

New curricula and missed opportunities: Crowded curricula, connections, and ‘big ideas’

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Key words

Mathematics education, teacher education, curriculum development.

Abstract

The recent review of Australian Curriculum represents an opportunity to significantly change the teaching of primary mathematics in Australia and perhaps elsewhere. The current curriculum document presents content in a linear and compartmentalized manner that dominates its structure and doesn't explicitly accentuate the links and connections found in the ‘big ideas’ of mathematics. As well, it pays lip service to the ‘big process ideas’ - the Proficiencies - which should be the vehicles for developing and making explicit links between and within the ‘big content ideas’. The nature of key content needs to be reconsidered and organised on the basis of the ‘big ideas’ of mathematics, and to emphasize the links and connections within and between them, as well as between them and real contexts. Such connections should be made explicit for children and hence teachers may need to hold their mathematical knowledge in different ways. The review document acknowledges the issue of the ‘crowded curriculum’ and makes recommendations such as increasing the depth of content whilst reducing its breadth. Exactly how this will be achieved remains to be seen but it is strongly suggested in this position paper that there needs to be a reorganization of content on the basis of ‘big ideas of mathematics’ or at least a substantial acknowledgement that this is important.

Background

During early 2014, the Australian federal government commissioned a review of the Australian Curriculum – this position paper is concerned solely with the *Australian Curriculum: Mathematics*. The original version of mathematics curriculum was endorsed in 2010 and the current Version 7.3 was the subject of the review process. When first developed and released, the *Australian Curriculum: Mathematics* represented a chance to take a fresh approach to the teaching of mathematics but what has happened indicates that it may have been an opportunity lost. Atweh, Miller and Thornton (2012) suggested that the lofty ideals of the Melbourne Declaration (Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA), 2008) on which the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority, 2012a) is supposedly founded have become subjugated to more utilitarian goals. Indeed it seems that the important high level proficiencies are not viewed as outcomes to be developed and assessed but are little more than actions in which students engage when learning content (Atweh, Miller & Thornton, 2012). This is supported by an analysis of the curriculum rationale which lacks guidance for teachers as to how to develop the proficiencies of reasoning and problem solving or how they can be the vehicles for developing the deep knowledge and complex problem solving seen as important in the Melbourne Declaration (MCEETYA, 2008). Indeed, the document Shape of the Australian Curriculum Version 4. (ACARA, 2012), calls for the development of ‘deep knowledge’ and the need to ‘deepen understanding’. However, the way in which the curriculum content is structured offers little hope that such ‘deep knowledge’ will be developed.

The review document released in October 2014 made some recommendations regarding the *Australian Curriculum: Mathematics*, one being that “the curriculum should be revised to ensure the focus is on essential knowledge, understanding, and skills associated with deep learning instead of attempting to cover too much content” (Australian Government, 2014, p. 176). This was echoed in the press release by Education Minister Pyne (2015) stating that issues with ‘overcrowding’ in the curriculum would be achieved by “reducing the quantity of content, adding more depth and less breadth”. Similar comments have been made in other national contexts. In discussing the United Kingdom National Curriculum, Oates (2013) noted that UK curriculum developers could learn from other jurisdictions and focus on “fewer things in depth” and to “focus on key concepts and content”. Similarly, in the United States context, the introduction to the *Common Core State Standards for Mathematics*, stated that “the standards must address the problem of a curriculum that is a mile wide and an inch deep” (National Governors Association Centre for Best Practices, 2010, p. 3).

As well, Siemon, Bleckly & Neal, 2012, p. 20) recently supported that call in stating that “A focus on the big ideas is needed to ‘thin out’ the over-crowded curriculum”.

In these days of high stakes testing and international scrutiny, teachers are under pressure to cover the content of curricula that seem to be increasingly crowded, a problem that has been noted in many quarters, particularly in the UK, USA and Australia. Perhaps the familiar linear model for organising curriculum content is no longer adequate and that what is needed is for teachers to hold and teach mathematics content knowledge in a different way. It could be along the lines of the ‘big ideas’ of mathematics, emphasising the links and connections within and between those ideas. Further, it is suggested that a generic model for organising such ‘big ideas’ may be helpful for teachers as they seek to make explicit to children the myriad links and connections that exist. It remains to be seen if the recommendations made by the curriculum review committee and endorsed by Minister Pyne will come to fruition in the form of content reorganization in terms of ‘big ideas’.

