observed, walked around the group, helped students with difficulty, and helped them develop a strategy to solve the problem. By looking at a group had constructed she asked the students to answer questions written on the worksheet. In front of the class the students read the result of their group discussion and the other group wrote the result on the white board. She showed 3 transparent cubes (small, medium, and large). Based on the students' solution she guided the students to determine the length of the sides of the cube and the formula for the volume of a cube. Below shows part of the interaction between teacher and students.

> *Teacher: What is a cube?*
> *Students: All the sides are the same.*
> *Teacher: How many cube units fit into the side of the smallest transparent cube?*
> *Students: 2 unit cube long.*
> *Teacher: How many cube units fit into the side of a medium transparent cube?*
> *Students: 3 unit cube long.*
> *Teachers: How many cube units fit into the side of the largest transparent cube?*
> *Students: 4 unit cubes long.*

Then she showed the meaning of the volume of the cube as being the total number of unit cubes it took to fill the large transparent cube.

> *Teacher: How many unit cubes are needed to fill the transparent cube that is 2 lengths of a unit cube?*
> *Students: 8 unit cubes.*
> *Teacher: How many unit cubes are needed to fill the transparent cube that is 3 lengths of a unit cube?*
> *Students: 27 unit cubes.*
> *Teacher: How many unit cubes are needed to fill the transparent cube that is 4 lengths of a unit cube?*
> *Students: 64 unit cubes.*
> *Teacher: Volume of smallest transparent cube is 8 unit cubes or 2 × 2 × 2 unit cubes. Volume of medium transparent cube is 27 unit cubes or 3 × 3 × 3 unit cubes. Volume of the largest transparent cube is 64 unit cubes or 4 × 4 × 4 unit cubes. If the side of a cube suppose s unit long so the volume of a cube $V = s \times s \times s$.*

Then the teacher explained how to determine the volume of a cube in standard units. She asked questions orally about how to determine the volume of each cube with sizes of sides 5 centimetres and 3 centimetres and students answered orally. Then students solved another problem on the white board. For example: *How many more unit cubes needed to fill a large cube of side 3 cm long?*



Student's solution: *The total unit cubes to fill a large cube is V = 3cm × 3cm × 3cm = 27 cm*^{3}. *So a large cube needs more unit cubes = 27 unit cubes – 4 unit cubes = 23 unit cubes.*

In the last part of the lesson the students were given individual tasks in a written test that could be solved by the same procedure as the teacher's procedure, such as: *Rasid measures the inside of a cube shaped tub. He finds that each is 70 cm long. How much water is needed to fill the tub?* While students were doing the task the teacher observed.

Lesson Study Group Meeting after Lesson 4A

A Group Meeting was held after the lesson at SD 8 M. All 6 teachers had participated in Group Meeting. Some comments and suggestions were as following:

- Classroom management – Students' motivation was good with the teacher frequently giving praise for the student responses. The teacher could have improved the lesson by distributing parts for the students' tasks when the group presented in front of the class. The group suggested that it would be better to make team members involved in presentation and make them more responsible for their tasks;

- Demonstration of procedures – When the teacher asked the students for the length of one side of the large cube many students tried to use a centimetre ruler to measure it. The teacher had intended that the students measure the length of a side using unit cubes, not a centimetre ruler and so it was suggested that instead of asking students, 'How long is the side of a large cube?' she could ask, 'How many unit cubes make up the length of the side of the large cube?' Also, Zu asked the class to construct a cube using only one dimension for each group. The group suggested that it would have been better to construct different cuboids so that students could be more challenged;

- Understanding mathematical relationship – At the beginning of the lesson the teacher reminded the students to mention various 2-D shapes, such as circle and triangle but the teacher did not ask or remind students about rectangles and squares. These shapes are important as prerequisite knowledge of the concept of a rectangular prism. The group suggested in this case that the rectangle and square were important prerequisites to construct and be used to count the volume of a cube and a rectangular prism. Students should be reminded of the previous concepts to build the new concept. For example, the formula for the volume of a rectangular prism is constructed by using the area of a rectangle as *the area of the base of the rectangular prism* multiplied by the height of a rectangular prism.

Based on group discussion and comments in the meeting after Lesson 4A the groups suggested changes for the next lesson are summarised in Table 4.11.

Table 4.11: Suggested Changes for Lesson 4B.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Classroom management	Encourage students to be more involved in their group activities.
• Demonstration of procedures	Use more specific questions to obtain the objective. Construct a cuboid in more than one dimension.
Mathematical content knowledge • Understanding mathematical relationship	Use the properties of a rectangle and a square related to the properties of a rectangular prism or cube.

Description of Lesson 4B

In Lesson 4B, taught to a different class a week later than Lesson 4A, Zu divided the lesson into three distinctive phases: the initial activity, a main activity, and an end activity. At the beginning of the lesson Zu asked the students to compare the characteristics of a cube and a rectangular prism by showing a cube and a rectangular prism model. After comparing the characteristics of the objects she explained the objective of the lesson. Then Zu continued to demonstrate how to construct a rectangular prism using 12 unit cubes and asked students its dimensions.

In the main activity Zu distributed 36 unit cubes and worksheets to each group. She gave an open task: *How many rectangular prisms can you make using 36 unit cubes*? Using unit cubes and working with a worksheet the students constructed rectangular prisms with different dimensions. Zu circulated the room monitoring each group's work. She reminded each group to make sure that their solution had more than one answer. Zu helped and allowed students to work independently. She prepared a table on the white board to make a list of the dimensions of the rectangular prisms. After working in a group one of the members wrote their group results and filled in the table on the white board.

With the whole class Zu guided the students' discussion to find the formula for the volume of a rectangular prism using the data in the table. She always gave praise and motivation to the students' responses. After finding the volume of a rectangular prism Zu gave a similar problem orally for students to solve and to make sure the students knew the procedure.

At the end of the lesson the students practiced solving problems individually using a work sheet in which they tried many more exercises that were very similar to the previous problem. During this activity the teacher observed and helped students with difficulties.

Lesson Study Group Meeting after Lesson 4B

A group discussion was held after the lesson. Comments and suggestions from the observers were as follows:

- Types of problems – The teacher taught the lesson that matched the group lesson plan but the students did not find it a challenging problem. They just applied the formula to solve a problem or answer a question. It would have been better to give the students a more challenging problem.

- Demonstration of procedures – When she observed the four groups of students constructing a rectangular prism using 36 unit cubes, they got many answers for the size of box models. She did not decide the absolute maximum of the box that could be constructed. Later in the Lesson Study discussion Zu disclosed that she was actually unclear herself about the differences between length and width and so was unable to explain this properly to the students. For example, when constructing a rectangular prism with the dimensions $6 \times 3 \times 2$ this means that the length is 6, the width 3 and the height 2. If the dimensions were $3 \times 6 \times 2$ it would mean that the length is 3, the width is 6, and the height is 2.

In the Lesson Study Group discussion process some points were made by the participants: (1) Length was longer than width; (2) Length was the side facing the reader, the vertical line is the height; and (3) Length was the horizontal dimension and the vertical line is the height. They constructed a rectangular prism using 36 cubes. During the process of constructing a rectangular prism they had conjectured: 1) The length was to face the reader; 2) Dimensions could be written as $l \times w \times h$, or $w \times l \times h$, or $h \times w \times l$; 3) Based on the dimensions of a rectangle concept, they suggested the length was longer than the width; and 4) In the horizontal position the rectangle was the base of a cuboid and the other dimension was the height of a cuboid. The conclusion was: The volume of a cuboid was constructed from a rectangle's area multiplied by the height of the cuboid. As a consensus for the dimensions of a rectangle they decided that the length was longer than width. The discussion moved to another topic for the next lesson. The team decided to teach the topic of area and perimeter of a parallelogram. Based on the group discussion and comments the suggested changes for the next lesson are summarised in Table 4.12.

Table 4.12: Suggested Changes for the Next Lesson.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Demonstration of procedure	Present the students with a challenging problem.
Mathematical content knowledge • Demonstration of knowledge	Demonstrate all possible dimensions of a cuboid with the same volume. Consider that the length is longer than the width.

A summary of the changes in pedagogical content knowledge in practice based on researcher observation is shown in the Table 4.13. For this table and those that follow, a √ symbol indicates one observed incidence of the particular criterion for every five minute period of teacher activity and students working.

Table 4.13: Pedagogical Content Knowledge as Demonstrated in Teaching Practice (Lessons 4A and 4B).

Teacher Using Activity	**Students Working**							
	Individual		**Pair**		**Group**		**Class**	
	4A	**4B**	**4A**	**4B**	**4A**	**4B**	**4A**	**4B**
KNOWLEDGE OF TEACHING								
Representation of Ideas and Use of Resources								
Concrete					√√√√	√	√	√√
Pictorial								
Real world								
Symbolic/abstract								
Demonstration of Procedures								
Tasks/problems					√	√		
Discussions					√√√	√√		
Student presentations					√√√		√√√	√√√√
Teacher presentations							√√√	√
Classroom Management								
Positive teacher statements	√	√			√	√√	√	√√
Negative teacher statements								
Encourages students having difficulty							√	√√
Encouraging students to have different points of view								
Encouraging students' participation					√	√√	√	√√
Encourage students to explain their reasoning					√	√	√	√√
Ask students to identify mathematical concepts								
Type of Problem								
Open						√		
Closed	√				√√√√		√	√√√√
Drill	√						√	√√√√
Applied								
Puzzle problem								
KNOWLEDGE OF STUDENTS' LEARNING								
Level of Teacher's Questioning								
Knowledge						√√	√√√	√
Comprehension						√√	√√√√	√√
Application								√
Analysis								
Synthesis								
Evaluation								
Diagnosing Learning Difficulties								
Oral question								√√√
Written question								
Written Test	√√√	√√√√					√√√	√√√√

Change in Pedagogical Content Knowledge

The pedagogical content knowledge for Zu developed over the two lessons. Changes were noted in the following:

- Using representation – In the first lesson Zu used the concept of the quantity of unit cubes needed to construct one cuboid. However, in the second lesson she gave a set quantity of unit cubes and asked the students to construct more than one cuboid;

- Demonstrating the procedures – In the first lesson Zu used a closed problem: to fill a large cube using unit cubes to establish the validity of the concept of volume. However, in the second lesson, she changed to a more open problem;

- Classroom management – In the first lesson Zu used more questioning techniques, was less encouraging of students and used less positive statements than the second lesson.

The description of change in pedagogical content knowledge for each of the above change was classified as knowledge of teaching.

Knowledge of teaching

In teaching practice Zu changed her teaching approach over the two lessons. She demonstrated changes in her teaching practice with regard to representation of ideas, demonstration of procedures, and classroom management.

1. Representation of ideas

Before Lesson Study Zu said that she had taught mathematics by giving an example to represent the concept based on the textbook format. During the Lesson Study Cycle Zu used the same model of representation to explain the concept of volume in both the lessons. She used unit cubes to introduce the volume of a cube and a cuboid in both the lessons. The standard unit of volume was introduced by the teacher, later. The unit cube was used as a basic model to construct a rectangular prism. In Lesson 4A, in building the volume of a cube, the teacher used a large plastic cube model that was filled by using unit cubes for each group. Each group of students constructed a cuboid of different dimensions for each group. They determined the elements of a cube, such as size, total of sides, and volume. From the result of each group the teacher concluded the formula for calculating volume of a cube.

In Lesson 4B the teacher demonstrated the volume of a cuboid by using cuboid models of different sizes for each group. Each group had 36 unit cubes and they used all the unit cubes to construct a rectangular prism. They constructed rectangular prisms of different dimensions and sizes with the same amount of unit cubes (36 unit cubes). For example, the teacher asked the students in two groups:

Teacher: How many rectangular prisms of different sizes can you make?
Group A: 9.
Teacher: How many rectangular prisms of different sizes can you make?
Group B: 12.

Then each group wrote the possible dimensions of all the rectangular prisms using 36 cubes in different sizes according to their findings. The teacher concluded with formula for calculating the volume of a cuboid based on the students' constructions. After Lesson Study she represented the concept not only by explaining the concept based on the textbook but also she used real objects familiar to students or in the classroom to help illustrate the concepts.

2. Demonstration of procedures

Demonstration of procedures to establish the validity of the concept was changed from a task based closed problem to a task based open problem. Zu taught using problem based tasks in the first part of each lesson. The students were given a problem at the start each lesson after the teacher had reinforced their previous knowledge gained from a prior lesson. In Lesson 4A Zu used a closed problem where she asked the students to fill in a large transparent cube using unit cubes with a different sized large cube for each group. In the second lesson the teacher posed the problem of how to use unit cubes (36 cubes) to construct as many cuboids as possible with varying dimensions, but the same volume. The students explored the variables of this problem. The groups discussed solving the problems on the worksheet. They presented the results in a table on the white board that had been prepared by the teacher. The teacher guided the students' discussions based on the students' results on the table. The volume of a cuboid was found through observing patterns and then discussing the formula for calculating its volume. For example: *What is the volume of all the cubes? What size is the cuboid?* This problem was an open problem and more challenging than the first lesson, which used a closed problem only.

Before Lesson Study, Zu demonstrated the concept of finding the volume of a cuboid. Students followed the teacher's explanation based on the examples that were given in the textbooks. The students solved the problems in the one way that was in accordance with the textbook. After Lesson Study the students demonstrated solving problems with the teacher as a facilitator. In solving the problem students used various methods and more than one way of solving it before they reached the conventional formula. It could be suggested that the way in which Zu's demonstration of procedures changed was a result of her reflections that were a part of the Lesson Study Cycle.

3. Classroom management

Zu's teaching changed to use less questioning techniques, was more encouraging of students and used more positive statements in the second lesson than the first. However, in both lessons she used questions, encouraged students, and made a positive statement to help students make progress with their ideas and to correct any misconceptions. In Lesson 4A Zu used questions to promote discussion that would help her identify the students' reasoning. In Lesson 4B Zu allowed students to work with and to use unit cubes to identify the characteristics of a cuboid in the group. She asked fewer questions. Students solved more problems individually and worked more with unit cubes. The students answered fewer questions orally and fewer questions were asked of the teacher and no one asked their classmates questions.

In both the lessons the teacher praised the students when they responded with the right answer. For example, when a student gave a correct answer the teacher gave her/him a sticker. When the class was divided into groups the teacher gave each group a name. The students learnt by actively working in the group as well as individually.

In Lesson 4B when the teacher asked them to compare the differences of a cube and a cuboid she held a cube model in her left hand and a cuboid in her right hand. The teacher offered help to the students when they needed help or had any difficulty with group working. The students were very happy to solve the problem. Zu had supported student attention by giving positive statements in both lessons. Zu encouraged more student reasoning and participation.

Before Lesson Study Zu did not pay attention to the difficulties of the students in her teaching. Students got the right answer but she did not inquire about their process of how they solved the problem. Assessing student achievement was based on the final score of the test. After Lesson Study she diagnosed students' difficulties by asking students at the end of the lesson to give feedback on what they found difficult. Also, she used a written test and observations of the students doing mathematics to access student achievement. Again this is a result of her involvement in the Lesson Study Cycle.

Change in Mathematical Content Knowledge

Zu's mathematical content knowledge did change over the two lessons. Demonstration of knowledge was changed as she developed a more accurate understanding of the concept of length and width of a rectangular prism.

In both the lessons she understood the formula for calculating the volume of a cuboid and a cube. She was able to compare the characteristics and the differences between a cube and a cuboid. Specifically, she knew that the cuboid had different lengths, widths, and heights but a cube had all its sides the same length. Both the shapes were constructed using unit cubes. The teacher also demonstrated knowledge of the differences between the properties of shapes and objects in two dimensions and three dimensions.

In Lesson 4A Zu lacked understanding of the meaning of the side of a cube. She counted the total sides of a cube, but she did not explain the meaning of side. Zu should have explained the position of the side of a cube. When discussing this in the group after the lesson, she was confused with the concept of length and whether or not the length was always longer than the width. This became an interesting discussion at Lesson Study Group Meeting. There were some interpretations of the length, such as the length being longer than the width, the length is the side facing the observer, the vertical line is the height, and the length is the horizontal position with the vertical line being the height on a cuboid. Finally, they came to the conclusion that the length is always longer than the width. So Zu had a greater understanding of the observation of the convention of length in 3-D objects. This gives a clear indication of the value of Lesson Study in helping Teacher Zu develop her mathematical content knowledge.

With regard to Zu's mathematical content knowledge for teaching mathematics, in her interview before Lesson Study, she had stated that her understanding of the problem was limited to the textbook. Zu was not confident and she had learnt mathematics only from the textbook. In her interview she explained:

> *I have lacked understanding of the mathematical concepts. I thought that how I was teaching was correct, as I have had no complaints from students.* (Interview Zu, 21 December 2011)

After the Lesson Study program Zu said that she was more confident in her understanding of the concepts of volume because she has discussed and shared with other teachers. As she explained:

> *I feel confidence toward understanding mathematical concepts that I have taught. I remember discussing that the length of a cuboid is longer than the width. The concept of length and width came from a rectangle. The rectangle is the base of a cuboid that has a height.* (Interview, 21 December 2011)

It was from her reflection of the discussion in the Lesson Study meeting that she developed powerful learning and understanding in her mathematical content knowledge.

Beliefs about Teaching and Learning Mathematics

According to the interview before Lesson Study Zu believed that mathematics was taught with the format of the teacher explaining the concept on the white board, giving an example and setting an exercise based on the textbook. The teacher was a learning resource. As she explained:

> *I teach mathematics using the textbook as a main resource. Each student has a textbook, so I explain the concept, give an example and problem exercises based on the textbook.* (Interview, Zu, 12 December 2011)

Observation of Zu in her teaching practice during Lesson Study found that she used problem based teaching, concrete representation, and group work to help students solve the problems. At the beginning of the lesson Zu presented a problem and distributed unit cubes for each group. She guided, observed, and asked the students question while they were doing mathematics. In Lesson 4A she presented the problem for the students to fill a large cube using unit cubes: they determined the total volume of a cube and the properties of element of a cube. In Lesson 4B the teacher gave an example of how to construct a cuboid using 12 unit cubes in different size. Then she gave students the task to construct a cuboid with 36 unit cubes in different ways. The students found many dimensions of a cuboid with the same volume. Then they determined the formula for volume of a cuboid. The teacher brought the problem to introduce the formula for calculating volume of a cuboid as the new concept.

After the Lesson Study Cycle Zu stated her belief was that mathematics should be taught by using problem-based teaching with the teacher as the facilitator and students working in groups. She also believed that a problem-based approach could be used to introduce the concept, not just in application to solve problems. In solving the problem the students became active and creative. They constructed a cuboid using unit cubes as real objects to see the characteristics of a cuboid and explored to solve the problem. Zu explained:

> *I believe that problems can help students to be creative in their thinking; they solve the problem using their own methods. With a problem solving approach, I give more positive responses, even though the students may give an incorrect answer. And I allow students to ask questions.* (Interview, Zu, 12 December 2011)

She thought that mathematics teaching should be creative and students should do mathematics and give their own ideas. Her beliefs developed noticeably as a result of the Lesson Study program.

Summary

In teaching mathematics during the Lesson Study program Zu changed her teaching approach from the first lesson to the second lesson. The way she taught was based on her improved pedagogical content knowledge.

- The teacher changed from using a one model representation to more than one model of different sizes to represent the concept of a cuboid;
- Demonstration of procedure to establish the validity of the concept was changed from a task based closed problem to a task based open problem;
- Classroom management was changed from asking questions to being more encouraging of students, and making more positive statements.

The mathematical content knowledge for Zu changed over the two lessons in demonstrating her knowledge. Demonstration of knowledge was changed as she developed a more accurate understanding of the concept of length and width of a rectangular prism.

Through the Lesson Study she changed her thinking about how to teach mathematics from a traditional textbook format to being able to use more of a problem based approach. The teacher believed in students solving the problem using their own way as this helped the students became active and creative.

Case 5: Lesson 5A and 5B by Teacher Sa

Teacher Sa had seven years teaching experience in primary schools. She taught Year Four students at SD 1M Bin in the inner city with a location about one kilometre from SD 8M Bin. There were about 30 students per class who come from the neighbourhood of the school. SD 1 M uses a bilingual program in Indonesian and English, especially for two subjects mathematics and science. This school has more facilities than normal schools. She was capable in English and had some previous professional learning experiences in teaching mathematics, but she did not know about Lesson Study as a professional learning tool. Lesson Study was introduced to her by the researcher. As a volunteer teacher in Lesson Study she taught Lesson 5A and Lesson 5B on the topic of perimeters and the area of a parallelogram. Lesson 5A was the original lesson that was designed by the group, and Lesson 5B was a revised lesson from Lesson 5A. Both the lessons were video recorded, audio recorded, and observed by another teacher.

Description of Lesson 5A

Lesson 5A was taught on 25 November 2010. Sa taught the topic of calculating the perimeter and area of a parallelogram with a Year Four bilingual class. She gave students grid paper, scissors, glue and straws. She also used a computer as a tool for teaching. She segmented the lesson into three distinctive phases: initial activity, main activity, and end activity. At the beginning of the lesson she sang a song about shapes and geometry that was modified from another song and showed these shapes on the white board. She asked the students to name the plane figures in both languages. Almost all students were able to answer orally with the right answer, both in Indonesian and English. She emphasised the shape of the parallelogram. Then she explained the objective of teaching: *To determine the perimeter and the area of a parallelogram.*

The teacher went on to do another activity. She began by dividing the class into groups of four and distributed handouts and grid paper. She asked students: *Draw a rectangle on the grid paper, and cut out the rectangle along the sides.* In a few minutes all students were working after she had given an example of what to do. The teacher continued to ask students: *On the rectangle, draw a line through one angle point to meet on the side in front of the angle.* As previously, the students were waiting for an example to do it. The teacher continued to ask students: *Cut the rectangle into two parts using a line: The parts are a right triangle and a trapezium. Rearrange them to become a parallelogram.* Students rearranged the right triangle and the trapezium to become a parallelogram, but almost all students had difficulty in making a rectangle on the grid paper. For the next step Sa asked the students to attach and to glue straws along the sides of parallelogram. The students measured the straws as a frame for the length of the sides of the parallelogram ABCD in centimetre units. They got two pairs of sides, which had the same length. She guided the students to see that the perimeter of the parallelogram was the total length of the four side lengths, by asking: *How long the sides of AB, BC, CD, and DA? Calculate the total sides of parallelogram ABCD.*

For the next step the teacher asked students to count the total unit squares on the parallelogram that they had constructed. Students compared the area of the parallelogram with the original rectangle, and also found the relationship between the elements of both shapes. The teacher observed and helped students having difficulty. Representative students from each group presented their results in front of the class. They showed how the parallelogram was constructed. Another group presented orally. For the next step, by using a dynamic figure construction, the teacher showed the relationship between the side of a rectangle OACD and a parallelogram ABCD. Then she guided the students to find the area of a parallelogram, by saying and asking: *Look at parallelogram ABCD and a rectangle OACD. Which sides are equal? Is the distance between two parallel sides of a parallelogram ABCD equal to the height of a rectangle OACD?*

Lesson Study Group Meeting after Lesson 5A

A Group Meeting was held soon after the lesson. Some comments or recommendations from the meeting are as follows:

- Representation of ideas – The teacher was surprised that the students did not know how to construct a parallelogram using grid paper. They were waiting for the teacher to give an example. They had no experience using grid paper. They usually drew parallelograms or other shapes directly on blank paper without grid paper. Using grid paper as a tool to draw a rectangle was a new experience for students, so they were confused. The group recommended that the next lesson could be improved by the use of a geoboard and rubber bands to construct the parallelograms. Also one teacher commented that using straws to represent a side of a parallelogram was difficult for many students. It would have been better to glue the straw on a different coloured paper to help students see it more easily;

- Demonstration of procedures – Constructing a parallelogram by drawing a rectangle, cutting the rectangle, and re-arranging it to become a parallelogram were time consuming. The group recommended that it would be better to use other procedures that used less time;

- Classroom management – The teacher gave the students the task to do in groups, but they worked individually. The students discussed in their group for too long, and many students worked individually in their group. The meeting recommended that this could be improved by worksheets being distributed with only one worksheet for each group, rather than to each student to avoid students working individually;

- Demonstration of knowledge – On the students' worksheet the problems were found to be unrealistic for the students. The dimensions used were inappropriate to the size of the objects measured, for example, the dimensions of a wet rice field were given in centimetres. The group suggested that it was better to be realistic with measuring units.

Based on group discussion and comments made in the meeting after Lesson 5A suggestions for changes were made and shown in Table 4.14.

Table 4.14: Suggested Changes for Lesson 5B.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Representation of ideas	Use the geoboard for each group to construct two dimensional shapes.
• Demonstration of procedures	Construct a shape using a shorter time period.
• Classroom management	Encourage students to be more involved in their group activities rather than working individually.
Mathematical content knowledge • Demonstration of knowledge	Use realistic contextualised problems.

Description of Lesson 5B

Lesson 5B was taught nine days later on 4 December 2010. Sa retaught the same topic with a different class at Grade Four. At the beginning of the lesson she and her students sang a song about shapes and geometry. She reminded the students of several of the polygons by showing the figure on the screen of computer and introduced the geoboard to construct each polygon. She presented the objective of the lesson: *To determine the perimeter and the area of a parallelogram*.

When working in a group the students constructed a parallelogram on a geoboard, they put string along the sides of the parallelogram. Sa told the students to show that the perimeter of a parallelogram was equal to the length of the string. To enrich students' knowledge about a parallelogram, she asked students to construct at least three parallelograms on the geoboard and draw them on dot paper.

With a dynamic figure, she showed how a parallelogram could be constructed from a rectangle, and that the area of a parallelogram was equal to the area of the original rectangle. The students noted the relationship between the elements of the rectangle and the parallelogram. After the students had understood the properties of the sides of a rectangle and a parallelogram and their relationship, Sa asked students to find the area of the parallelogram ABCD on dot paper. She guided students to calculate the area of each parallelogram by counting square units. While students were working on the worksheet, she observed and helped any students having difficulties. Some students were confused about the height of a parallelogram and finding the area of a parallelogram. Also, a few students had difficulties drawing a parallel line on dot paper. After calculating the area of parallelogram, Sa asked students to draw a parallelogram with an area of 6 square units on dot paper. After the students had solved many tasks, the teacher reviewed how to calculate the formula for the area of a parallelogram and the formula for calculating the perimeter of a parallelogram. In the next step, Sa gave the above to do as an individual task.

Lesson Study Group Meeting after Lesson 5B

A Group Meeting was held soon after the lesson. The lesson went well, and so the observers made few comments. Some comments were noted:

- Representation of ideas – Due to a lack of geoboards, students needed to share and this caused difficulties;
- Classroom management – Students answers were not presented to the whole class. It was better if students could have presented the solution to the whole class.

Based on the discussion at the Group Meeting a few suggestions were summarised in the Table 4.15.

Table 4.15: Suggested Changes for the Next Lesson.

Aspect of the Lesson	Suggestions for the Teacher
Pedagogical content knowledge • Representation of ideas	Prepared dot paper as a substitute for the geoboard.
• Classroom management	Present students' answers in the whole class.

A summary of change in pedagogical content knowledge in practice based from the researcher's observations are shown in the Table 4.16. For this table a √ symbol indicates at least one observed incident of the particular criterion for every five minute period of teacher activity and student working.

Table 4.16: Pedagogical Content Knowledge as Demonstrated in Teaching Practice (Lessons 5A and 5B).

Teacher Using Activity	Student Working							
	Individual		Pair		Group		Class	
	5A	5B	5A	5B	5A	5B	5A	5B
KNOWLEDGE OF TEACHING								
Representation of Ideas and Use of Resources								
Concrete	√	√			√√	√	√√√	√√
Pictorial	√	√			√√√	√√	√√	√√√
Real world					√	√	√	√
Symbolic/abstract					√	√	√	√
Demonstration of Procedures								
Tasks/problems					√√	√√	√√	
Discussions					√√√√√	√√√√√	√	√
Student presentations					√	√	√√√	
Teacher presentations					√		√	√√√
Classroom Management								
Positive teacher statements	√√	√			√	√√	√	√√
Negative teacher statements								
Encourages students in difficulty	√√	√			√√√√	√√	√	√
Encourage student to have different point of view						√√	√	√
Encourage students to participate	√	√			√	√	√	√
Encourage student explain their reasoning								
Type of Problem								
Open					√	√√		√√
Closed	√	√			√	√	√	√
Drill	√	√			√	√	√	√
Applied					√	√	√	√
Puzzle problems								
KNOWLEDGE OF STUDENTS' LEARNING								
Level of Teacher Questioning								
Knowledge	√	√			√	√	√√√	√√
Comprehension					√√√		√	√√√
Application					√		√	√
Analysis								
Synthesis								
Evaluation								
Diagnosing Learning Difficulties								
Oral questions	√	√			√√	√	√	√
Written questions	√	√			√	√	√	
Written Tests	√√√	√√			√√√	√	√	√√

Change in Pedagogical Content Knowledge

Pedagogical content knowledge for Sa changed over the two lessons. The changes were noted in the following ways:

- Representing the idea – Sa changed from cutting and removing shapes for constructing a shape in the first lesson to using a geoboard and dot paper in the second lesson;
- Demonstrating a procedure – Sa moved from a long process to demonstrate a procedure in the first lesson to a shorter process in the second lesson;
- Classroom management – Sa changed from having students working individually in a group setting in the first lesson to encouraging students to participate in working together in a group in the second lesson;
- Type of problem – Sa used closed and open problems in the first lesson, then she moved to more open problems in the second lesson;
- Using teacher questioning – Sa used more knowledge questions in the first lesson, but changed in the second lesson to asking more exploratory questions that also explored the concepts more broadly.

The descriptions of these changes in pedagogical content knowledge for each change have been classified under the headings knowledge of teaching and knowledge of students' learning.

Knowledge of teaching

Sa showed changes in her teaching over the two lessons in the way she represented ideas, demonstrated procedures, managed the classroom, and chose types of problems.

1. Representation of ideas

In the first lesson Sa showed how to construct a parallelogram using two shapes by cutting a rectangle into a right angled triangle and a right-angled trapezium. The students re-arranged them to construct a parallelogram. The teacher had difficulty managing the students' activities. She set students to work in groups but they worked individually. During the activity most of the students had difficulty and could not follow the steps of the task from drawing, cutting, and arranging shapes. In Lesson 5B the teacher showed how to construct a parallelogram by placing a rubber band on the geoboard. She set students to work in a group that was easier to manage than in Lesson 5A. Also the students constructed the new shape more easily using geoboard than by arranging shapes.

2. Demonstration of procedures

Her teaching changed from being a long process to explain the procedure to a short process in Lesson 5B. In Lesson 5A students used grid paper to construct a rectangle. The rectangle was cut according to the straight line from one vertex of an angle to the side in front of the angle. One was a right-angled triangle. The other was a right-angled trapezium. The students re-arranged the two shapes to become a parallelogram. One student was successful and constructed a parallelogram for the first time but it took a long time. Then the students cut and glued straws along the lengths of the parallelogram making a straw frame. By measuring the length of the straws, the students knew the perimeter of the parallelogram was equal to the length of the straws used in the frame of a parallelogram. The teacher needed to use a long process to help students understand the perimeter of the trapezium.

In Lesson 5B the students were faster to construct a parallelogram using a geoboard rather than cutting grid paper as in Lesson 5A. A few of students had difficulty in constructing a parallelogram because using rubber bands and geoboard was something new. The teacher successfully guided them in the group. The students had more opportunity to try and construct a parallelogram. The students could see clearly that the length of the piece of string along the sides would match the perimeter of the parallelogram. The teacher needed only a short time for the procedure of placing rubber band on a geoboard. The students were more creative and productive to construct 2-D figures. So the students had more opportunity to explore how to construct 2-D figures.

Sa lacked a problem solving approach to teaching mathematics before the Lesson Study Cycle. She taught step-by-step as she explained the procedure, gave examples, and the students did exercises from the textbook. As a teacher, she demonstrated how to solve the problem. She explained the process and used more examples from the textbook. After Lesson Study she taught mathematics differently using a problem solving approach in limited topics. In her interview she explained:

> *I have taught using a problem solving approach. Re-teaching the Lesson Study topic to a different class made me comfortable with the teaching steps. For the new topic, I taught measurement. I needed extra time to prepare a lesson plan and worked hard, but I found it easier to implement in teaching practice.* (Interview, Sa, 24 December 2011)

She began to demonstrate a better understanding of a problem solving approach. For example, she used an alternative method to solve the problem and not only followed the textbook format and procedures. The students solved the problem using various methods, so they had a deeper understanding based on their experience. For example, using dot paper square of size 5 × 5 students constructed parallelograms of different sizes then they determined the area of each size. Students were active and demonstrated the problem solving process. The teacher observed and helped the students to introduce the procedures.

3. Classroom management

Sa changed her style of classroom management in her teaching. She changed from students working individually in a group setting to encouraging students to participate by working together in a group. Almost all students were active in their learning in both the lessons. In Lesson 5A each student tried to construct a parallelogram using grid paper and straws. Even though the students were seated in groups they worked individually using their grid paper. Sa did not encourage students to participate in the group. But she guided and encouraged students having difficulties. She asked the students to report by using one of the individual student's results as a representation of the group work.

In Lesson 5B she encouraged students to participate more in each group. She gave a point such as giving a score of one hundred for students who solved the problem immediately, for example: *How many parallelograms with an area of 4 square units could be made on dot paper?* She guided and encouraged students having difficulties. They discussed in a group how to construct a parallelogram, and how to determine the perimeter by using string around the parallelogram on a geoboard. The students presented their group findings in front of the class. Also, Sa asked the students to identify the formula for finding the area of a parallelogram by connecting the elements of a rectangle and a parallelogram. In these activities students explored how to solve a mathematical problem much more than in Lesson 5A. The change in her style of management could be attributed to her reflections as part of the Lesson Study Cycle.

4. Type of problem

Sa's teaching plan was based on problems in both lessons. There was a change to use a more open problem in the second lesson. In Lesson 5A she began with an open task that was given to groups: *Draw a rectangle of any size of your choice on grid paper.* After students had drawn their rectangle, Sa then gave another task: *On your rectangle draw a line starting from one angle and ending on the side opposite the angle and cut according to the line into two parts, a right triangle and a trapezium.* Students had confused in drawing this because drawing a rectangle with any size on grid paper was a new experience for them. She read the task aloud and gave key points to students to understand the problem. She gave hints and an example. The students were able to construct a parallelogram. Based on their construction, the teacher guided them to calculate the perimeter and the area of a parallelogram. She asked students to measure the length of frame of parallelogram made of straw as perimeter. To guide the students calculate the area of a parallelogram she asked students to count the total square that covered the parallelogram.

In Lesson 5B Sa gave more open problems in the beginning of the lesson, during the lesson and at the end of the lesson. In the beginning of the lesson she gave an open task: *Construct a parallelogram of any size on geoboard.* She walked around the group and helped students who had difficulties. During the lesson Sa gave an open task such as: *Construct a parallelogram with an area of 4 square units on the geoboard or dot paper.* The students had more opportunity to explore and solve the problem using their own methods. Using this later change of task type to one of a more open nature.

Knowledge of students' learning

Sa's knowledge of student learning changed during the period of the Lesson Study Cycle. This was evident in the level of her questioning and how she diagnosed learning difficulties in the students.

1. Level of teacher questioning

Sa used more questions and questions of different levels. In both the lessons she asked the students mostly questions based on knowledge - 5 questions in Lesson 5A and 6 knowledge questions in Lesson 5B. For example, while she was showing the figure, she was asking the students: *What was the name of figure?* In the whole class format she also asked about the properties of a rectangle and a parallelogram. She used 5 comprehension questions in Lesson 5A and 8 comprehension questions in Lesson 5 B. For example, when she explained the connection between the side part of a rectangle and a parallelogram, she asked: *What are sides the same length between a rectangle and a parallelogram?* She asked them 2 application level questions in Lesson 5B and 1 question in Lesson 5A. So in Lesson 5B, she used more questions and more questions of different levels than in Lesson 5A. This was related to giving more open tasks in Lesson 5B, which had resulted from her involvement in the Lesson Study Group Meeting.

2. Diagnosing learning difficulties

Diagnosing students' learning difficulties also changed. She used more oral questions to find this out in Lesson 5B. She also identified students' difficulties based on the tasks. At the beginning of Lesson 5A almost all students had difficulties in constructing a shape using grid paper. By her giving an example the students were able to construct a rectangle. Also, they had difficulties in drawing a line on a rectangle to make it a right triangle and a trapezium. The teacher guided and asked questions of the students. They waited for the teacher to give the example before they worked.

In Lesson 5B when she gave the task to construct a parallelogram on a geoboard not all groups were able to construct the parallelogram. They had problems making parallel lines using rubber bands. She asked oral such as questions of the students about how to construct a pair of parallel lines using rubber bands and they were then able to construct a parallelogram. Then she continued to construct a parallelogram on dot paper for each group. She observed the groups and found that some students had difficulties in constructing a parallelogram. She asked students oral questions, such as: *Could you make a parallel line?* By counting the dots on this paper they were able to make a parallel line.

By using a geoboard or dot paper she gave various open problems with the solution being not only parallelograms of different sizes but also parallelograms of different sizes but having the same area. For example, she gave a task on a worksheet: *Construct parallelograms with an area of 6 square units using dot paper.*

Sa understood more clearly after the Lesson Study discussion about how to diagnose learning difficulties in the students. This came about from the discussion where the other teachers observed what the difficulties were and what might be causing them. Sa implemented these suggestions into her second lesson by asking the students more open ended questions and assessing their answers and then rephrasing questions and explanations and setting tasks at different ability levels.

Changes in Mathematical Content Knowledge

Sa demonstrated some changes in her teaching practice over the two lessons, which possibly reflected some changes in her mathematical content knowledge.

- Understanding of procedure – Sa developed her understanding of the procedure for calculating the area of a parallelogram;

- Demonstrating knowledge – Sa demonstrated more accuracy in making mathematical statements in the second lesson.