A lack of connection

The third aim of the *Australian Curriculum: Mathematics* states that students should “recognize connections between the areas of mathematics and other disciplines” (ACARA, 2012a , p. 3). The word ‘connections’ is the key element of this paper in that it is suggested that the mathematical content should be reorganized around the ‘big ideas’ of mathematics and the connections within and between them. However, while the word is used in one of the three aims of the *Australian Curriculum: Mathematics*, and is used frequently in describing content, there is little indication of HOW teachers can make such connections explicit. There needed to be a stronger overarching statement to aid teachers in assisting their students to make connections. It is suggested here that the answer lies in teachers thinking about mathematics as a connected and linked enterprise and in ways other than is presented in the prevailing linear curriculum structure.

The crowded curriculum and the case for ‘big ideas’

Atweh & Goos (2011, p. 223) noted in their discussion of the *Australian Curriculum: Mathematics* that “the identification of content into the traditional mathematical fields . . . may be convenient in a syllabus but it does not lend itself to dealing with real-world applications that often require cross-disciplinary approaches”. The identification of connections and the explicit teaching about them could be the key to dealing with the issue of the crowded curriculum though the call for a focus on ‘big ideas’ is not new. In its *Principles and Standards for School Mathematics* which have underpinned the *Common Core State Standards*, the National Council of Teachers of Mathematics (NCTM) claimed that “Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise (NCTM, 2000, p. 17). Similarly, the Australian Association of Mathematics Teachers (2009 a, b) released two discussion papers which used the term as one of a trio of aspects of mathematical knowledge and understanding – mathematical concepts, mathematical actions and ‘big ideas’. Yet, the *Australian Curriculum: Mathematics* does not discuss ‘big ideas’. So what are these ‘big ideas’ of mathematics and how might they help teachers think about mathematics and teach it more effectively?

What do the ‘big ideas’ look like?

Recently, ‘big ideas’ in mathematics have been variously described. Charles established a definition of a ‘big idea’ as “a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole” (Charles, 2005, p. 10). He contended that ‘big ideas’ are important because they enable us to see mathematics as a “coherent set of ideas” that encourage a deep understanding of mathematics, enhance transfer, promote memory and reduce the amount to be remembered (Charles, 2005, p. 10). As well, he noted how a focus on ‘big ideas’ enables teachers to understand how mathematical topics and ideas are connected across years and grade levels rather than be considered as separate entities. Charles provided a set of twenty one ‘big idea’ statements such as the following:

The Base Ten Numeration System – The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

Patterns – Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. (Charles, 2005, p. 13, 17)

In the Australian context, Siemon, Bleckly & Neal (2012) noted, as did Charles regarding performance of US students, that evidence from international testing reflected a decline in Australian student performance in mathematics. They pointed to the need to ‘thin out’ the overcrowded curriculum by focusing on the ‘big ideas’ and promote a more connected view of mathematical topics and ideas. They acknowledged the work of Charles and others and felt that ‘big ideas’ need to be articulated better so that they “translate to learning trajectories that could be used to inform teaching and support mathematics learning over time” (Siemon et al., 2012, p. 24). They offered a set of six linked ‘big ideas’ developed from earlier work by Siemon (2006), namely “Trusting the Count, Place Value, Multiplicative Thinking, Partitioning, Proportional Reasoning and Generalising” (Siemon et al., 2012, p. 25).

Both Charles and Siemon et al. made important points about the need for ‘big ideas’ to be considered in ways that are different to the current style of organising curricula. The prevailing linear model can be represented in Figure 1 where content is compartmentalised in learning areas, content strands, and year levels. Divisions between year levels as well as between strands and learning areas are often uncrossed.

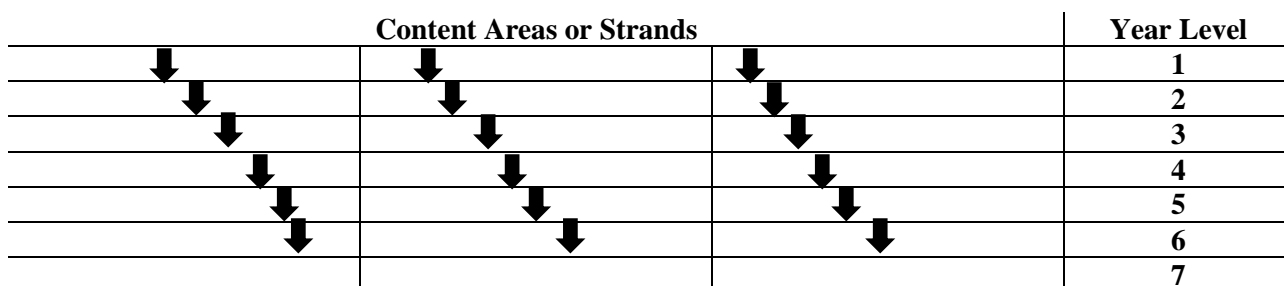


Figure 1: *The linear curriculum model*

The model for a curriculum would look quite different if ‘big idea’ thinking was used to organise content. If connections cross content strands, learning areas, and year levels were not only acknowledged but used as the basis for planning and teaching, the curriculum would become much more fluid, more connected, and more likely to facilitate ‘sense making’ by children. It would also reduce the perception of a ‘crowded curriculum’ for teachers. Figure 2 is an attempt to represent such a curriculum design. The horizontal arrow represents the connections between mathematical concepts and ideas across different content strands, such as making links between fractions, decimals and percentages and the description of probability. The vertical arrow represents the development of key ideas across year levels as is shown later in Figure 3.

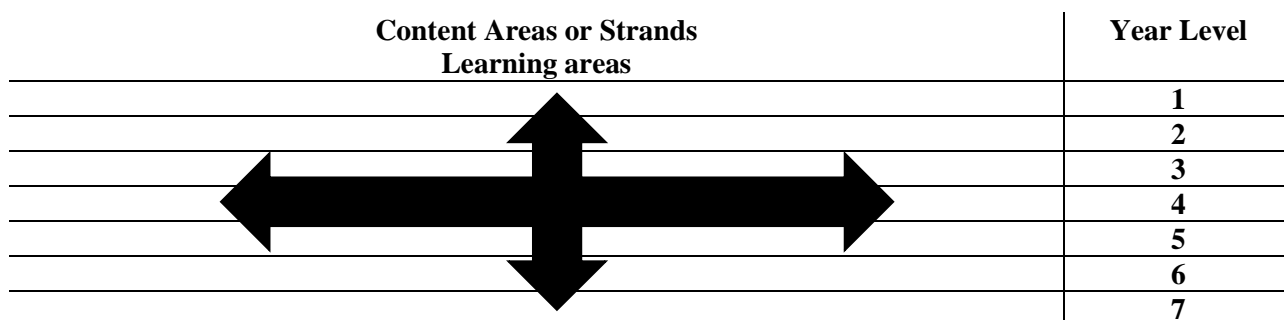


Figure 2: *Model for a ‘fluid’ curriculum based on ‘Big Ideas’ and key concepts*

Figures 1 and 2 are useful in demonstrating the different ways of viewing and organizing content in a curriculum but the developmental nature of ‘big ideas’ needs to be more adequately depicted. Siemon et al. described the importance of teachers knowing how ‘big ideas’ developed over time in order to understand typical learning trajectories which are not considered in terms of year levels but levels of conceptual understanding. Figure 3 (Hurst & Hurrell, 2015) is developed from the paper by Siemon et al. (2012) and attempts to show how ‘big ideas’ of number develop and underpin one another. The six ‘big ideas of number’ are those identified by Siemon et al. and the ellipses are purposely overlapped to emphasize that key components of a ‘big idea’ such as multiplicative thinking develop alongside components of ‘trusting the count’ and ‘place value’. For example, the notion of part-part-whole understanding is one such component.