1. Understanding procedure

Sa changed in her use of procedures to calculate area problems. In Lesson 5A she used a strategy of cutting, drawing and rearranging to construct a parallelogram. Then she demonstrated how to calculate the perimeter by making a frame of straws as long as sides of a parallelogram. She measured the straws as being the perimeter of a parallelogram. The students then constructed a parallelogram on grid paper but needed a long process to do so.

In Lesson 5B her practice had changed and she used a rubber band and geoboard to construct a parallelogram. Next she introduced the perimeter of a parallelogram by placing a piece of string as the sides of a parallelogram to show the perimeter of a parallelogram. With a geoboard and dot paper, she used an improved strategy to construct a parallelogram in various tasks. For example, she presented a problem: *Construct a parallelogram with an area 6 square units: Draw as many as you can on the dotted paper.* The students were able to construct a parallelogram in more than one way using geoboards and dot paper.

2. Demonstration of knowledge

In Lesson 5A Sa gave examples of concepts less accurately than she did in Lesson 5B. She asked students to draw a field with the size 180 metres × 120 metres but in the shape of a parallelogram. She was also less accurate giving the dimensions of the problem: *The perimeter of a garden in the shape of a parallelogram is 24* *centimetres. If the first side is 5 centimetres long. How long is the second side?* This problem is not realistic because the size of the garden is very small. In Lesson 5B she presented the problem more accurately using standard units that related better to the real situation.

Before she participated in the Lesson Study program she discussed her mathematical content knowledge during an interview. With regard to understanding concepts, she said it was sometimes difficult to understand on her own. She explained:

> *The concepts are sometimes difficult for me to understand on my own. If I have difficulty with any concepts, then I tend to leave the problem and move to the other problems.* (Interview, Sa, 24 December 2011)

She never developed problems and just followed a problem procedure from the textbook. In solving the problem, she used one way of solution according to the textbook and used examples based on the textbook. After the Lesson Study program, the concepts were easier to understand for Sa, because of the discussions and sharing ideas with other teachers and solving problems together. Problem solutions were developed based on the textbook and sharing with other teachers. With regard to understanding the problems she explained:

> *The problem is easily understood by discussing together with other teachers. The concept is easy to understand because of discussing and sharing ideas with other teachers and solving together. Such as, understanding the concept of length and width is discussed in a Lesson Study Group Meeting.* (Interview, Sa, 24 December 2011)

Problems were solved using various methods and in more than one way. Sa identified students' misconceptions through discussion and observation and followed the thinking process of the student solving the problem. Sa felt more confident about her mathematical knowledge and felt that she had a deeper understanding of mathematical knowledge after discussion and sharing with other teachers in the group. Demonstrating knowledge to students was more accurate because of her personal understanding of mathematical ideas.

Beliefs about Teaching and Learning Mathematics

According to Sa's interview before Lesson Study she believed that mathematics was taught with a teacher-centred approach in a traditional setting, viewing all the students as having the same average knowledge for that level. She suggested that the teacher did not have to pay attention to differentiation to cater for in students' learning. In teaching through problem solving she thought that the students would lack an understanding of the problem and have difficulty with solving the problem. She thought that this approach would need extra time. She explained:

> *In general, a teacher teaches with a teacher centred approach, pays less attention to students' individual needs and in a class setting, students have to memorise much of the formula and mathematical procedures and many of the student exercises.* (Interview, Sa, 24 December 2010)

Observation of her teaching practice showed that she taught with well-prepared teaching materials, used a group setting, and used a problem based approach. She used problem-based teaching at the beginning to build the new procedure of how to calculate the area of a parallelogram. She also used computer tools to represent the ideas more dynamically. In Lesson 5A she used grid paper as tool to construct a rectangle but the students had difficulty in the early stages because they had never used it before. By giving an example, they were able to solve it. In Lesson 5B she gave the students an open problem and dot paper to construct a parallelogram. She took more time, paid more attention to know what students were doing, and what students were thinking. The students faced the problem sometimes in a group and sometimes as individuals. If the students solved the problem by themselves, they needed more time. The teacher was sometimes not very patient and she wanted to help students too soon. She was over eager to help them, rather than allow the students to learn through their own mistakes. Generally Sa's beliefs about teaching before Lesson Study were inconsistent with her classroom practice.

After the Lesson Study sessions she believed that mathematics could be taught in interesting ways in a group setting. She suggested that in this way students had opportunities to share and discuss and to present their own findings. She believed that open problems made students think creatively. She thought that doing problem solving made students active, creative, and confident. She explained:

> *Using problem solving makes the students solve the problem in various methods so they have a deeper understanding based on their experience. It deepens students' understanding and the students improve through solving the problem, and the teacher guides and introduces the new concept.* (Interview, Sa, 24 December 2011)

Her beliefs reflect changes in her teaching following Lesson Study and changes can be attributed to her reflections in involvement in Group Meetings.

Summary

Changes were noted in the following aspect of Sa's teaching, which reflect development in her pedagogical content knowledge:

- Constructing a parallelogram changed from cutting and removing shapes to placing a rubber band on geoboard and using dot paper;
- Her demonstration of procedure changed from a long process to a shorter process;
- Students working changed from working individually in a group setting to encouragement of students to participate in working together in a group;
- Choosing the problem changed to the use of more open problems in the second lesson.

Regarding mathematical content knowledge, Sa developed her understanding of procedures and demonstration of knowledge.

- She improved her knowledge of understanding of procedure in using geometrical models for calculating the area of a parallelogram;
- She had a more accurate understanding of mathematical ideas, mathematical structure and connections between ideas.

Initially, she believed that mathematics should be taught in a traditional setting and she viewed the students having the same average knowledge. She thought that teaching mathematics through problem solving took too much time and she lacked an understanding of how students solved problems and could not help when they had difficulty. Her beliefs changed after the Lesson Study program. She now believed that problem solving helped students develop more ideas, become more creative and develop deeper understanding.

Observer Teachers' Cases

As was mentioned earlier, the Lesson Study Cycle project involved twelve teacher participants from inner city and suburban areas in Bengkulu. During the Lesson Study Cycle not all the participants had an opportunity to become a volunteer teacher who taught the lesson planned by the groups. There were five volunteer teachers Ha, Wi, Mi, Zu, and Sa. The rest of the participants were made up of seven observer teachers. The three teachers from Group I were La, Yu, and He, and four teachers from Group II were Ma, Yi, Si, and Ri shown on Table 4.17.

Table 4.17: The Observer Teachers in Lesson Study Group I and Group II

No	Teacher	Grade	School	Group
1	La	IV	SD 9P	I
2	Yu	V	SD 1P	I
3	Hen	IV	SD8P	I
4	Ma	V	SD 1M	II
5	Si	V	SD 4M	II
6	Yi	IV	SD 4M	II
7	Ri	V	SD 8M	II

The observer teachers were involved in Group Meetings and classroom observations of the lessons. The results from the lesson observations, video recordings, group discussion post teaching, and interviews are grouped here according to seven cases, with each case being one of the seven observer teachers.

Case 1: Teacher La

La had 19 years of teaching experience in primary schools. She lacked experience of professional learning and Lesson Study as a professional learning vehicle. During the Lesson Study meeting La was an active participant in the Group Meeting and the classroom observation. According to her interview her changes in pedagogical content knowledge were in the representation of ideas and diagnosing students' difficulties.

Initially, La used teaching aids, but these were limited in representing the ideas. She taught mainly based on the textbooks. To explain procedures La used concrete and pictorial models based on the examples given in the textbook. After Lesson Study La used real objects around the classroom to represent the concept and textbooks as additional resources. In addition, La used her own teaching aids, such as a corner of the wall to show an example of an angle.

She demonstrated procedures by explaining and defining it in a traditional class setting. Based on the textbook, she explained an example and asked the students to do a similar exercise. After Lesson Study La taught with fewer explanations, gave students problems and allowed students to solve the problems according to their own methods. La guided the students in solving problems to understand the new concept or idea.

La said that problem solving was very important, but the students had difficulty understanding the problems, and they lacked the ability to ask questions. After the Lesson Study meeting La believed that the teacher not only explained the concepts, but also the teacher needed to listen to and observe students doing mathematics. The students solved the problems with various methods, and the teacher guided the introduction of the new concept. La said that group work made it easier to observe and identify student learning.

Case 2: Teacher Yu

Yu had 4 years teaching experience in primary schools. She had limited professional learning and was unfamiliar with Lesson Study as a method for professional development. Yu actively participated in Group Meetings and classroom observations. She explained that changes in her pedagogical content knowledge were in the representation of ideas and demonstration of procedures. Previously, Yu had not used teaching aids to help her represent the concepts and ideas because she lacked understanding of how to use them and also there were not enough resources for the number of students. Yu taught based on textbooks and curriculum statements only. After Lesson Study she used teaching aids, such as length measuring tools. She also used manipulative objects and real life things from around the classroom, or objects familiar to the students.

Initially, Yu demonstrated the concepts by explaining the procedures. The students listened and followed the explanation and did exercises from the white board individually. After Lesson Study the students worked in groups, and then the students demonstrated their results in front of the class. The teacher guided them to find the new concept.

Before Lesson Study Yu believed that the teacher explained the concept and gave an example, then the students worked on similar problems and exercises to solve the problem. After Lesson Study Yu thought that problem solving was a good approach, but that using this approach needed more time. However, she did agree that problem solving improved student thinking in solving story problems.

Case 3: Teacher Hen

Hen had 11 years teaching experience in primary schools. He had experienced professional learning about teaching and learning in general rather than mathematics in particular. He was unfamiliar with Lesson Study as a method for professional development. He actively participated in Group Meetings and classroom observations. As a result of observations and group discussions during Lesson Study he changed in his teaching in the way that he represented ideas and diagnosed students' difficulties.

Initially, he had used limited manipulatives that were available at the school and used general examples from the textbook. He told, explained, and defined the concept for the students. After Lesson Study he added more or new manipulatives made by teachers or students or selected from the environment around the students. The students were involved in the demonstrations through the teacher's questions.

He identified students' difficulties by just looking at the last activity they had undertaken. He also assessed students' learning by using a final test or exercise. After Lesson Study, he diagnosed students' difficulties through group discussion, asking questions, and students' answers. From the process of discussion, he identified what the problem was. To assess student learning he used observation in class, and oral and written tests.

Before the Lesson Study program he said that mathematics was taught by explaining the concept, defining the concept, finding the formula, and solving the problem. The students solved the problem and completed more exercises. After Lesson Study, he believed that the problem solving approach gave an opportunity for students to solve the problem according to their own methods. He thought mathematics should be taught in a joyful way, because students find it easier to comprehend and grasp concepts when learning is enjoyable.

Case 4: Teacher Ma

Ma had 25 years teaching experience in primary schools. He had been involved in many professional learning opportunities in mathematics and also as a teacher facilitator with the Teacher Association (KKG). In KKG Ma stated that as facilitator he asked the participants to bring classroom problems from each school and then they discussed how to solve their problems, such as organising a Mathematics Olympiad. Ma had not experienced Lesson Study as a professional learning medium before. During Lesson Study Ma actively participated as he had more experience than most of the teachers about teaching and learning mathematics. Ma was involved in discussions in Group Meetings. His change in pedagogical content knowledge was in his representation of ideas and demonstration of procedures.

In representing the idea of his teaching, Ma used the limited materials available at the school, including textbooks, and manipulatives. For example, he explained:

> *I used an eraser tool from the white board to show an edge of a cuboid before Lesson Study. It was better after as I used a frame of a cuboid model.* (Interview, Ma, 24 December 2011)

After Lesson Study Ma used teaching aids that were available at the school and also he prepared as many alternative materials as needed.

In Ma's demonstration of procedures he did not initially use a problem solving approach. Ma used a step-by-step procedure, explained the concepts, gave examples, and set the students to complete the exercises. As a teacher Ma was active to demonstrate the problem solving process. After the Lesson Study Cycle meeting Ma taught using problem solving in limited topics. The students solved the problems by various methods so that they had a deeper understanding based on their experiences. The teacher guided them in introducing the new concepts and the students were active in demonstrating the problem solving process within the group. Students were also active in presenting the concept to the class.

Case 5: Teacher Si

Si had 33 years teaching experience in primary schools. She had a few limited professional learning experiences in teaching and learning in primary schools. She did not know about Lesson Study as professional learning. Si was an active participant in Lesson Study, both in Group Meetings and classroom observations. Her changes in pedagogical content knowledge were about demonstration of procedure and diagnosing students' difficulties.

In her demonstration of procedures Si explained the concept based on the textbook by writing and telling the students in front of the class to solve the problem.

The students solved the problems in drill exercises based on the textbook. After Lesson Study Si involved manipulatives to represent the concepts; students solved the problem in various methods, and demonstrated the problem solving process using teaching aids. She guided students to solve problems.

Diagnosing students' difficulties received no attention during this time prior to Lesson Study. Accessing student learning was based on a final score on a written test in the last lesson. After the Lesson Study Si used questions and students answers to look at students' difficulties. Si not only used a final score of a written test, but also she looked at the processes used by the students to solve problems.

Si said that she thought mathematics was taught by explanation and the teacher wrote and guided the students in front of the class to solve the problem. She thought that students needed to have drill exercises and solve the problems by involving manipulatives. After the Lesson Study program Si said that when using a problem solving approach students were more creative and showed a greater improvement in their knowledge content when an open ended problem solving approach was used by the teacher.

Case 6: Teacher Yi

Yi had ten years teaching experience in primary Schools but had not had any professional development, so this was the first time that Yi had participated in Lesson Study. She actively participated during Lesson Study both in classroom observation and Group Meetings. Previously, she had limited teacher knowledge in using teaching aids to describe a concept. In teaching she told and explained the procedure based on the textbook. After Lesson Study she used teaching aids and the manipulatives available at the school.

She demonstrated with more explanation, used examples based on the textbook by talking and using pictures in the textbook. After Lesson Study, she paid attention to the students doing mathematics and guided students through the introduction of the new concept.

Prior to Lesson Study she had not paid attention to the students' difficulties. Diagnosis of student learning was based on purely a final score on a written test. After Lesson Study she diagnosed students' difficulties by using a question and answer technique. She assessed student learning not only based on a final score, but also she looked at the process of students solving the problem. Students also explained how they solved the problem.

In her belief about teaching mathematics she stated that the teacher wrote on the white board and guided the students to solve the problem then students solved problems based on the textbook. After the Lesson Study program she said that teaching mathematics should be done with a problem solving approach whereby students solved problems in their own way. Using this approach the teacher motivates students doing mathematics and it becomes a creative process.

Case 7: Teacher Ri

Ri had 20 years teaching experience in primary schools. She had been involved in professional learning, such as Teacher Association (Kelompok Kerja Guru) (KKG) and other workshops. She had not had experience in Lesson Study as professional learning before. During the Lesson Study she actively participated in Group Meetings and classroom observations. Ri's changes in pedagogical content knowledge were in her representation of ideas, demonstration of procedures, and diagnosing students' difficulties.

In the representation of ideas Ri explained the procedures as shown in the textbook, and the students read the textbook. Ri was afraid to move away from the procedures in the textbook. She used teaching materials available at the school. After the Lesson Study meeting she guided students to try to solve the problems using their own methods. Ri used teaching materials at the school, but modified the teaching material. She also based examples on the environment around the students.

After Lesson Study the students demonstrated how they solved the problem; the teacher listened and followed the students doing mathematics. Students were actively learning and solving the problems using various methods and finding more than one solution.

Ri had not paid attention to diagnosing students' difficulties as students had only little opportunity to do mathematics. She assessed student learning on the basis of a final score on a written test. After Lesson Study she followed the process of how students solved the problem so she could see the students' difficulties. To assess student learning she used a final written test and observation of the students doing mathematics.

Ri believed that the teacher explained the procedure, gave an example, and then students did exercises that she suggested. Using manipulatives in teaching mathematics was important for the students to gain a clearer understanding. After Lesson Study Ri believed that the use of a problem solving approach made it easier to get students involved in doing mathematics.

Summary

The non-volunteer teachers changed in pedagogical content knowledge and their beliefs about teaching and learning mathematics. Changes in pedagogical knowledge happened in the representation of ideas, demonstration of procedures and diagnosing students' difficulties. Representation of ideas or concepts were changed from using general examples based on the textbook with limited manipulative availability at the school to adding resources made by the teachers, students or taken from the immediate environment. Demonstration of procedures changed from teaching mathematics by explaining and defining the procedure in a traditional class setting to teaching with fewer explanations and then allowing students to solve the problem through group work using their own methods and also guiding students to understand the new concept or ideas. Diagnosing students' difficulties received more attention after the Lesson Study Cycle. Teachers' beliefs about teaching and learning mathematics changed from a belief that mathematics was taught by explaining the concept, giving examples and the students doing exercises, to a belief that the students should be active in solving the problems using their own methods and with guidance by the teacher.

Chapter Summary

This chapter presents the results from qualitative data collection from this study. The participants were classified into two categories. There are five teacher participants as volunteer teachers, and seven teachers as observer teachers. The results from the qualitative case study for five volunteer teachers were described and summarised at the end of each participant case in Chapter Four. The findings of the project will be discussed in Chapter Five with the presentation of the conclusions recommendations and implications in Chapter Six.

CHAPTER FIVE

Discussion

Introduction

This research project has focused on describing the impact of professional learning through the use of a Lesson Study Cycle on the teaching related to problem solving of a small group of Indonesian primary school teachers. Findings from the study have provided valuable information about the impact of professional learning in changing teaching practice. This chapter restates the aims, objectives, and results obtained in the study. The main sections of this chapter discuss the findings from the study and their implications according to the theoretical framework underpinning the study.

Research Aim and Objectives

The particular objectives of this study were to describe changes in teachers' pedagogical content knowledge, teachers' mathematical content knowledge, and teachers' beliefs about teaching and learning mathematics. In general, the study sought to evaluate the effect of Lesson Study in bringing about such change in teachers' practices in primary school mathematics.

The main research question was: *To what extent does professional learning based on problem solving and a Lesson Study approach effect: Teachers' pedagogical content knowledge, teachers' mathematical content knowledge, and teachers' beliefs about teaching and learning mathematics?*

This was supported by a subsidiary question: *In what ways does Lesson Study bring about change in teachers' practice in teaching mathematics through problem solving in an Indonesian primary school context?*

Findings of the Study

The results of the study were presented in Chapter Four under the broader categories: changes in teachers' pedagogical content knowledge, changes in teachers' mathematical content knowledge, changes in teachers' beliefs about teaching and learning mathematics, and the value of the Lesson Study Cycle approach in bringing about changes. Triangulation in data gathering was achieved by developing data from a range of sources, namely classroom observations, videotaping of Lesson Study Group Meetings, and semi-structured interviews conducted during and after the Lesson Study process. A number of key findings which emerged from the results were that changes occurred in: representation of ideas, demonstration of procedures, classroom management, use of different types of problems, knowledge of student learning, level of teacher questioning and diagnosing learning difficulties, understanding of procedures, understanding of mathematical relationships, demonstration of knowledge and teacher beliefs. Those findings are summarised under the following headings.

- Change in pedagogical content knowledge;
- Change in mathematical content knowledge;
- Change in teacher beliefs about teaching and learning;
- Value of Lesson Study in bringing about changes.