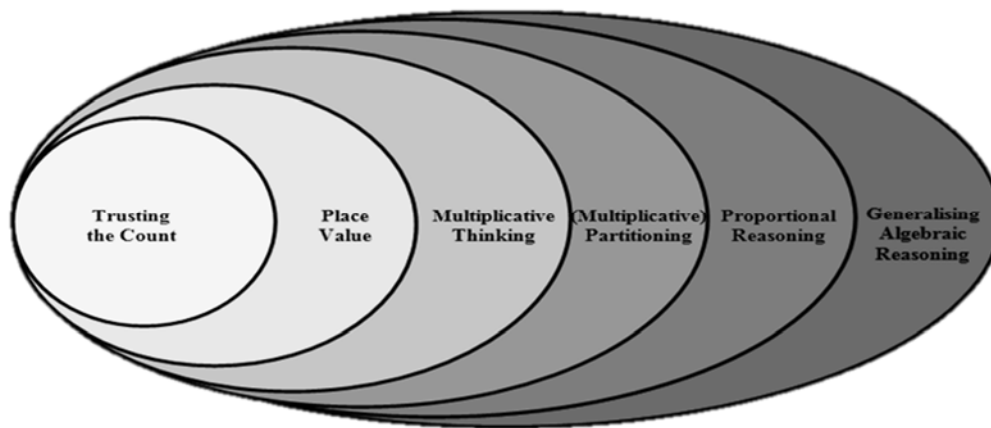


Figure 3: *Developmental nature of 'big ideas' of number*

However, the discussion of 'big ideas' did not begin with Charles (2005) or Siemon et al. (2012). In her seminal work, Ma (1999, 2010) did not specifically use the term 'big ideas' but described 'knowledge packages' that constitute teacher knowledge being the way in which ideas are organized and developed. She described such 'knowledge packages' as containing different ideas, some of which might be more important than others, as well as developmental sequences and 'concept knots' that represent the vehicles for connecting and linking ideas that are related to one another (Ma, 2010, p. 78). There is further discussion of Ma's work later in this paper. If teachers can think of mathematics in this way and make such connections and links explicit for their students, it is possible to overcome the 'crowded' and 'mile wide-inch deep' view of curriculum. However, in order for this to succeed, the *Proficiencies* (ACARA) need to be at the heart of teaching and not be simply paid 'lip service' in curriculum statements.

'Big ideas' of mathematics were also discussed by Clarke, Clarke & Sullivan (2012) who noted that, while the ideas themselves are significant, their real value lies in stimulating teachers to deconstruct their own conceptual structures and to them thinking differently about their mathematical content knowledge. In other words, it is about teachers reorganizing the way in which they hold mathematics content knowledge and the way in which they present it to their students. These 'big ideas' will be called 'big content ideas' for the purpose of this position paper as there are other 'big ideas' that need to be considered. These other ideas generally reflect the *Proficiencies* (ACARA, 2012a) and the *Common Core Standards for Mathematical Practice* (NGA Center, 2010) and will be called 'big process ideas' which include ideas like problem solving, reasoning, inferring and constructing arguments. This set of ideas also includes the ability to recognize and apply mathematics in context, which relates closely to the notion of 'contextual knowledge' contained in the numeracy model developed by Willis (1998) and described by Kemp & Hogan (2000). These ideas are also closely linked to the 'disposition' referred to in Willis's definition of numerate behavior. The view presented here re-conceptualizes how content knowledge could be organized and taught and emphasizes that teachers need to deeply understand the mathematics, and to make links and connections explicit to their students. It also emphasizes the processes that enable teachers to do the latter.

Linking 'big ideas' to numeracy

The term 'numeracy' has been used in various ways to discuss mathematics and mathematical knowledge and it is generally considered to mean much more than just 'mathematics'. Most definitions of numeracy include reference to content knowledge, recognition of mathematics in contexts, application of mathematics, and a disposition to use mathematics in everyday life. Kemp & Hogan acknowledged the work of Willis in describing numeracy as being "intelligent, practical mathematical action in context . . . [and that] being numerate, at the very least, is about having the competence and disposition to use mathematics to meet the general demands of life at home, in paid work, and for participation in the community and civic life" (Kemp & Hogan, 2000, p. 5). ACARA (2012b) describes numeracy in the following way: "Numeracy involves students in recognizing and understanding the role of mathematics in the world and having the dispositions