The discussion of the results in this chapter will include links that emerged between categories. It is evident from the results already presented that changes were more evident in relation to teachers' pedagogical content knowledge than mathematical content knowledge. Also, these changes were noted to a different extent with the five volunteer teacher participants Ha, Wi, Mi, Zu, and Sa. The following discussion analyses these changes and the different extent to which they occurred and explores reasons. As well, there are possible implications for action at several different levels, namely individual teacher, school, professional learning organisers, and the national government as the policy maker in the system.

Change in Pedagogical Content Knowledge

The area in which greatest change occurred was the pedagogical content knowledge of the teachers. Specifically this change occurred in one of two categories, these being knowledge of teaching and knowledge of student learning shown in Table 5.1. As can be seen, change was not uniform across all five volunteer teacher cases.

Table 5.1: Change in in Pedagogical Content Knowledge after the Lesson Study Cycle.

Change in Pedagogical Content Knowledge	**Volunteer Teacher**				
	Ha	**Wi**	**Mi**	**Zu**	**Sa**
Knowledge of Teaching					
Representation of Ideas	√	√	√	√	√
Demonstration of Procedures	√	√	√	√	√
Classroom Management			√	√	√
Type of Problems	√				√
Knowledge of Student Learning					
Level of Teacher Questioning	√	√			√
Diagnosing Learning Difficulties	√	√			√

Knowledge of teaching — Representation of ideas

By representations in teaching Ball (1988, p. 167) argued that the word representations means models of a wide range that convey something about the subject matter to the learner. In her argument she gave an example of a story about hungry people sharing pizzas as a representation of division of fraction constructed to help students comprehend the mathematical concept. In teaching, the teachers are constantly engaged in the process of constructing and using representation of subject matter knowledge. Some representations are provided by teachers from textbooks and worksheets or through the use of manipulatives, or other teaching materials. Representation of idea in this study identified four general modes of representation: concrete, pictorial, real world, and symbolic or abstract representation. The changes that occurred in this study included teachers using different types of representations not previously used, using types more often than was done previously, or using particular representations for different activities.

The findings indicated that the practice of all five teacher participants had changed regarding their representation of an idea in classroom practice following the Lesson Study Cycle. Through representations that the teachers selected and the ways they used them, the teachers conveyed massages about special content knowledge in their lessons. There were various changes in representation of ideas for each participant. Observations showed the Teacher Ha used pictorial representations in Lesson 1A and used more concrete representations in Lesson 1B. This change followed suggestions made at a Group Meeting that stated the teacher should allow students to cut the rectangle from grid paper, and arrange the shape to be a trapezium in Lesson 1B. His changes in Lesson 1B allowed students to take ownership of their experience by cutting, comparing, and calculating the area of a trapezium. In addition, Wi's practice changed from using more simple or general representation to a concrete representation by using a numerical scale for measuring angle. She made this change as after Group Meeting, which suggested that Lesson 2B needed to provide more manipulatives and teaching aids to measure angles. For example, in Lesson 2A she compared two angle figures to introduce a smaller and a larger angle. She introduced an acute angle and an obtuse angle by telling and showing the figures in the whole class. In Lesson 2B, she introduced an acute and an obtuse angle by comparing the right angle or an angle of 90^0. She demonstrated that angle could be expressed in numerical representations.

Another teacher Mi's practice changed in constructing a cuboid from using only a simple model with a few unit cubes in Lesson 3A to incorporating various models with many unit cubes in various colours in Lesson 3B. Her changes followed a Group Meeting suggestion that the teacher should show the cuboid with different sizes for sides to develop better understanding of the meaning of length, width, and height. Using more unit cubes helped students demonstrate their own solutions to the problem. This is an interesting result of the Group Lesson Study Meetings in that discussion revealed the lack of manipulatives and raised awareness of the need for schools about the idea of becoming a supportive learning community. In terms of this, Mi's lesson was able to include the use of unit cubes from Ha's school. This relates to previous research that teachers often work interdependently and collaboratively when planning and reflecting with other teachers (Murata, 2011). Also Taylor, Anderson, Wagner, and West (2005, p. 21) found that Lesson Study seems very suitable because it does not require a complex or expensive infrastructure in terms of resources.

In Lesson Study Group Two, Zu and Sa also changed in how they represented ideas. Zu changed from using a one model representation to more than one representation of the procedures. She made this change in Lesson 4B as part of suggestion of a Group Meeting that Lesson 4B needed to emphasise representation of the previous concept to build the new concepts. In Lesson 4B she used different amounts of unit cubes to construct cuboids of many different sizes. However, in Lesson 4A these unit cubes had been used to construct one cube. By using 36 unit cubes, she represented cuboid models in many different sizes with the same volume. It was helpful for students trying to understand the volume of a cuboid to learn not only by a formula but also by calculating volumes by working with the unit cubes. The other teacher, Sa changed from cutting and removing shapes to the use of a geoboard and dot paper to construct a two dimensional shape. Her change was suggested by the Group Meeting that in Lesson 5B the teacher would be better to use a geoboard for each group to construct two dimensional shapes.

In fact, the use of manipulatives as representation models helped students be more active in solving the task or problem according to their own solution, such as constructing two dimensional shapes on geoboards in Sa's lesson. In this lesson, she not only gave the procedure for the formula to solve the problem but also she gave alternative solutions in various ways, such as counting unit squares. The teachers allowed students to use manipulatives to solve problems or tasks in groups. These findings are consistent with previous studies (Shulman, 1986; Ball, 1988; Rowland et al., 2009) that the ways of representing the subject matter to make concepts more comprehensible to learners should be appropriate for students' age and students' thinking. Also, teachers should understand the concepts in sufficient depth, and in different situations, including how to use concrete materials (Ball, 1990; Martin, 2007).

Also, representation models using manipulatives helped to involve students in discussions and to improve student thinking. For example, when students constructed a parallelogram in Sa's lesson the students worked together to construct a parallelogram using a rubber band in each group. It was a different situation than if they had to draw a shape using paper and pencil only. They had experience of how to make parallel sides in real situations using a rubber band.

However, the use of manipulatives as representation models was new for most students and a few teachers in the Lesson Study Cycle. In traditional teaching the teachers taught the lesson based on textbook formats giving explanation, examples and exercises. Some of them had never used manipulatives. One of the reasons was that they had large classes and limited manipulatives. The students learned from a textbook. According to the interview before Lesson Study the teachers noted that they had limited opportunity to use different representations in traditional teaching, and a few teachers lacked an understanding of how to use them to represent the concepts. The teachers had not had much experience in using manipulatives. Before the commencement of the Lesson Study Cycle the researcher had trained the participants in teaching with a problem solving approach and the use of manipulatives. During the Lesson Study Cycle the teacher participants involved manipulatives in lesson plans. The teachers became aware of the fact that manipulatives could be used to help students understand the concepts. At the end of the Lesson Study Cycle one of the teachers explained in an interview:

> *I see positive value in Lesson Study. I learned how to use manipulatives to explain concepts or procedures and steps of the teaching and learning process.* (Interview, Zu, 10 December 2010).

One year after the Lesson Study program the teachers stated that their lessons changed in regard to the representations and tended to involve manipulatives or teaching aids. One of the teachers explained:

> *In representing the concept, I explain the concept following the text book format without manipulatives. After Lesson Study I not only explain from the text book but also I use real objects around the classroom and manipulatives.* (Interview, Zu, 11 December 2011)

However, while they knew that manipulatives helped students to develop better understanding of procedures and the concepts, the use of real objects in their classes was still limited by availability in schools. To further change practice regarding representation of ideas, teachers need support through provision and availability of manipulative resources in their schools.

Knowledge of teaching — Demonstration of procedures

Teachers demonstrate and explain procedures in many different ways. Demonstrating a procedure in this study was evident when the teacher demonstrated the tasks or problems, set up discussions, and presented alternative solutions for a problem or when students gave presentations.

Based on the observations during the Lesson Study Cycle, this study found that the practice of the five volunteer teachers changed regarding how they demonstrated procedures. In presenting tasks, Ha changed from using a teacher centred approach to a student centred active learning approach. In Lesson 1A he used an algorithmic or procedural approach and the students followed his presentation step by step as he explained the procedure. His approach could be identified as teaching for problem solving (Van de Walle et al., 2010) in which the skill to find the formula was taught first and then the formula was used to solve a similar problem. In Lesson 1B he set students to work in groups and began with a problem for them to solve. He allowed student discussion, and he paid more attention to student presentation in their group work. Based on their results he introduced the new procedures. He changed as a follow up to a suggestion in a Group Meeting that suggested he should demonstrate the task to allow students to be active in their learning. In Lesson 1B his teaching approach could be identified as moving more towards teaching through problem solving in which problems are presented as more open and students learn by exploring the problem situation (Van de Walle et al., 2010). Ha's demonstration of procedures changed to provide opportunities for students to solve a problem in meaningful contexts, use their own methods, use manipulative materials, and work in a group. These points reflect some of the criteria for teaching through problem solving (Hyde & Hyde, 1989).

Another teacher, Wi, taught mathematics with a more student centred approach in Lesson 2A but used more teacher explanation in Lesson 2B. She tended to follow a set procedure in Lesson 2B, explaining how to use a protractor and the students completed a short activity. Even so, this was a change in her way of demonstrating a procedure. It resulted from a Group Meeting suggestion that she needed to demonstrate the problem or task in a more challenging way. The students did the problem after following the teacher's explanation. Wi's teaching tended to prescribe the procedure. All students worked with the same objective at the same time in the same manner. While this change might have represented a return to a more traditional approach, it resulted from discussion at a Group Meeting.

The practice for another teacher Mi, when demonstrating a procedure, changed from using static viewing to dynamic viewing. Static viewing means looking at an object in a static position without moving, such as a picture of an angle, whereas dynamic viewing involves moving the object or the viewer. In Lesson 3B she focused on showing the object of a cuboid with dynamic viewing to emphasise the meaning of dimension of length, width and height. The suggestions from the Group Meeting suggested that Lesson 3B needed to involve students using more unit cubes. In fact, she did more than the group's suggestion. She used unit cubes to demonstrate a cuboid in a dynamic way to show the deeper meaning of length, width, and height. She demonstrated a cuboid in many positions and asked students to identify the attributes. Her demonstration provided students with a better context to make sense of the concept. To learn mathematics students need to describe their thinking verbally (Edens & Potter, 2007), and Wi's students were able to do this more easily through her use of dynamic viewing.

Changes in the demonstration of procedure also occurred in Group Two with teachers Zu and Sa. In Lessons 4A and 4B Zu's instruction changed from using a task-based, closed problem approach to using tasks based on open problems and demonstrating procedures to establish the validity of a concept. She allowed students to construct cuboids that had the same volume but with different dimensions. Her change resulted from a previous suggestion of the Lesson Study Group Meeting that Lesson 4B needed to involve construction of a cuboid in more than one way. Her teaching change provided opportunities for students to solve the problem in a meaningful situation and to construct for themselves personal understanding of the concept.

Another teacher, Sa changed her demonstration of a procedure from it being a long process to becoming a short process in constructing shapes to explain an idea. She illustrated how to construct 2-D shapes using a geoboard and dot paper. This took much less time than using paper cutting and rearranging shapes to make a parallelogram in the first lesson. In using paper cutting the teacher asked students to draw a rectangle on grid paper but not all the students did the task. They waited for the teacher's example because the use of grid paper was something new. This is related to the notion of contingency which concerns the teachers' response to such unplanned, unexpected classroom events (Rowland et al., 2009). This shows how some aspects of the teachers' pedagogical content knowledge were more developed than others and some aspects changed more than others. Her change was in response to a previous suggestion of the Group Meeting to construct a shape using a simple procedure. Her demonstration provided students with a focus for constructing 2-D figures.

With regard to the demonstration of procedures, the five teachers changed in different ways because teaching mathematics using a problem solving approach was something new for them compared to traditional teaching approach with the format of explaining the concept and giving a problem example based on the textbook. Their changes were conducted carefully depending on their level of knowledge of what was suggested in the group discussion and how it related to their teaching practice.

Knowledge of teaching — Classroom management

Classroom management is the process of ensuring that the classroom lesson runs smoothly without, among other things, disruptive behaviour by the students. It is closely linked to the issue of motivation, discipline and respect. In this study, classroom management included positive or negative teacher statements, encouragement when students had difficulty, encouraging students to express different points of view, and encouraging students to give reasoning for their answers.

The findings showed that not all of the five volunteer teachers made changes to their practice with regard to classroom management. Most teachers appeared to be somewhat insecure as they were using an approach that was relatively new and they also had observers in their classrooms. Nevertheless, during the lessons they managed to control their classes and successfully managed to try teaching using a problem solving approach. For example, Mi changed her questions, made more positive teacher statements, and offered more encouragement to student thinking. She motivated students by asking questions more frequently in the second lesson. She also gave more positive statements for student responses, both for the right and wrong answers. In her lesson she was less attentive to students with low achievement. As such, she did not change to completely satisfy the Group Meeting's suggestion, which was to select individual students to respond when asking question in the whole class. The teacher tended to pay attention toward students who had high achievement.

The others who changed their classroom management style were Zu and Sa. Zu's lessons changed by asking fewer questions, encouraging students more, and offering more positive statements. She was strategic in her used of praise and rewards to inform students about their behaviour rather than controlling student behaviour. She also explained to students that a correct answer would earn a reward. That is, she encouraged collaboration in selecting rewards and defining appropriate behaviours that will earn rewards. She changed as a follow up to the suggestion from the Lesson Study Group Meeting previously that in Lesson 4B students should be more involved in their group activities. Another teacher, Sa, changed from having students working individually in a group setting to working together in a group. This happened because in the second lesson manipulatives were distributed in groups so all members in the group could be involved in solving the problems. That is they had opportunities to work with manipulatives because there were enough for each student. This was consistent with the Group Meeting suggestion that for the next lesson students should be more involved in their group activities.

Three teachers' lessons began with problems to introduce the ideas or procedures. The ideas were constructed mostly based on students' sharing, group discussion and students' presentation following a teacher question, and encouraging students with positive statements. However, the teachers challenged students with mostly lower level questions. For example, students had to construct parallelograms of different sizes on a geoboard. The teacher asked the students to calculate the perimeter of the parallelogram. This is consistent with a previous study by An, Kulm, and Wu (2004) that teaching focused on student thinking by building on students' mathematical ideas. Suggestions about classroom management for three teachers were different, but the suggestions in Group Two were almost the same for two teachers Zu and Sa, even though they had different reason to make changes. It had been suggested to both Zu and Sa to encourage students to be more involved in the group activities. In both lessons the students worked in a group setting but in Sa's lesson students mostly worked individually and in Zu's lesson a few students did not become involved in discussion. As a result of suggestions from the Group Meeting, the students in both lessons were more involved in group discussion.

Knowledge of teaching — Type of problem

The choice of problem types could be seen as an element of mathematical content knowledge. In this study, the problem type is considered to be an element of pedagogical knowledge, because the emphasis is on the teacher's choice to use a particular problem type in teaching situations. Thus, it is considered to be a pedagogical decision. In teaching mathematics, most problems could be classified as closed problems or open problems. Closed problems were well known by the students and teachers and could be found in the text book. A closed problem has one correct answer with the way of solution clearly out lined. These are often seen as textbook exercises. An open problem is different as it has no formulation and no fixed procedure that guarantees a correct solution. It possibly has no single method of solution and may have more than one answer. Johnson, Herr, and Kysh (2004) noted that a problem is different from an exercise.

Observations indicated that during Lesson Study the five teacher participants used problem-based teaching with closed or open problems. Most teachers tended to use more open problems. Students played an active role in their learning as the teacher allowed them to solve problems in their own way. It was noted that less change occurred in the practices of the volunteer teachers from the first lesson to the second lesson regarding the use of different problem types than for other criteria. Observations showed that two teachers, Ha and Sa, changed with regard to the type of problems used. Ha's teaching moved from teacher explanation using closed problems in Lesson 1A to using open problems in Lesson 1B. Ha used closed problems to encourage greater understanding of the concept in the first lesson, then changed to an open problem in the second lesson. This was in response to the Group Meeting where it was suggested that he should develop more challenging problems to determine the area of a trapezium. Lesson Study Group I decided to develop open problems as challenging problems and this developed from the professional learning about problem solving conducted by the researcher prior to the Lesson Study Cycle. The use of open problems was looked on as something new for all the teachers as a teaching approach. After the open problem was used in Ha's lesson in the Lesson 1B, the other lessons that followed in the group used open problems. This demonstrated that the Group Meetings had a cumulative effect.

Most of the other teachers used both open and closed problems in both the first lesson and the second lesson. This is important as it shows how the discussion about the first pair of lessons (1A and 1 B) resulted in changes in practice for the lessons that followed as taught by other volunteer teachers in the group in the next lesson. The teachers tried to use open problems when they planned the first lesson. It is clear that the other teachers learned from Ha's experience while they observed in his classroom. In other words, his teaching practice influenced the other teachers in planning the following lessons.

As an example, Mi started Lesson 3A with an open problem. This shows the cumulative effect of the Lesson Study Group Meetings to that point as Mi had been involved and had reflected on her teaching as to how she might start her topic. Even though she had not had her lesson observed and evaluated by the Group Meeting, she had learned from the previous Group Meetings (1A/1B and 2A/2B) and had started with an open problem as a result of discussions in those meetings.

Therefore, the Lesson Study Group Meetings that followed lesson observations did influence the practice of volunteer teachers regarding problem type choice. There is little evidence of the volunteer teachers changing practice from lesson A to B to the same extent as Ha did from Lesson 1A to 1B, because they had already incorporated more open problems in the first lesson of the series as a result of observing Ha's teaching and engaging in the Lesson Study Group Meeting. Most other volunteer teachers used open problems in their teaching as a way to introduce the concept or formula. After finding the concept or formula all the teachers tended to give similar tasks as practice. The practice was to use routine problems in which the problems could be solved using an operation or formula in the same way that Yan and Lianghuo (2006) classified problems from text books.

As was the case with Lesson Study Group I, the use of open problem for Sa's lesson had been influenced by Zu's lesson in Lesson Study Group II. Even though the type of problem was not discussed after on Lesson 4A, Zu taught by using open task in Lesson 4B. She selected open problems from a draft lesson previously designed by the researcher.

Sa used more open problems in Lesson 5B than Lesson 5A. In Lesson 5A, as a result of involvement in the previous Lesson Study Group Meeting, she used open task beginning with: *Draw any a rectangular on grid paper.* In the Lesson Study Group Meeting that followed she said that she was surprised by her students' slow responses and that they waited for her to give them answers. It was noted in the meeting that open problems were relatively new for both students and teachers, as was the use of grid paper. The Group Meeting suggested that open problems should be used not only in the beginning of the lesson to draw a parallelogram but also during and at the end of the lesson to challenge the students more. The teacher Sa followed up this suggestion in Lesson 5B by giving individual tasks, as shown in Figure 5.1.

Task 1: Construct a parallelogram at every 5 × 5 dotted paper with different sizes

(a) *(b)* *(c)*

Task 2: Determine the area of each parallelogram that you created in (a), (b), (c)

Figure 5.1: Tasks with 5 × 5 dotted paper.

Task 1 is an open task that consists of three open problems. Each problem has more than one answer. Task 2 is a closed task based on the three open problems. Each problem has one answer that can be solved in more than one way. To solve these problems the students needed to find a new strategy, which could be drawing in any way to find the answer. This problem could be categorised as a non-routine problem (Yan & Lianghuo, 2006). It is different from Task 2, which requires the students to calculate the area of a parallelogram by counting square units or using a formula. It could be called a routine problem. Stanic and Kilpatrick (1989) noted the problem type is mostly used as a task or exercise in a mathematics lesson until the students have knowledge of a specific technique or skill. Teacher Sa used more open problems throughout the lesson with the result that students were more active and asked more questions, and also more students explored the procedures.

When teaching starts with problems it gives students the chance both to consolidate and extend what they know and to stimulate their learning. Unfortunately the teachers in this study had limited knowledge and experience of problem-based teaching to modify the open problems.

With regard to the use of problems in real life situations, the teachers in this study tended to use applied problems with the objects around the class. They selected problems from the textbook or modified them, such as the problem measuring the angle that was constructed between the door panel and the wall in Wi's lesson. The students drew angles according to the door panel and wall on the floor, then measured the angle using a protractor. Generally the real problems were presented in groups during the lesson. However, at the end of each lesson the teachers predominantly used drill exercises and applied formulae to solve problems selected from the textbook. For exercises or practice problems, students were shown a technique, and then given problems to practice on, until they had mastered the technique (Schoenfeld, 1992, p. 13). It was important to note that even though the teachers were beginning to use problem-based teaching, they generally used drill exercises and practice problems in their teaching.

Knowledge of students' learning — Level of teacher questioning

One of the best ways teachers can help students participate is by asking questions that encourage students to think and share. By using careful questions the teacher can guide the students through the discovery experience, which will have far greater effect on their learning than a lecture experience in which a teacher provides information for students. In this study the level of teacher questioning in helping students' learning included knowledge, comprehension, application, analysis, and evaluation questions.

Findings indicated that three teachers Ha, Wi, and Sa changed in their levels of teacher questioning. Most changes in question types used by the teacher were the result of features of their teaching. They were not due directly to suggestions from the Lesson Study Group Meetings. However, these changes might have resulted indirectly from Group Meeting suggestions, such as using teaching based on problem solving and using manipulatives with students working in group settings. While the students were solving the problems, the teacher was walking around the students and asking them about their difficulties with understanding procedures. This was an indirect consequence of the Group Meeting. A Group Meeting also suggested that the teacher should try to ask questions that were appropriate for students individually and avoid asking questions needing a simultaneous answer when seeking to diagnose student learning.

A change from knowledge level to comprehension level in Ha's situation almost forced him to use a different way of questioning. Similar cases can be seen by Wi's and Sa's lessons that reflected changes in the level of teacher questioning. Wi's lesson changed from using questions from lower levels in different levels to more comprehension and higher level questions. This change occurred because of her class situation in practice, and her questions tended to encourage students to understand about the concept and to motivate students to be involved in learning.

The other teacher Sa also used more questions and more questions of different levels. This change did follow a direct suggestion from a Group Meeting but occurred because of her classroom situations. It naturally followed in order to deal will student difficulty. All questions were used by the teachers to see what students had already learned, to help students understand the procedure, to see the students understood the concepts, and to see if the students could reason about the procedure.

Knowledge of students' learning — Diagnosing learning difficulties

To diagnose whether or not students are experiencing difficulties, teachers need to know what mathematics the students know and be able to identify evidence of their mathematical difficulties. This study considered that diagnosis of learning difficulties included teachers' use of oral questions, written questions, and written tests.

It is important to note in regard to both aspects of knowledge of student learning discussed here, that the practice of three teachers, Ha, Wi, and Sa, did change. It is also noted that changes did not occur as a result of particular suggestions from Group Meetings and it is suggested here that they occurred in response to the classroom situation in each case. However, the focus of the lesson Study Cycle was the use of problem-based teaching as opposed to a traditional style that the teachers had been using. It was because of the change in teaching style that made the teachers more aware of the importance of diagnosing student learning difficulties that their practice changed. For example, Ha found the students had difficulties in counting square units in calculating the area of trapezium. He used more alternative ways to help students better understand the concepts. Ha not only identified difficulties by using oral questions but he also used written questions as part of the group discussion.

Another teacher, Wi, used more questions at different levels to diagnose students' difficulties. For instance, when students had difficulty understanding the meaning of angle, she asked more questions about parts of an angle. She changed her practice naturally in response to the students' difficulty.

Sa used more oral questions in diagnosing learning difficulties. In Lesson 5B she identified that students were confused with the new manipulatives used to construct shapes. Her change in diagnosing learning difficulties naturally followed the students' difficulty. When students were working in groups they were faced with new teaching aids that made than ask questions because of their lack of familiarity and experience in using the aids.

It was noted that only three out of five volunteer teachers changed regarding their knowledge of student thinking. The changes were related to the level of teacher questioning in diagnosing learning difficulties, where they asked more questions, more different levels of questions, and higher levels of questions. They used questions to evaluate what students knew, to elicit what students thought and to help them construct conceptual knowledge. Most questions were lower order questions with predetermined short answers. The changes were related to students being faced with new teaching aids, and students working in a group, both new situations, which meant that teachers had to observe and monitor students' difficulties. Interviews indicated that before the Lesson Study Cycle the teachers paid little attention to diagnosing learning difficulties. They had taught in a traditional class setting with a view that students learned as a whole class, not as individuals, and they assessed student learning based on a final test score. After the Lesson Study program all teachers paid more attention to student learning processes and based their evaluation of student learning on ongoing observation, not just a final test score.

Change in Mathematical Content Knowledge

The second important aspect of teacher's knowledge that changed after experiencing the Lesson Study meetings was mathematical content knowledge. Specifically, this change occurred in one of three categories: understanding of procedure, understanding of mathematical relationships, and demonstration of knowledge as shown in Table 5.2.

Table 5.2: Change in Mathematical Content Knowledge after the Lesson Study.

Change in Mathematical Content Knowledge	Ha	Wi	Mi	Zu	Sa
Understanding Procedure	√				√
Understanding of Mathematical Relationships	√	√			
Demonstration of Knowledge		√	√	√	√

Understanding procedure

Haake and Su (2005, p.3) noted that a procedure is a process that runs in stages to produce a solution, with a theoretical foundation that shows why each step in the procedure works. They also noted that a procedure gives step by step instructions towards a solution in which each step should be intuitive (e.g. it should be easy to comprehend), plausible (e.g. should be simple to argue, and manageable (e.g. must be straight forward to compute) (Haake and Su, p. 14). The inference is that teachers need to have a clear and rich understanding of procedures and be able to adapt practice to help students.

Findings based on the observations of five volunteer teachers showed that two teachers, Ha and Sa, changed in their understanding and demonstration of procedures and they changed in different ways. These changes were a result of two things, the change from a traditional approach to a problem based approach and discussions at Group Meetings Ha's understanding and demonstration of procedures changed from algorithmic to exploratory as his teaching became less teacher centered and more student centered, as was described in Chapter Four. He always understood the procedure for finding the area of trapeziums but the way in which he demonstrated it changed as his teaching approach changed. In Lesson 1A he used a step by step procedure to determine the formula for area of trapezium. He understood the meaning of area and knew the formula for area of a trapezium as ½ × (a + b) × h. He knew this on the basis of his knowledge of the relationship between rectangles and trapeziums. He demonstrated the procedure while the students listened and followed his instructions. The teacher transferred his knowledge to the students as the traditional approach views the teacher as a resource for student learning.

This was different to Lesson 1B where he explored in order to determine the formula for area of trapezium. By dividing any rectangle into two equal parts, he considered the relationship between the area of a trapezium and the area of a rectangle, and was able to make a generalisation that the area of a trapezium was ½ × (a + b) × h. He openly constructed the new knowledge from previous knowledge and was able to give an alternative solution. He guided students to understand based on sharing and exploration.

He changed in his understanding and demonstration of procedure in order to adapt his teaching approach in which an open problem was presented at the beginning of the lesson. The Group Meeting had discussed the development and use of open problems, and for students to develop alternative solutions. It was natural for Ha to need to use alternative ways of demonstrating procedures to fit with the change in approach.

The other teacher, Sa, changed from solving a problem in one way to using more than one way or method. She did so to support the process of thinking of her students. She understood the meaning of area and how to calculate the area of a parallelogram using a formula. By cutting off a triangle from one end of a paper parallelogram, she re-arranged the pieces into a rectangle, so that the area of a parallelogram was seen as length × height. She developed a problem to apply the formula, such as to calculate the area of a parallelogram in both the lessons. By using dot paper and geoboard, she was able to develop an open problem in the second lesson, such as: *Construct parallelogram with the area of 4* cm^2. She understood that constructing as a shape in these ways might help students to understand the concept. Her change demonstrating the procedure occurred because her teaching approach had changed. The Group Meeting had discussed developing and using open problems and the change in Sa's teaching followed naturally from this.

In summary, the findings indicated that changes in understanding and demonstration of procedures tended to relate to changes in teaching approach. Both the teachers had knowledge of using concrete examples to guide students' thinking. They understood the need to change when introducing the area of a trapezium from an algorithmic to an exploratory approach and in introducing the area of a parallelogram to move from one solution to more than one solution. This is an example of specialised content knowledge, which is a specialised form of common content knowledge for teaching and the findings are consistent with the theory of subject matter knowledge (Ball et al., 2009).

Understanding mathematical relationships

It has already been noted that the greatest changes to practice occurred in relation to pedagogical content knowledge. The changes to mathematical content knowledge were less but still important. It is suggested that changes in the demonstration of mathematical content knowledge resulted from changes in the teaching approach, and changes in pedagogical content knowledge. Some of the changes were as a result of particular discussion in Group Meetings but most changes in demonstration of mathematical content knowledge probably resulted from teachers reflecting on the need to change to accommodate a change to a problem based approach.

Two teachers, Ha and Wi, changed in how they demonstrated their understanding of mathematical relationships. Ha already understood relationships between the properties of a rectangle and a right-angled trapezium such as the width of a rectangle was the same as the distance between the two parallel sides of a trapezium. In Lesson 1A he showed a pictorial model dividing a rectangle into two equal parts of right-angled trapeziums and he proved the formula for area of a trapezium using an algorithmic procedure. In Lesson 1B he explored a rectangle by dividing it into equal parts in various shapes, instead of using one shape as he did in Lesson 1A. Through the comparison of areas between the trapezium and a rectangle and the relationship for each side, he determined the formula for calculating area of a trapezium. For example, as in Figure 5.2, he indicated the height of a right-angled triangle was the same as the width of a rectangle, and the base of a right-angled triangle was the length of a rectangle.

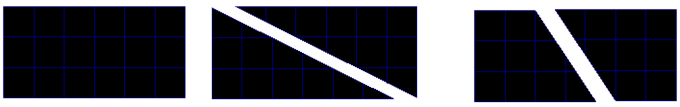


Figure 5.2: A rectangle divides into equal parts in different ways.

Ha understood the mathematical relationships and changed his demonstration of them to allow students to explore the relationship between the properties of a rectangle, a right-angled triangle, and a right-angled trapezium. He used grid paper and cutting of shapes as appropriate tools for helping students understand. This reflected his move towards a problem based approach rather than a procedural approach.

The other teacher Wi had a limited knowledge of angles and the relationship with how to measure them. However, her understanding developed as she realised the need to be able to better demonstrate angle concepts to her students. Initially, Wi did not link mathematical ideas to show relationship between them. She tended to introduce them separately in a procedural way rather than through a problem solving approach. In Lesson 2A she introduced an angle by using examples and compared two angles to show the size of angle and the smaller and bigger angle. This did not really help students to see the relationship between angles and how to measure them. In Lesson 2B she introduced concepts of comparing angles, right angles, obtuse angles, acute angles, and how to use a protractor to measure an angle. Then she used a hand clock model to show the size of an angle and to connect to the notion of turning. She did not pay attention to the properties as shown in Figure 5.3 where the two angles were the same size, but the angle COE looked bigger than angle AOB. However, the use of the hand clock model would have helped students to see the relationship between turning and angle size.

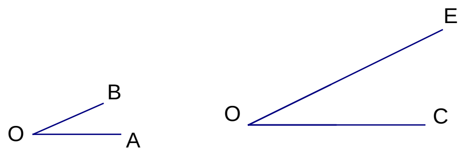


Figure 5.3: Two angles the same size.

Although changes in understanding and demonstrating mathematical relationships were not specifically suggested in the Group Meetings there had been discussion about developing open problems. Ha and Wi changed because of the need to help their students better see the mathematical relationships. Ha had known the relationship between, and the properties of, rectangles and trapeziums in order to determine the area of trapezium. Wi had limited personal understanding of the relationship between angle and measurement of an angle. However, it could be said that this understanding developed, as she had to demonstrate it differently to help her students learn about angles. This in turn was a consequence of the professional learning and the decision to use more of a problem based approach.

Demonstration of knowledge

Another aspect of teachers' mathematical content knowledge is the way in which they demonstrate their knowledge. Some examples of demonstration of knowledge of teachers may include their ability to write mathematical expressions, use mathematical language, and build students' understanding of key concepts. Teachers' knowledge is what they know and how they think about it (Ball, 1988).

During the Lesson Study Cycle it was evident that most of the volunteer teachers were unclear about aspects of mathematical knowledge needed to teach the various topics. This was highlighted during discussions in Group Meetings where teachers were able to develop their own mathematical content knowledge though sharing ideas with other group members. The findings showed that four of teachers Wi, MI, Zu, and Sa changed in their demonstration of knowledge. First, Wi gave different definitions of angle to help students understand. In Lesson 2A she introduced an angle by using an example of a real object, as well as pictorial, and symbolic versions. Then in Lesson 2B she defined an angle as *the intersection of two rays that meet at one point*. She drew an angle as shown in Figure 5.4.

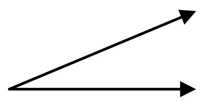


Figure 5.4: An angle.

According to her definition the angle could be drawn as shown in Figure 5.5 but it was not an angle. It is more than one angle.

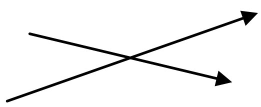


Figure 5.5: More than one angle.

Regarding the Figure 5.4 the definition of an angle should be *the meeting of two rays their end points.*

Wi's definition was not clear. She did not show the difference between a ray, a segment, and a line on the side of an angle. She just showed the element of the angle, side of the angle, and point of the angle. However, she gave a good example of angle with real objects around the classroom. The definition of angle was not discussed in the Group Meeting. The reason for that is not clear. It could be assumed that all teachers lacked sufficient understanding to enable them to define an angle, or it may be that they were comfortable with Wi's definition. Whatever the reason, there is an indication that the teachers were probably not confident about correctly defining angle. This is strengthened by the fact that one of the group members was confused the meaning of *unit angle* and *angle unit* as well as how to measure angles with non-standard units. However, in her lesson Wi was able to introduce a degree as a standard unit and compare angle sizes using small angles as nonstandard units.

In writing the name of an angle she was inconsistent. She sometime used three capital letters or one capital letter. Although both names of the angle were correct, the students become confused. In a Group Meeting, one of the participants, Ha, commented to the group:

> *Giving the name of an angle should be in capital letters in order of the alphabet. One angle has two names, for example angle ABC is the same as angle CBA. Giving the name to an angle by using one capital letter will make students confused. It also becomes ambiguous with inner area or outer area of an angle.* (Comment group discussion, Ha, 28 October 2010)

A second teacher, Mi, demonstrated a more accurate mathematical understanding. She calculated the volume of a cuboid by counting the unit cubes and using the formula. However, her writing of the formula of a cuboid as $V = L \times W \times H$ in capital letters indicated that she had lacked understanding of the convention in using a mathematical statement. The capital letter L, W, and H used respectively as length, width, and height, should by convention be written in lowercase letters as l, w, and h. This suggestion came from the other teachers in the Lesson Study Group Meeting. In the second lesson she wrote the mathematical statement correctly.

A third teacher, Zu, developed a more accurate understanding of the concept of length and width of a rectangular prism. In Lesson 4A she showed that she understood the properties of a cube and a cuboid. She compared the differences and similarities of the properties of a cube and a cuboid. Also she compared the differences between two dimensional shapes and three dimensional objects, even though she did not focus on a rectangle shape as a prerequisite to constructing a cuboid. She directly used a unit cube to construct a cuboid and defined the meaning of the volume of a cuboid as the amount of unit cubes it occupies. However, in Lesson 4B, she gave an open problem: *How many rectangular prisms can you make using 36 unit cubes?* The students wrote various dimensions of a cuboid, such as, 9 ×

2 × 2; 6 × 3 × 2; and 2 × 6 × 3 to show many possible answers. In this lesson, Zu focused on determining the formula of a cuboid based on the data table, rather than by counting unit cubes.

However, when Zu presented the various dimensions of cuboids with the same volume, she was not clear about the meaning of length, width, and height of a cuboid as shown by her reflection in the Group Meeting. The other teachers gave some interpretations: *Length is longer than width; Length is to face the reader; The vertical line is height; and Length is the horizontal position; The vertical line is the height on a cuboid.* One of the teachers commented:

> *The biggest number is length. Length is to face with the reader is wrong statement. My opinion is that length is the longest size as part of side of a rectangle.* (Comment Group Meeting, Ma, 19 November 2010)

Finally, they concluded that the length is always longer than the width. It showed that not only Zu but also the other teachers had a lack of understanding of the conventions about length. They needed to know that length is the part of a rectangle that is longer than width, and a rectangle is the face of a cuboid. So in a cuboid the length is always longer than width. This was a good example of a learning community in action where the conclusion was constructed based on teacher participants' opinions and knowledge and sharing ideas in the Lesson Study format.

A fourth teacher, Sa, demonstrated a more accurate understanding of mathematical ideas. In Lesson 5A she drew a field with the size 180m × 120 m but the shape was a parallelogram. She drew a field with the right dimensions but with the wrong figure. The real field is a rectangle shape with the size 180 m × 120 m, but she drew a field in three dimensions on the white board so that it looked like parallelogram. It should have been drawn in two dimensions so the original and the drawing figure were the same shape - a rectangle. In this case the perimeter of a rectangle and the perimeter of a parallelogram are the same. Her presentation could have led to some students developing a misconception.

Also, on a worksheet she wrote the measurement of a garden as a parallelogram with the perimeter of 24 cm and the first side being 5cm long. She gave unrealistic measurements because the garden size was too small. She knew her mistake after the Group Meeting, where the other teachers suggested the units were incorrect. This happened because she used copy and paste on her computer when she modified the problem. In Lesson 5B she drew in the correct figure and used the right units. She followed up with changes based on the group's suggestion. This finding is consistent with a previous study (Mewborn, 2001) that many elementary teachers do in fact lack a conceptual understanding of the mathematics they are expected to teach.

The above examples of how four teachers demonstrated their knowledge shows two things. First, it shows that many of them were not clear about aspects of their mathematical content knowledge needed for teaching. Second, it shows the strength of the Group Meeting in the Lesson Study Cycle as a forum for discussion where teacher misconceptions can be clarified.

Change in Teachers' Beliefs about Teaching and Learning Mathematics

The third important aspect of teaching that changed after the Lesson Study meetings was teachers' beliefs about teaching and learning mathematics. Teachers' beliefs are hard to describe. Beliefs are complex, dynamic and situated, so that beliefs expressed in an interview with a researcher may appear to contradict those expressed in conversation with a teaching colleague or enacted in the classroom (Forrester, 2008, p. 25). The portrait of teachers' beliefs about teaching and learning mathematics was connected with the observations of teaching practice during the Lesson Study Cycle and interviews with teacher participants before and after the Lesson Study Cycle.

The findings about the five volunteer teachers showed variation in their beliefs about teaching and learning mathematics. Initially, most of the volunteer teachers believed in a traditional format for learning mathematics which included placing more emphasis on explanation of new procedures or formulae followed by the text book work and practice exercises.

Findings showed that all the teachers were depending on mathematics textbooks. They stated that their lessons were based on a textbook format. They mainly followed the order and instructions of textbooks and used drill exercises. All of their students had textbooks and teaching was based on content and curriculum knowledge without a focus on student mathematical thinking (An, Kulm, & Wu, 2004).

Regarding a problem solving approach some teachers believed that there would be difficulty in understanding of the problem by students and that using a problem solving approach needed more time, as Wi explained:

> *Teaching mathematics using a problem solving approach needs too much time. The students find it difficult to understand the story problem*. (Interview, Wi, 21 December 2011)

However, one volunteer teacher Mi, said that mathematics should be taught not only with a problem solving approach but also using multiple methods and students should be active in their learning. For student learning, all the teachers believed that the students would understand the formula better after applying the formula to solve problems. However, the problems they chose were generally closed in nature and little more than repetitive exercises in which a formula was used. They believed that students would learn mathematics as they solved many problems and discussed their solutions. For example, Mi thought that by giving an example problem solution to the students they would be able to solve the new problems with the same procedure. Also, they believed that by having students solve the problem using manipulatives it would give them deeper understanding of the idea.

The use of manipulatives in teaching was thought to develop deeper student understanding. But their teaching practice was still focused on a textbook format and use of procedures. They showed inconsistencies between their practice and beliefs. They viewed problem solving as students solving many problems of the same type. True problem solving of an open nature was thought to be too difficult and time consuming, that is teaching through problem solving.

After the Lesson Study Cycle the teachers' beliefs changed. Most teachers believed that mathematics should be taught by asking questions at the beginning of a lesson and by giving problems or tasks at the beginning. They suggested students should be active in their learning and that for this to occur, the use of manipulatives was necessary. Ha described that in problem solving there is an emphasis on the process and the deeper understanding rather than on memorising only. If the students forgot the formula they could recall what they had done. As he explained after Lesson Study:

> *With the problem solving approach students are doing a mathematics activity and do not memorise. With the old approach if a student forgot the formula that was the finish. In a problem solving approach, if students forget the formula they will recall what they have done and imagine what they did with manipulatives. They get the solution of perimeter of a rectangle without memorising only the formula.* (Interview, Ha, 23 December 2011)

The other teachers believed that problem based teaching could be used to introduce a new concept and not just be used as an application at the end of the lesson. As a motivator and facilitator, the teacher would allow students to solve problems according to their own method in order to construct their knowledge and understanding.

The teachers believed that a problem solving approach could provide students with opportunities to understand basic ideas and support their learning so that the students would become independent learners. However, the teachers' beliefs could be different from their actual teaching practice. It could be argued that the teachers' beliefs expressed in the interview may appear to contradict what occurred in their teaching practice. For example, one of the teachers, Ha, explained:

> *I have taught mathematics using a problem solving approach after recommendations from Lesson Study, but only little topics in this lesson because a problem solving approach needs additional time and needs more preparation before teaching and learning. My experience of participating in Lesson Study has made me improve my knowledge of teaching and learning mathematics.* (Interview, Ha, 12 December 2011)

Another teacher Sa also explained about her teaching after the Lesson Study Cycle:

I have taught using problem solving approach. Re-teaching topics in Lesson Study in a different class makes it easy and comfortable with teaching steps. For a new topic, I taught measurement. I needed extra time for preparing a lesson plan and work hard, but it is easy to implement it in teaching practice. (Interview, Sa, 24 December 2011)

Most teachers believed in teaching mathematics using a problem solving approach. However, they showed limited use of teaching with a problem solving approach after the Lesson Study Cycle. They felt that problem solving needed too much preparation and too many activities to construct children's knowledge. It is clear that professional learning for teaching takes time. This finding reflects the theory of professional learning that any attempt to bring change in the classroom practices of teachers and change in their beliefs and attitudes is a long-term process. It acknowledges the fact that attempts to change teachers' learning occur over an extended time frame (Guskey, 1986; Reimers, 2003). With the assumption that attempts to change teachers' beliefs require a strong commitment from teachers, Guskey (1986) noted that significant change in teacher's beliefs is likely to take place only after changes in student learning outcomes are evident.

Value of the Lesson Study Approach in Bringing about Changes

Lesson Study is not a single uniform method of professional learning. For instance, Lesson Study in Japan takes various forms. The Lesson Study Cycle in this project involved planning the lesson, teaching the lesson and involvement in a Group Meeting after the lesson. This reflection session was repeated after the replanned lesson was taught. From the results of observations during the Lesson Study from September 2010 to December 2010 and interviews after Lesson Study, a number of changes in pedagogical content knowledge, mathematical content knowledge, teachers' beliefs, and level of teacher collaboration were noted, as a result of Lesson Study.

Lesson Study - Bringing change in teachers' knowledge

Findings showed that the Lesson Study Cycle brought changes in teachers' knowledge and this was evident with regard to teacher pedagogical content knowledge, mathematical content knowledge, and teacher beliefs.

Change in pedagogical content knowledge

The change of teachers' pedagogical knowledge included changes in knowledge of teaching and changes in knowledge of student learning. Changes in pedagogical content knowledge were greater than changes in teachers' mathematical knowledge. Changes in knowledge of teaching and students' learning brought about by Lesson Study will be discussed in the section that follows.

Change in knowledge of teaching

Change in knowledge of teaching occurred in each phase of the lesson cycle: planning the lesson, teaching practice, and group discussion. In planning the lesson both Lesson Study groups decided to use problem solving as a teaching approach as this was the focus of the program. The groups spent time discussing their lesson plans. When designing the lesson plans, Group One decided to choose three basic competencies as set out in the Indonesian syllabus: area of a trapezium, angle with nonstandard units and degrees, and volume of a cube and a rectangular prism. Group Two decided to choose two basic competencies: volume of a cube and a rectangular prism, and perimeter and area of a parallelogram. The lesson plans were to be more student-centred and taught in a group setting. They included involvement of manipulatives and development of open problems. In modifying the lesson plans, the teachers worked as a group. They had valuable discussions that allowed the individual knowledge of teaching and knowledge of students' learning from the participants to come forward. Their individual knowledge was expanded by the collective knowledge of the group as has been shown earlier in this discussion. In designing lesson plans the teachers shared ideas and sources of learning. For example in Group One, one of the teachers, Ha, brought an amount of unit cubes from his school to lend to Mi for her Lesson, in response to the realisation that student understanding would be enhanced if more resources were available.

Teaching practice during the Lesson Study cycle indicated that five volunteer teachers implemented the revised lesson plan for the following lesson. Group discussion after the lesson indicated an opportunity to develop teachers' knowledge through reflection and feedback for the discussion of the topic taught at each grade level as well as the teaching approach and knowledge of students' learning. Reflections of the five volunteer teachers indicated that four of the five teachers were not confident in teaching with observers in the classroom. The four teachers were not confident to teach during Lesson Study because they lacked experience of professional learning and knowledge of mathematical problem solving. The one teacher, Ha, who was confident, had had much more professional learning than the others. This suggests that the participants might not have gained as much as could have otherwise been gained from the Lesson Study Cycle and that regular involvement in professional learning is desirable.

Results showed that the group gave an opportunity to develop the idea of a 'problem' for the teachers. They used mostly open problems to introduce the new concept after the Group Meetings whereas they had not used open problems in teaching and learning mathematics before the Lesson Study program. Traditionally, they had used the textbook as the only resource. Even so, the teachers' construction and use of open problems was very limited. To create open problems, they modified closed problems from textbooks as a result of collaboration in the Group Meetings. For example, in Sa's lesson, she modified a problem from the closed problem on the text book: *Find the area of a parallelogram with the side 3 cm long and where the distance between two parallel sides is 2 cm long.* The problem become an open problem: *Construct the parallelogram with an area of 6 square centimetres*. The Lesson Study Group Meetings also had an important role in contributing to changes in personal knowledge of teaching. Most teachers developed their knowledge of teaching due to the suggestions of the Lesson Study Group during their meetings. As mentioned easier Lewis (2000) had noted that Lesson Study can spread knowledge of new content and new approach.

Change in knowledge of students' learning

The change in teachers' knowledge of students' learning occurred in planning the lesson, in teaching practice, and in group discussion. When designing the lesson plans the teachers described in more detail the steps of teaching and learning than they did with a traditional lesson plan. For example, in Group One, Ha's lesson plan was changed from the short lesson to more a detailed lesson plan. The original lesson plan contained two pages with main activities of teaching and learning described in six, short simple sentences and a general statement. If the lesson plan has been used by another teacher it would have been difficult to follow. The revised lesson plan was six pages long and included samples of student handouts. The main activities were more detailed and described step by step as to how to teach and how the students might learn.

Teachers in Group Two took time to prepare the lesson about the area of a parallelogram. The teachers decided to present a set of teaching aids: grid paper, scissors, glue, and ruler to investigate the area of parallelogram using a problem solving approach. The process of planning the lesson helped the teachers to think critically about how students need to learn. Classes were also set in groups to make the teachers pay more attention toward students and their learning, as opposed to considering the whole class in more traditional plans.

The three teachers demonstrated greater knowledge of student learning as they naturally responded to a situation where students had difficulty. The change from teacher-centred to student active learning made the teachers pay more attention to student thinking, working in a group, and asking about any difficulties.

Change in mathematical content knowledge

Findings based on the classroom observations reflected changes in mathematical knowledge, which included: more understanding of mathematical exploration, mathematical procedures and mathematical processes and better demonstration of knowledge. These findings were supported by comments in interviews after the Lesson Study program, and reported changes in mathematical knowledge included: increased knowledge of mathematical problem solving and more knowledge in developing various problems.

Change in demonstration of mathematical content knowledge occurred in planning the lesson, teaching practice, and reflection and feedback in discussions after the lessons. In planning the lesson, participants in both Lesson Study Groups reflected changes in mathematical knowledge. They learned about and selected open problems from draft lesson plans that had been designed by the researcher and other sources. The researcher sometimes guided how to modify a problem, as an open problem was something new for the teachers. Also the teachers worked collaboratively to design the lesson plans. In teaching practice, some teachers showed a change in mathematical content knowledge. These changes were discussed in a Group Meeting after the lesson. In the Lesson Study Group Meeting, the teachers were building mathematical knowledge by identifying resources for learning, for representing mathematical problems and for developing problem solving. For example, there was interesting discussion in the Group Meeting after Zu's lesson which used an open problem: *How many rectangular prisms can you make using 36 unit cubes?* The participants had many answers because they had different understandings about the length of a rectangular prism. They simulated the construction of a rectangular prism, and gave a range of views. Interestingly, discussion about mathematical content only seemed to happen when the teacher had difficulty with aspect of teaching practice as happened in Zu's lesson described earlier.

Bringing change in teachers' beliefs

Reported change in beliefs of teachers included: more confidence in teaching mathematics, more student centred teaching, more asking of questions, more allowance for students to think in different ways and more active learning. Teacher efficacy is defined as a teacher's judgment of his or her ability to bring about student learning or development (Puchner & Taylor, 2006). With regard to planning the lessons, all teacher participants stated in their interviews that better lesson plans were prepared by sharing and discussing ideas. Most teachers stated their belief that their teaching practice was improving as a result of engaging in the Lesson Study Cycle.

Teachers' Reflection on Lesson Study

The interviews and reflections after Lesson Study Cycle indicated that most teacher participants perceived the Lesson Study Cycle positively. Lesson Study is something new as professional learning in the Indonesian primary school context. In the interviews conducted before the first Lesson Study meeting all teacher participants indicated that they did not understand the function of Lesson Study as a professional learning vehicle. Lesson Study was not new to Indonesia (Marsigit, 2006) but its use appears not to have been transferred from the secondary context. This Study began with the workshop for introducing Lesson Study, and continued with Lesson Study Group Meetings in Groups One and Two. During the Lesson Study Cycle, five teachers from the twelve original teachers had an opportunity to become a volunteer teacher. The five teachers who acted as volunteer teachers included three teachers from Group One and two teachers from Group Two.

Findings from an earlier Lesson Study meeting showed that most participants had no interest in becoming a volunteer teacher as part of a Lesson Study Group. Their reasons were generally of a personal nature such as, not being confident in mathematics or in their teaching when it was observed by colleagues. They felt that they were being evaluated and lacked confidence in their teacher knowledge. The teachers needed to be convinced that Lesson Study would not be an invasion of their classrooms. They had to feel confident that Lesson Study was only a tool that had implications for the improvement of, not only their teaching, but also for learning that takes place in their classroom. Comments in interviews (December 2011) after participating in Lesson Study showed that they felt confident and had improved in their teaching of mathematics.

Conducting Lesson Study Meetings occurred in collaboration with the teachers. From time to time Group Meetings moved from one school to another school according to their discussion focus and the person who was teaching. The participants in both groups had a high commitment to the project through Group One showed more commitment than Group Two. All participants in Group One attended the Lesson Study meetings according the time schedule. This was different from Group Two in which a few participants were absent from Lesson Study meeting. In fact, the teachers in Group Two in the inner city had busier schedules with extra activities outside teaching in the classroom. For example, at the same time when Lesson Study meetings were scheduled one of the teachers in the inner city group had another professional development commitment.

Moreover, Lesson Study improved collaboration among teacher participants. In particular, teachers learned about teaching mathematics using problem solving by observing each other and sharing ideas. In fact, through group discussion the participants increased their asking of questions and in collaboratively developing a lesson plan using teaching aids. They were more open minded in sharing with each other, and more confident in their teaching practice. For example, in Ha's Group Meeting the participants shared ideas about how to teach the concept of area of a trapezium, how students could learn this concept, how to develop student handouts or worksheets, and how to organise student grouping. As he explained after a Lesson Study meeting:

> *I can improve teaching and learning with colleagues both in design of lesson plans and implementation of lesson plans in teaching practice.* (Interview, Ha, 29 December 2010)

One year after Lesson Study meetings one of the participants explained:

> *I am very interested in Lesson Study because Lesson Study can improve teaching and learning to develop best practice. We are able to discuss in a group, and can be open minded for teachers to get knowledge.* (Interview, Ma, 24 December 2011)

Normally, most of the teachers taught mathematics in whole class situations without grouping of the students but during the Lesson Study all participants put students in groups. The new experience in being involved with this model of professional learning had a positive impact on their teaching and improving their style of instruction. This is demonstrated in Group Meetings following the second lesson. When it went well observers only reported that, and noted that there was no further discussion needed.

Summary

Lesson Study was a new experience in professional learning for the teacher participants. It had positive impact for teacher participants, in the ways that it changed in pedagogical content knowledge, mathematical content knowledge and teacher beliefs. Traditionally, most of the teacher participants had taught mathematics in whole class settings, used a textbook format, and no student grouping. Following Lesson Study, all participants changed in the way they taught by using grouping, using manipulatives, and choosing open problems.

The findings show that the greatest change occurred in pedagogical knowledge especially in knowledge of teaching and knowledge of students' learning. The use of manipulatives to represent ideas and help build conceptual understanding was relatively new to the teachers. Also the teachers had new experiences in choosing open problems during Lesson Study.

Regarding the demonstration of procedures, five teachers showed development. Teaching mathematics using problem solving was something new compared to a traditional teaching approach using a text book format that involved explaining the concept, and giving practice examples. Their changes were implemented in response to what was suggested in group discussions and how it might affect their teaching practice. Regarding the level of teacher questioning, most questions were categorised as knowledge questions or lower order questions with predetermined short answers. The teachers showed less development in using higher order questions, through there was some change as their teaching began to use more of a problem solving approach. In mathematical content knowledge, the teacher change tended to relate to change in teaching approach and was reflected in understanding of procedures, understanding of mathematical relationships and demonstration of knowledge.

Teacher beliefs about teaching changed from having an emphasis on the explanation of the new procedure or formulae followed by practice examples to using a problem solving approach. However, while most teachers believed in the benefits of teaching mathematics using a problem solving approach, they showed limited knowledge of problems and how to use them. It is clear that the Lesson Study Cycle as described here did have a positive impact on the professional development of the participant teachers in the ways described and discussed here.

CHAPTER SIX

Conclusions and Implications

Introduction

In a traditional setting teacher professional learning is driven by an expert or trainer to transfer new knowledge to the teacher participants based on research information. It has become more common for professional learning programs to involve teacher participants in more active ways to reach the aim of improving teacher knowledge and practice. As a professional learning model, Lesson Study is driven by teacher participants to disseminate knowledge among the teachers based on problems or needs in the classroom. Lesson Study is different to traditional professional learning approaches as it generally occurs within the school contexts of participants. The present study evaluated the effectiveness of professional learning based on Lesson Study and a problem solving approach for Indonesian primary school teacher working in small groups.

This study is significant because it evaluated the effectiveness of a professional learning program in order to provide information about provision of professional learning to develop best practice for primary school teachers. It helped the teachers change their practice through Lesson Study while they are teaching in their classroom. Lesson Study could help teachers to develop teaching materials based on content appropriate for local cultures and which relate to the development of the curriculum (KTSP) at the school level. The other significance is that educators who have been involved in this Lesson Study program through observation of teaching practices and Group Meetings are now able to recognise the difficulties associated with teaching methods based on problem solving and to understand the complexity of changing practice. Also teacher educators are better able to understand the factors that affect primary school teachers in their teaching practice.

The study aspires to provide a better understanding of what happens in classroom practice when teachers begin teaching mathematics using a problem solving approach having previously taught in a traditional way. This study can contribute to the knowledge about models for teacher professional learning and help to inform teachers, teacher educators, schools and departments of education, when developing policy. As a result of informing authorities of these outcomes, it may be possible to redefine the importance of links between teachers' knowledge and implementation of professional learning.

This chapter presents the conclusions and implications of the study about professional learning based on problem solving and Lesson Study for primary school teacher in an Indonesian context. It begins with a discussion of some important points of conclusion about teachers' pedagogical content knowledge, mathematical content knowledge, teachers' knowledge in teaching mathematics using a problem solving approach, teachers' beliefs about teaching and learning mathematics, and the effectiveness of professional learning based on the Lesson Study model.

Teacher Pedagogical Content Knowledge

This study found that teachers' pedagogical content knowledge developed with regard to the planning of lessons, teaching of the lessons, and assessing students' learning. The Lesson Study Cycle had changed teacher practice to become more student centred, more based on use of problems, and with more allowance for students to think in different ways. In a way of representing ideas, the teachers used various teaching aids or manipulatives, to represent ideas in order to make learning meaningful for students. Shulman (1986, p.6) had identified pedagogical knowledge as including "the most useful forms of representations of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstration". This study found that the teachers built their pedagogical knowledge based on the lessons and their practical experiences. The teachers shared the planning of lessons, as well as reflections and group discussions. Their teaching and learning had changed to include use of a variety of manipulatives, using a variety of ways of demonstrating procedure, and the use of open problems in teaching. Also they had become more effective in observing student learning and student thinking.

Most volunteer teachers demonstrated better preparation of lessons and better teaching practice after the Lesson Study Cycle. In preparing the lesson, the teachers had the opportunity to work collaboratively and that encouraged teachers in implementing the lesson plan. The volunteer teachers initially did not feel confident in their teaching practice with observers in the classroom, but they felt more confident after the Lesson Study program. Their skill, knowledge and teaching practice were improved following reflection and suggestions made in Lesson Study Group Meetings. Lesson Study Group Meetings after each lesson provided the participants with the opportunity to share their experiences, discuss classroom observations and make and respond to suggestions from observers and the researcher.

The teacher participants changed to become aware of the fact that manipulatives should be used to help student understanding of concepts. All teachers used manipulatives as a result of Lesson Study as their teaching approach changed from traditional teacher centred teaching to students being able to demonstrate knowledge in different ways.

Teacher Mathematical Content Knowledge

The teachers' mathematical knowledge developed to reflect deeper understanding of concepts and procedures that also improved their confidence in teaching. Ball, at al. (2009) noted that mathematical knowledge for teaching meant the mathematical knowledge needed to carry out the work of teaching mathematics. In the Lesson Study process, the teacher participants discussed and decided what content should be taught and how deeply that content should be covered. The improvement in their mathematical knowledge was limited to the topics about area of trapezium, area of parallelogram, measuring angle, and volume of cuboid. The Lesson Study model was shown in this study to be able to highlight shortcomings in aspects of teacher mathematical content knowledge. The Group Meetings were shown to be effective ways of helping teachers to develop their knowledge and to become more effective.

Teacher Knowledge in Problem Solving

Following Lesson Study, the teacher participants changed their practice in general and became more aware of the use of problem solving. They began to understand a problem solving approach and how it gave students opportunities to solve problems in their own ways. Students worked in groups and used manipulatives when teachers began teaching with a problem solving approach. Teachers listened and observed student responses more often and were less dominant in giving explanations. Teachers began to use open problems as tasks for initial exploration to introduce the new concept or procedures. However, even though the teachers began to use a problem solving approach in teaching mathematics, in terms of assessing student learning, they still tended to use closed problem exercises at the end of the lesson. The teachers acknowledged that a problem solving approach could provide a vehicle for students to construct their own ideas and to take responsibility for their own learning. However, this understanding of what constituted a problem was still limited and it is felt that had the Lesson Study Cycle continued for a longer time, further development of teacher knowledge of problems might have been possible.

Teachers' Belief about Teaching and Learning Mathematics

Changes in the teachers' beliefs about teaching and learning mathematics were positive effects of the Lesson Study Cycle. The Lesson Study Cycle was able to change teachers' beliefs about teaching and learning mathematics to non-traditional beliefs using a problem solving approach. After the Lesson Study program most teachers believed that mathematics should be taught not only using the teacher's explanation but also by asking questions and giving a problem or a task in the beginning. They believed that problem solving emphasised the process and deeper understanding rather than students relying on memorising. Problem solving could be used to introduce a new concept, not just in application of it. The change occurred within a short time from September to December 2010 during Lesson Study Cycle. The five teachers who were volunteer teacher participants in Lesson Study gained a new insight into how their own teaching practice affected their classroom, how their student learned from the lesson, how they interpreted the ideas, and this led them to re-examine their beliefs about teaching and learning mathematics. The change from traditional teaching based on teacher explanation for introducing the concept to problem based teaching for introducing a new concept is not an easy task. Yet, those involved did change their thinking to a notable extent.

Although the teachers believed a problem solving approach provided a particularly rich vehicle for students' thinking so that the students could become independent learners their teachers' beliefs were not always reflected in their teaching practice. After Lesson Study Cycle they used problem solving as a teaching approach in limited ways. They believed that problem solving needed more time for preparation of teaching materials and teaching aids. They felt that if teaching mathematics used a problem solving approach the lessons would not allow them to reach the curriculum targets. Therefore, to successfully implement problem solving a longer Lesson Study Cycle process is needed.

Lesson Study Bringing Change

The Lesson Study Cycle brought positive change in teacher practices and knowledge and was an effective model of teachers' professional learning as it helped develop teachers' pedagogical knowledge and mathematical knowledge. Draft lesson plans from a teacher educator were discussed and adapted to school lesson plans during Lesson Study activities. By designing a lesson plan, and implementing it with classroom observation and discussion after the lesson, the teachers built a new lesson plan. As a result, the teachers could improve knowledge of mathematical problem solving, knowledge of teaching and knowledge of student learning. Although the Lesson Study Cycle brought changes in teachers' knowledge, they still had limited knowledge of problem solving.

To be effective in teaching mathematics using a problem solving approach, the teachers not only need to extend lesson plans and to prepare the materials but also need to have greater problem solving experience themselves. Teachers' problem solving experiences could include understanding types of problems, mathematical problem solving skills and strategies and developing a range of problems. These are very important for teachers to experience before they could adequately deal with teaching using a problem solving approach. Reflection on teaching practice, knowledge of teaching and students' learning are essential parts of good practice and can help bring about change in students' learning outcomes and teachers' beliefs. The Lesson Study Cycle certainly encouraged reflection and did bring about positive changes in a comparatively short time.

Implications

The results of the study, suggest some implications for planning at the school level, for planning professional learning, for policy makers, and for further research and development. The implications of this study are as follows:

School level

One of the implications at the school level is that Lesson Study would help teachers develop and prepare for teaching based on content appropriate for local situations and societies. Reflection and group discussion at school level could address local school and community needs and issues. It has great potential for the improvement, of teachers' teaching and student learning at the school level rather than on a large scale. It would be appropriate to use Lesson Study to implement curriculum (KTSP) where the school personnel have authority to develop curriculum corresponding with the local situation and society. A school could offer Lesson Study as a program for professional learning and as a process to improve teaching and learning for the classroom practice of its teachers. Consequently, such use of the Lesson Study Cycle would require the determination and support of the school administration in the initial stages, and the ongoing commitment on all teachers involved.

Professional learning

Lesson Study could be of potential benefit for continuing teachers' professional learning. It could provide teacher participants with an insight into the use of a learning community to design and develop teaching materials and practice based on their own experiences. It could support teachers to become producers as well as consumers of teaching materials. In the Lesson Study teachers learned something new from their peers and applied it to improve their teaching of mathematics. In the future Lesson Study could become a preferred model of professional learning that has the combined support of Teacher Associations (KKG) and government. Currently, KKG a teacher association works collaboratively with teachers to solve problems that arise from teaching and learning. In fact, KKG conducts meetings on a weekly basis in some schools. One result is the design of lesson plans that are not necessarily implemented, but kept as administrative documents at the school.

Although the Indonesian government supports teachers' professional learning in problem solving, many of the teachers return to their schools and use traditional teaching without change even after their professional learning. One of the reasons may be that there are no external resources or support for teachers who were just *doing it alone* in their school. Also, traditional professional learning is driven by outside experts and trainers and is provided for teachers based on research but with little input from teachers. Another reason for the lack of implementation of Lesson Study might be a lack of resources and the learning might not be based on needs of students. Through Lesson Study it might be possible to form learning communities for continuing the implementation of the results of the professional learning, which is driven by the teachers. It is the latter feature, that is, driven by teachers that may determine the success of the professional learning.

Policy makers

There are at least two implications of Lesson Study for policy makers. First, Lesson Study has improved teachers regarding planning the lesson, teaching the lesson, assessing students' learning, and demonstrating deeper understanding of concepts or procedures that improve their confidence in teaching. It has had positive effects in the way teachers participate. Also Lesson Study as an effective model for teachers' professional learning in helping develop teachers' pedagogical knowledge and mathematical knowledge. As a professional learning model Lesson Study could be used to help teachers be more reflective and to improve the quality of student learning as a result of greater learning pedagogical competence of primary teachers (Depdiknas, 2007). Lesson Study has been conducted successfully in secondary schools where it has improved teaching practice, teaching methodology, teacher competencies, and students' achievements (Marsigit, 2006). Hence, policy makers within the Department of Education in Indonesia should be encouraged to support the use of Lesson Study in the future planning of programs for improving primary teachers' knowledge.

Second, Lesson Study could initially be implemented in schools in a relatively remote area with less facilities and external intervention. This study showed that meetings for the Group I of the Lesson Study from the suburban area were better attended than the meetings of Group II from the inner city. In Group I all members attended the meetings on time. Although The Group I lacked facilities they had a good attitude about collaboration. They shared teaching aids, textbooks and other resources. Although there was a minimum of external intervention, they discussed among themselves and the learning was powerfully teacher driven.

Further research and development

One important aspect of this study was the use of a problem solving approach in the Lesson Study. This result of this study showed that for problem solving to be an effective approach, teachers need to not only write extended lesson plans but also to have greater problem solving experience if they are to improve practice. Using open problems was something new for most teachers. This study found that teachers needed greater knowledge of problems and problem type as they were moving from a traditional teaching style to teaching with problem approach. There needed to be more time provided than was available in this Lesson Study Cycle for teachers to sustain the learning they gained from it. Results are encouraging and further research is needed over a larger time frame.

This study placed teachers in a central role having wider responsibility. This becomes an important aspect in developing teaching conducted in local classrooms. Teacher participants conducted classroom observation that would bring sharing knowledge about teaching and learning practice as part of the normal ways teachers would interact professionally. Therefore, the use of the Lesson Study Cycle would improve mathematics teaching quality, improve teacher quality of professional interaction from teacher to teacher, which would automatically improve teacher collegiality and their need to improve the teaching quality. This study would help teachers to see their classrooms in different ways through conducting research where the teachers could investigate and verify what worked for their students. Subsequent areas of interest in this study and opportunities for future research are the use of professional learning that promote teachers in personal professional growth among teachers.

Limitations of the Study

This study had several limitations. The first limitation relates to the number of participants. There are more than 100 primary schools in four district areas in Bengkulu with at least six teachers in each school. It is difficult to involve such a large number of primary teachers to participate in professional learning as in this study. Hence, this study used a case study approach that involved a small group of primary school teachers from six schools in a small district area. It would have been difficult to include all or most primary school teachers to participate in Lesson Study program and so it was limited in scope, which means that the sample might not be representative of other schools in the district. Therefore, when considering the findings of this study with regard to other schools and other districts, one must be careful not to make broad generations.

Secondly, interviews after Lesson Study meeting indicated that one of the limitations of Lesson Study was time constraints. Although most teachers had a high level of commitment to participate in Lesson Study, they had a problem with making time to come together to design lesson plans, observe and discuss. At the same time, the teachers had difficulty in arranging meetings, as they had to teach in their classes while observing in Lesson Study. As classroom teachers, they were responsible for teaching all subjects in their classes without the teacher changing. This could have been a distraction for the teachers and might have limited the effect of the professional learning.

Third, this study used interviews with teacher participants for collecting data, as well as observations of lessons and Group Meetings. This meant that data gathered were weighted towards the teacher participants in the Lesson Study. Interviews with students in the teachers' classrooms, and classroom observations after the Lesson Study Cycle, were not conducted by the researcher. Although the study findings showed positive effects on the teachers about teaching mathematics, it would have given more insight had interviews been conducted with both teachers and their students. Future research using a wide range of data gathering techniques could be conducted.

Fourth, although the study was conducted in a short time with positive effects on teacher pedagogical knowledge and mathematical knowledge, the continuity or sustainability of this project will depend on each of the schools involved. The schools have to allocate budget resources and time. It is very important to take these factors into consideration when planning further extension of professional learning programs at school level. Budgeting and time constraints are clearly important issues that can affect the implementation of Lesson Study in schools.

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Appendices

Appendix A: Classroom Observation

Name : Date :
Class observed : Time :
Observer : School :

Directions

Below is a list of effective teaching behaviours of teaching mathematics through problem solving that may occur during a class. This form is to be used as a guide, not a list of teaching requirements. The observer and the teacher use this list prior to the observation as a basis to discuss selected areas on which to focus.

Teacher Activity	Student Working				Note Students Activities/Student Response
	Indv.	In pair	Group	Class	
Type of problem					
- Use open problem					
- Use closed problem					
- Use drill exercise					
- Use applied problem					
- Use puzzle problem					
Resources to illustrate problem					
- Use concrete representation					
- Use pictorial representation					
- Use real world representation					
- Use symbolic/abstract representation					
Level of Teacher's Questioning					
- Use knowledge question					
- Use comprehension question					
- Use application question					
- Use analysis question					
- Use synthesis question					
- Use evaluation question					
Demonstrate of Procedures					
- Tasks/problems					
- Discussions					
- Student presentations					
- Teacher presentations					
Classroom Management					
- Use positive teacher statement					
- Use negative teacher statement					
- Encourages students to answer difficulty					

Teacher Activity	Student Working				Note Students Activities/Student Response
	Indv.	In pair	Group	Class	
Classroom Management					
- Encourage student different point of view					
- Encourage student thought and participants					
- Encourage student give reasoning					
Diagnosing learning/Assessment					
- Use an oral question					
- Use written question					
- Use written Test					

Appendix B: Lesson Study Meeting Documentation

Name : Date :
Group observed : Time :
Observer : Place :

Directions

Below is a list of individual and group participation in the Lesson Study meeting after implementation of the lesson plan (research lesson). This form is to be used as a guide, to observe Lesson Study Group Meeting.

Aspects	Indicator	Member of groups							Note/Comments Example
		T 1	T 2	T 3	T 4	T 5	T 6	R	
Content	What the group is talking about								
Background of the research lesson	Group members present the goal of the research lesson, designed, and what they learned								
Reflection/ feedback from the data research lesson	Presentation and discussion of data from the research lesson in the following:								
	Teachers look for type of problem								
	Teachers look for resources to illustrate problem								
	Teachers look for type of question								
	Teachers look for classroom organization on student working								
	Teachers look for classroom discourse								
	Teachers look for assessment								
Revision	Teachers revise for type of problem								
	Teachers revise for resources to illustrate problem								
	Teachers revise for type of question								
	Teachers revise for classroom organization on student working								
	Teachers revise for classroom discourse								
	Teachers revise for assessment								
Conclusion									

Aspects	**Indicator**	**Member of groups**							**Note/Comments Example**
		T 1	**T 2**	**T 3**	**T 4**	**T 5**	**T 6**	**R**	
Process	How the group is handling its communication								
Communication	Who talks? How long? How often? How interrupt?								
Helping	Teachers support or assistance to each other								
Listening	Teachers engage in effective listening								
Participating	Teachers contribute to the Group Meeting								
Persuading	Teachers exchange, defend, and think critically about their ideas								
Questioning	Teachers interact, discuss, and asking questions among all of the group members								
Respecting	Teachers encourage and support the efforts and ideas of others								
Sharing	Teachers offer their ideas their ideas and report their findings to each other								

Appendix C: Interview Before Lesson Study

Interview before Lesson Study

Time of Interview : Date: Place:
Interviewee :
Position of Interviewee:

Introduction

The purpose of this study is to investigate teachers' professional learning for teaching problem solving in mathematics in Indonesian Primary Schools. Your information will be helpful for professional learning to enhance teachers' personal knowledge about problem solving, their pedagogical content knowledge of problem solving and their practice of teaching problem solving in real life contexts. All interviews will be transcribed, duplicated and the original name will be erased. Only the researcher will know the real names of the people involved in the study.

Questions

1. Name of interviewer (optional) :
2. School :
3. How many years you have been teaching mathematics?
4. In what ways have you been involved in professional learning prior to lesson study?
5. What is your understanding of the current professional development for improving your teaching mathematics problem solving?
6. Which parts of the professional learning have been most helpful in improving your teaching of problem solving in your classroom?
7. Do you know Lesson Study? If yes
   a. Why are you interested in Lesson Study for professional learning?
   b. What do you hope to achieve by doing Lesson Study?
   c. In what ways has the Lesson Study model influenced your professional learning?
   d. Which of the critical features you just described are most difficult to implement in your classroom, and why?
   e. What were some of the benefits of Lesson Study for you and your classroom practice?

f. What were some of the challenges/ problem/issues you and your Lesson Study colleagues faced in conducting Lesson Study?

8. Do you know mathematics problem solving? If yes, how do you know your knowledge with mathematics problem solving has developed? (e.g. in understanding problem solving strategy, design or selecting problem base tasks)

9. How do you build your own knowledge and skills in mathematics problem solving?

10. Have you planned the lesson teaching mathematics through problem solving? If yes, tell me:
    a. How do you plan the lesson teaching mathematics through problem solving? (e.g. involve teaching strategy, provide students to explore and apply mathematics in problem solving)
    b. How do you teach mathematics through problem solving. What have you been doing as a teacher in your classroom? (e.g. in challenging students' thinking, providing assistance when students need it, providing feedback for students)

11. How did the professional learning affect you as a teacher?

12. Currently, how confident do you now feel about teaching mathematics through problem solving?

Appendix D: Interview at the end of Lesson Study

Time of Interview : Date: Place:
Interviewee :
Position of Interviewee:

Introduction

The purpose of this study is to investigate teachers' professional learning for teaching problem solving in mathematics in Indonesian Primary Schools. Your information will be helpful for professional learning to enhance teachers' personal knowledge about problem solving, their pedagogical content knowledge of problem solving and their practice of teaching problem solving in real life contexts. All interviews will be transcribed, duplicated and the original name will be erased. Only the researcher will know the real names of the people involved in the study.

Questions

1. Name of interviewer (optional):
2. School:
3. How many years you have been teaching mathematics?
4. Before following Lesson Study, in the last two years what kind of professional learning did you help in improving teaching mathematics through problem solving?
5. Before following Lesson Study, How did you prepare teaching mathematics? (eg.design lesson plan, preparing material)
6. Before following Lesson Study, how did you feel confidence in teaching mathematics through problem solving?
7. During the Lesson Study, what was you the most difficult to apply in teaching practice? Which part? Why?
8. During the Lesson Study, what did you feel useful in the teaching practice in the classroom?
9. During the Lesson Study, what was you learned about teaching mathematics through problem solving? (e.g., inter of problem solving content knowledge, strategy solving problem, strategy teaching problem solving?
10. After participating in Lesson Study, how to design lesson plans using problem solving approach? (e.g. involving teaching strategy, giving students exploration, and applying problem solving)
11. During the Lesson Study, what are the challenges/ problems/ obstacles/ issues for as teachers in implementing Lesson Study?

12. After participating in the Lesson Study, what impact/change did you feel? (e.g. content knowledge problem solving, pedagogical knowledge problem solving, teaching practice about problem solving)
13. After participating in Lesson Study, how did you understand about content and teaching problem solving?
14. After participating in Lesson Study, how did you feel confidence in teaching mathematics problem solving?

Appendix E: Interview after Lesson Study Program

Time of Interview : Date: Place:
Interviewee :
Position of Interviewee:

Introduction
The purpose of this study is to investigate teachers' professional learning for teaching problem solving in mathematics in Indonesian Primary Schools. Your information will be helpful to inform professional learning to enhance teachers' personal knowledge about problem solving, their pedagogical content knowledge of problem solving and their practice of teaching problem solving. All interviews will be transcribed, duplicated and the original name of the interviewee will be erased. Only the researcher will know the real names of the people involved in the study.
Questions

1. Name of interviewer:
2. School:
3. How many years you have been teaching mathematics?
4. Before following the Lesson Study cycle,
    a. What kind of professional learning have you undertaken to help improve your teaching of mathematics? (e.g. KKG workshop,)
    b. What professional learning have you completed that relates to problem solving
5. Have you been teaching maths using a problem solving approach after Lesson Study Professional learning? Describe what you have done
6. How do you prepare for teaching mathematics? (eg. design lesson plan, preparing material) give some examples

Item	Before Lesson Study	After Lesson Study
a. Lesson Plan		
b. Resources (manipulatives, text books)		
c. Task/ problem		
d. Example		
e. Class management (e.g. group, whole class, etc.		
f. evaluation		
g. supporting		

7. Do you feel confident in teaching mathematics through problem solving?

Item	Before Lesson Study	After Lesson Study
a. Mathematic content knowledge		
b. Pedagogical content knowledge		
c. Curricular knowledge		

8. What do you believe about teaching and learning mathematics through problem solving?

Item	Before Lesson Study	After Lesson Study
a. Student learning		
b. Teacher teaching		

9. How has your teaching practice changed in your mathematics class after using a problem solving approach?

Item	Before Lesson Study	After Lesson Study
a. Content knowledge		
1. Understanding concept		
2. Understanding problem		
3. Develop problem : type of problem		
4. Solve problem		
5. Select strategy		
6. Demonstrate knowledge		
7. Give example		
8. Identify misconception		
b. Pedagogical content knowledge		
1. Teaching problem solving approach		
2. Demonstrate problem solving process		
3. Represent the concept		
4. Use example		
5. Diagnosis student difficulty		
6. Anticipate student thinking		
7. Classroom organisation		

Item	Before Lesson Study	After Lesson Study
8. Access student achievement		
c. Curricular knowledge		
1. Select goal/competency		
2. Select topic		

10. During the Lesson Study,
    a. What was the most difficult thinking to do in teaching mathematics? Which part? Why?

    b. What aspects did you were feel useful in teaching mathematics in the classroom?

    c. What did you learn about teaching mathematics through problem solving? (e.g., inter of problem solving content knowledge, strategy solving problem, strategy teaching problem solving?

    d. What were the challenges/ problems/ obstacles/ issues for you in implementing Lesson Study?

11. What did you think about Lesson Study?

Item	Before Lesson Study	After Lesson Study
a. Are you interested in Lesson Study?		
b. Have you achieved by doing Lesson Study		
c. In what ways has the Lesson Study model influenced your professional learning?		
1. Preparing material		
2. Teaching practice		
3. Developing knowledge		
d. What were some of the benefits of Lesson Study for you?		
e. What were some of the challenges/ problem/issues for you in Lesson Study?		
1. time		
2. cost		
3. learning community		

Appendix F: Example of Lesson Plan and Student worksheet

Lesson Plan

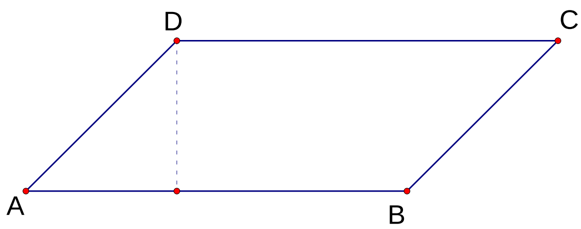
Subject : Mathematics
Class / Semester : IV/1
Time allotment : (2 x 35 minutes)/1 meeting
Competency Standard : Using the concept of perimeter and area of a Simple shape of geometry in problem solving
Basic competency : Finding the perimeter and the area of parallelograms and triangles

Indicators:
1. Finding the formula for perimeter of a parallelogram
2. Applying the formula for perimeter of a parallelogram
3. Finding the formula for area of a parallelogram
4. Applying the formula for area of parallelogram

I. Learning Objectives:
1. Given a geoboard, students are able to construct a few parallelograms
2. By using string, students are able to calculate the perimeter of a parallelogram
3. By using string and geoboard, students are able to construct parallelograms with the same perimeter in different size
4. Students are able to determine the formula for perimeter of a parallelogram
5. Students are able to use the formula of area and perimeter of a parallelogram to solve problem in the real life

II. Material:

Parallelogram



Perimeter of parallelogram ABCD = total length of four sides of a parallelogram
= AB + BC + CD + DA
Area of parallelogram ABCD = base x height

III. Learning methods:

Discussion

Problem Solving

IV. Learning procedures:

1. Pre-activity (10 minutes)

   The teacher shows the picture of garden as a parallelogram, and the teacher asks students:

   - How long is the perimeter of the garden?
   - What is the area of the garden?

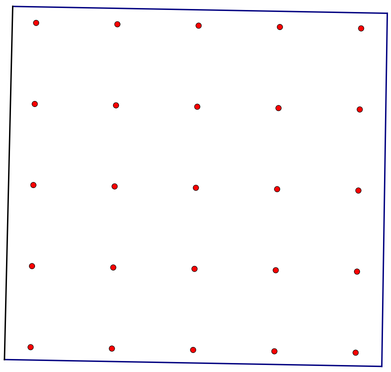
   The teacher states the learning objectives

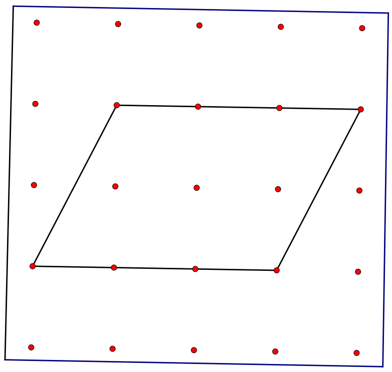
2. Main activity (50 minutes)

   The teacher divides the group of four or five students

   The teacher gives a geo board, rubber bands, and worksheets for each group

   Students in groups construct a parallelogram on the geo board 5 x 5 by using rubber bands





By using the string, students browse the trace all sides of the parallelogram constructed and cut the string along the perimeter of parallelogram

The teacher gives some questions such as: what conclusions about the perimeter of a parallelogram?

Furthermore, students are asked to construct 4 parallelograms on a geo board 5 x 5.

The Teacher asks for height and length of the base of parallelogram constructed, and students are asked its area.

Students calculate the area of each parallelogram.

The teacher asks students to make parallelograms with area 6 square units as many as they can on a geo board, and then draw this parallelograms on dotted paper.

The teacher guides the students working in group how to construct them.

Students write the results of group work on worksheets.

Students present the results of group work in front of the class.

Each student calculates the perimeter and area of the garden in the form of parallelogram.

3. Post-activity (15 minutes)
The teacher and students make conclusions together.
The teacher conducts evaluation.

V. Learning aids/references

1. Worksheets.
2. Geo board
3. Rubber band and string
4. Buku pelajaran matematika untuk sekolah dasar 4A. Khafid, Nur Aksin, Suyati. Penerbit Erlangga
5. Gemar matematika 4 YD Sumato, Heni kusumawati, Nur Aksin, Bse
6. Matematika 4 untuk kelas 4 SD MI, RJ Soenarjo, Bse

VI. Evaluation

1. Technique: tests and non-test
2. Instrument attached

Bengkulu, December, 2010
Team of Lesson Study Group I:

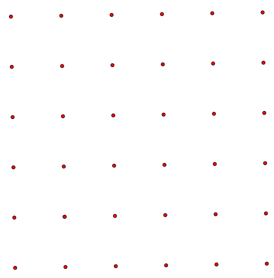
1. Ma – SD 1 M
2. Sa – SD 1 M
3. Si – SD 4 M
4. Yu – SD 4 M
5. Zu – SD 8 M
6. Ri – SD 8 M

Attachment

Test Instrument

Name: Class:

1. Construct two paralleograms whose area are 4 cm^2 in different size on doted paper!



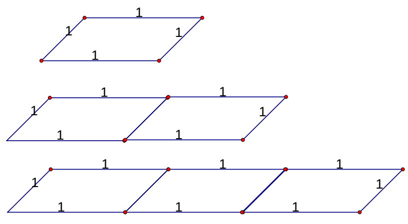
2. For a paralelogram whose sides are 5 cm and 10 cm, which of the following is true

   a. The area equals 50 cm^2

   b. The area is greather than 50 cm^2

   c. The area is less than 50 cm^2

4. The parallelogram tables are a arranged so each person sit on each side of table shown in the figure. One parallelogram table was sit for 4 people, two parallelogram tables were sit for 6 pople, and three parallelogram tables were sit for 8 people. If 10 parallelogram table were arranged according to previous patterns, hom many would people be sit?



Student Worksheet

Name: ....Class: IV ....School: ....

1. Construct parallelograms at every 5 x5 dotted paper with different sizes

a) 	b) 
c) 	d) 

2. Calculate the area of each parallelogram that you created in a), b), c) and d) above!

Answer:

3. Construct parallelograms which the area are 6 square units as many as you can on the dotted paper

a)

b)

c)

d)

Appendix G: Participant Information Sheet and Consent Form

Participant Information Sheet

Curtin University of Technology

School of Education

Participant Information Sheet
Professional Learning for Teaching Problem Solving in Mathematics in Indonesian Primary Schools

Dear Teacher,

My name is Agus Susanta. I am currently completing research for my PhD in education at Curtin University of Technology, Australia.

The Purpose and the aims of the research:

The purpose and the aims of the research are to investigate teachers' professional learning for problem solving in real life contexts in mathematics in Indonesian Primary Schools. The study will investigate enhanching of teachers' teaching problem solving and using professional development via Lesson Study to support the learning and skills for teaching problem solving for upper level in primary schools. Data will be collected on: teachers' personal knowledge of problem solving, teachers' pedagogical knowledge of problem solving; teachers' practice of teaching problem solving in real life contexts; teachers' self-efficacy, and beliefs about mathematics and mathematics teaching.

Your role

I am interested in finding out about professional learning gained through Lesson Study. I would like to observe your participation in small group professional learning workshops in your school area. You will be involved in professional learning workshops through Lesson Study. Each group of six primary teachers will meet regularly to plan, design, implement, evaluate and refine lessons for a unit of work. Teaching problem solving will be the focus of this Lesson Study. You as a participant will have opportunities to discuss, work together, design the lesson plan, and become a volunteer to teach the lesson in your own class. Classroom observation using a written observation schedule will be conducted by researcher. Interview and videotaping will be conducted on the Lesson Study meeting. Teachers sharing experiences in Lesson Study, implementing of teaching material in class, and having professional learning will be helpful in improving the teaching of problem solving in mathematics. I will ask you to comment on how your involvement in the

Lesson Study project has impacted on your professional learning as a teacher, and on what you see as key features of how your teaching has been developed.

Participant requirements

The participants of the study are 12 primary teachers; a group of six primary teachers in inner city schools and six primary teachers in suburban schools. You will need to have at least 1 year teaching experience and 2 years of primary schools teacher program studies in the university. You need to spend 2 hours per week to participate in the study during one semester.

Confidentiality and security

Participation in this study is completely voluntary, and participants may withdraw at any time without prejudice or negative consequences. Teachers must have written consent from the principal of their school to be able to participate in this study. All interviews will be transcribed, duplicated and the original with name will be erased. Only the researcher will know the real names of the people involved in the study.

Risks/benefits

The benefits to participants are the following: improve teachers' capacities to develop teaching material for primary school children based on content appropriate with local culture and society; provide opportunity for teachers to participate as a learning community to develop teaching materials and practice; and provide teachers with written guidance for how to teach mathematics problem solving based on the Indonesian curriculum. Informed consent will be obtained from the teachers, and the principal of the school, and permission will be obtained Head of Department of Education in Indonesia. The teachers will be informed of all aspects of study that might be expected to influence willingness to participate. The researcher will agree to sign a confidentiality agreement.

Contact details

The participants will be informed of procedures for contacting the researcher within a reasonable time period. If the participants have any further questions or concerns about this study, please contact the researcher Agus Susanta by phone +61412599332, email: agussusanta@postgrad.curtin.edu.au or supervisor A/Prof. Len Sparrow; l.sparrow@curtin.edu.au or co supervisor A/Prof. Sandra Frid; S.Frid@curtin.edu.au. Or the secretary of the Human Research Ethics Committee (phone +6189266 2784 or hrec@curtin.edu.au or in writing C/-office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth WA 6845). Thank you very much for your involvement in this research, Your participation is greatly appreciated. This project information letter is for you to keep.

Sincerely,
Agus Susanta
ID 14114212
agussusanta@postgrad.curtin.edu.au

Curtin University of Technology

Curtin University of Technology

School of Education

Lembar Informasi Peserta

Pembelajaran Profesi tentang Pemecahan Masalah dalam Pembelajaran Matematika di Sekolah Dasar di Indonesia

Yth. Guru

Nama saya Agus Susanta. Saya sedang mengerjakan riset untuk program PhD bidang pendidikan di Curtin University of Technology, Australia.

Maksud dan Tujuan Riset:

Maksud dan tujuan riset ini adalah untuk menyelidiki pembelajaran profesi guru pada pemecahan masalah matematika sekolah dasar di Indonesia. Dalam penelitian ini akan diselidiki peningkatan guru dalam mengajar pemecahan masalah dan melalui pengembangan profesi Lesson Study untuk mendukung pembelajaran dan ketrampilan mengajar problem solving untuk kelas atas sekolah dasar. Data yang dikumpulkan berupa: pengetahuan personal guru pada pemecahan masalah, pengetahuan pedagogi guru dalam pemecahan amasalah, praktek pengajaran, percaya diri guru, dan percaya tentang matemetika dan pembelajaran matematika

Tata cara

Saya tertarik pada penemuan tentang pembelajaran profesi ditingkatkan melalui Lesson Study. Saya akan mengobservasi peserta dalam kelompok pembelajaran professi di sekolah anda. Anda akan dilibatkan dalam profesi pembelajaran di dalam kelompok. Setiap grup dari 6 guru kelas atas, secara teratur bertemu merancang rencana pembelajaran, melaksanakan,mengevaluasi dan memperbaharui untuk suatu unit atau topik. Pengajaran pemecahan masalah akan menjadi focus pada lesson study. Anda sebagai peserta punya kesempatan berdiskusi, bekerja sama, mendesain rencana pembelajaran, dan menjadi sukarelawan untuk mengajar pada kelasnya. Observasi kelas mengunakan intrumen tertulis akan dilaksanakan oleh peneliti. Wawancara dan pengambilan video ketika pertemuan lesson study akan dilakukan. Guru-guru saling tukar pengalaman, mengimplentasikan bahan ajar di kelas akan membantu dalam pengembangan pengajaran pemecahan masalah . Saya akan minta komentar tentang bagaimana keterlibatan di dalam Lesson Study apakah ada pengaruh terhadap pengembangan profesi sebagai guru, dan cirri-ciri apa yang dapat dilihat bagaimana kemajuan anda mengajar setelah itu.

Persyaratan peserta

Peserta dalam Lesson Study adalah 12 guru sekolah dasar dari kelompok 6 guru di kota dam 6 guru di luar kota. Sebagai persyaratan anda punya pengalaman mengajar paling sedikit 1 tahun dan 2 tahun belajar di perguruan tinggi. Kegiatan akan dilakukan selama satu semester dan anda diminta menyediakan waktu 2 jam perminggu untuk kegiatan ini.

Kerahasiaan dan keamanan

Peserta dalam penelitian ini bersifat suka rela, dan peserta bisa keluar sewaktu-waktu tanpa ada konsekuensi negatif. Peserta harus menuliskan surat perjanjian dari kepala sekolah masing-masing untuk bisa ikut dalam kegiatn ini. Semua peserta interviu akan direkam dan diduplikasi, dengan tidak mencantumkan identitasnya. Hanya peneliti yang mengetahui siapa yang terlibat dalam penelitian ini.

Resiko/manfaat

Manfaat terhadap peserta sebagai berikut: Meningkatkan kapasitas guru dalam pengembangan bahan ajar untuk matematika di sekolah berdasarkan isi yang sesuai dengan budaya masayarakat; menyediakan panduan guru bagaimana mengajar dengan pendekatan pemecahan masalah sesuai dengan kurikulumnya. Kesepakatan akan didapat dari guru-guru, kepala kepala sekolah, dan surat ijin akan diperoleh dari Diknas pendidikan Indonesia. Guru- guru akan dinformasikan semua aspek yang menjadi keinginan untuk berpartispasi. Peneiti setuju untuk menandatangani surat perjajanjian.

Siapa yang bisa di hubungi

Peserta diberitahukan cara untuk berhubngan selama periode penelitian. Jika peserta ada pertanyaan lebih lanjut, silahkan hubungi peneliti Agus Susanta nomor telepun +61412599332, email: agussusanta@postgrad.curtin.edu.au atau pembimbing A/Prof. Len Sparrow; l.sparrow@curtin.edu.au pembimbing kedua A/Prof. Sandra Frid; S.Frid@curtin.edu.au. Atau ke seketariat penelitian Human Research Ethic Committee (phone +6189266 2784 or hrec@curtin.edu.au or in writing C/-office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth WA 6845)

Terimakasaih atas partsipasinya dalam penelitian ini, keikutseratataan anda adalah sesuatu yang sangat berharga. Surat tentang informasi ini harap anda simpan.

Hormat kami,

Agus Susanta
ID 14114212
agussusanta@postgrad.curtin.edu.au

Participant Consent Form

Curtin University of Technology

School of Education

Professional Learning for Teaching Problem Solving in Mathematics in Indonesian Primary Schools

Teacher Consent Form

- I have read this document, or have had this document explained to me in a language I understand, and I understand the aims, procedures, and risks of this project, as described within it.
- For any questions I may have had, I have taken up the invitation to ask those questions, and I am satisfied with the answers I received.
- I understand that my contributions to this research will be reported in a thesis, and in published journals, provided that I am not identified in any way.
- I understand that my involvement is voluntary and I can withdraw at any time without affecting my career.
- I understand that a summary of findings from the research will be made available to me upon its completion.

Signature of teacher	:.................................
Date	: ....
Name of teacher	: ....

Curtin University of Technology

School of Education

Pembelajaran Profesi tentang Pemecahan Masalah dalam Pembelajaran Matematika di Sekolah Dasar di Indonesia

Surat Perjanjian Guru

- Saya telah membaca dokumen ini atau telah memahami dokumen ini dalam bahasa yang saya mengerti, dan saya mengerti maksud, prosedur, dan resiko proyek ini, seperti yang telah dijelaskan sebelumnya.
- Untuk pertanyaan yang diberikan ke saya, saya membuka diri untuk pertanyaan tersebut dan bersedia menjawab pertanyaan yang saya terima .
- Saya memahami bahwa kontribusi saya dalam riset ini akan dilaporkan dalam thesis, dipublikasikan dalam jurnal, dan identitas saya tidak dipublikasikan dalam bentuk apapun .
- Saya memahami bahwa keterlibatan saya merupakan suka rela dan saya dapat mundur setiap saat tanpa mempengaruhi karir saya .
- Saya memahami bahwa rangkuman penemuan dalam riset akan disediakan untuk saya setelah riset ini lengkap.

Tanda tangan guru :…………………..

Tanggal :…………………..

Nama guru :………………….