and capacities to use mathematical knowledge and skills purposefully”. Kemp & Hogan noted that being numerate required a blend of three different types of knowledge, an idea also developed by Willis (1998). Mathematical knowledge is knowing, understanding and using the mathematical ideas that are typically contained in a school curriculum. Contextual knowledge involves knowing what mathematics means in a particular context and how the mathematics is shaped by a context. Strategic knowledge involves working through problem situations, analyzing data, often dealing with conflicting mathematical requirements and making decisions about the mathematics involved (Willis, 1998; Kemp & Hogan, 2000).

It seems to follow that if students develop a richer and more connected view of mathematics as modelled by their teachers, then their mathematical knowledge will be stronger, their ability to recognize its use in a range of contexts will be more purposeful, and their ability to make strategic decisions about how mathematics is used will be more precise. As such, they will likely be more numerate with a greater possibility of fluency and transfer occurring between mathematical situations and contexts.

‘Big content ideas’ of Mathematics

There is not necessarily any one particular way in which content ideas can be linked around ‘big ideas’ or even what those ideas might be as links may be conceptualized in different ways by different people. Hence, it may be problematic as to how curriculum writers might incorporate and represent ‘big idea thinking’. Some suggestions about how this might be done are discussed later. Notwithstanding, it is knowing about the links that is important. For example one key underpinning idea is the notion of pattern, which pervades mathematical knowledge. Pattern is evident in all content areas of the primary/elementary curriculum and can be a powerful idea if teachers can make it explicit to their students. It is clearly a ‘big idea’ and some examples of it are offered here:

- Knowing about patterns provides us with predictability and enables us to generalize.
- The same pattern structure can be found in many different mathematical forms and real-life contexts.
- A pattern can be repeating, growing and concentric and is governed by a rule.
- There is a regular pattern in the way in which counting numbers are said and this is repeated.
- There is a repeated cyclical 100-10-1 pattern in reading and writing numbers beyond the hundreds.
- We can skip count forwards and backwards by any number and see a pattern generated.
- Number lines are ways of representing the patterns in counting numbers.
- Patterns exist in basic number facts, and associated factors and multiples.
- Some shapes can be arranged in tessellating patterns when others cannot.
- There is a pattern in the way we use language prefixes to name measurement units.

Teachers can use skilful and purposeful questioning of their students to make explicit the connections with patterns. For instance, in teaching about number facts, students could be asked ‘What patterns can you see in the 3, 4, 5 and 6 times tables? How are they the same and different?’ When engaging in solving problems with multiple solutions, they could be asked ‘Can you see a pattern in the solutions you’ve found so far? Do you think there might be a pattern?’ Once children realize the significance of looking for and using patterns, the transfer of learning is increased, the cognitive load is lessened, and the curriculum looks less crowded.

Using Ma’s (1999) ‘knowledge package’ and ‘concept knot’ as models for organizing content knowledge, Figures 4 and 5 are offered as examples of how a number of ideas can be connected around a central idea. Figure 4 shows how the use of the multiplicative array as a representation can provide the link between important mathematical ideas and Figure 5 depicts one way of connecting mathematical ideas along the lines of Ma’s notion of concept knots and knowledge packages. Ma noted that the teachers involved in her research had developed their own particular ways of connecting the same mathematical ideas and that each individual teacher’s way of thinking about the links and connections was as valid as the next. This is underlined by the earlier comment made by Clarke, Clarke & Sullivan that the value of ‘big idea thinking’ is in the way that it stimulates each teacher to deconstruct and reconstruct his/her own conceptual structures to clarify meaning, links and connections.

Neither Figure 4 or Figure 5 is intended to be an exhaustive set of ideas but rather to show that a number of mathematical ideas can be linked together in order to develop a conceptual understanding of related ideas. Single-directional arrows are used in Figure 4 to indicate that the multiplicative array (in this case, a 5 X 3 array) is a powerful representation that teachers can use to make the various links and connections explicit to their children. Multi-directional arrows are used in Figure 5 to indicate that the connections between the ideas can emanate or be realized in more than one way.

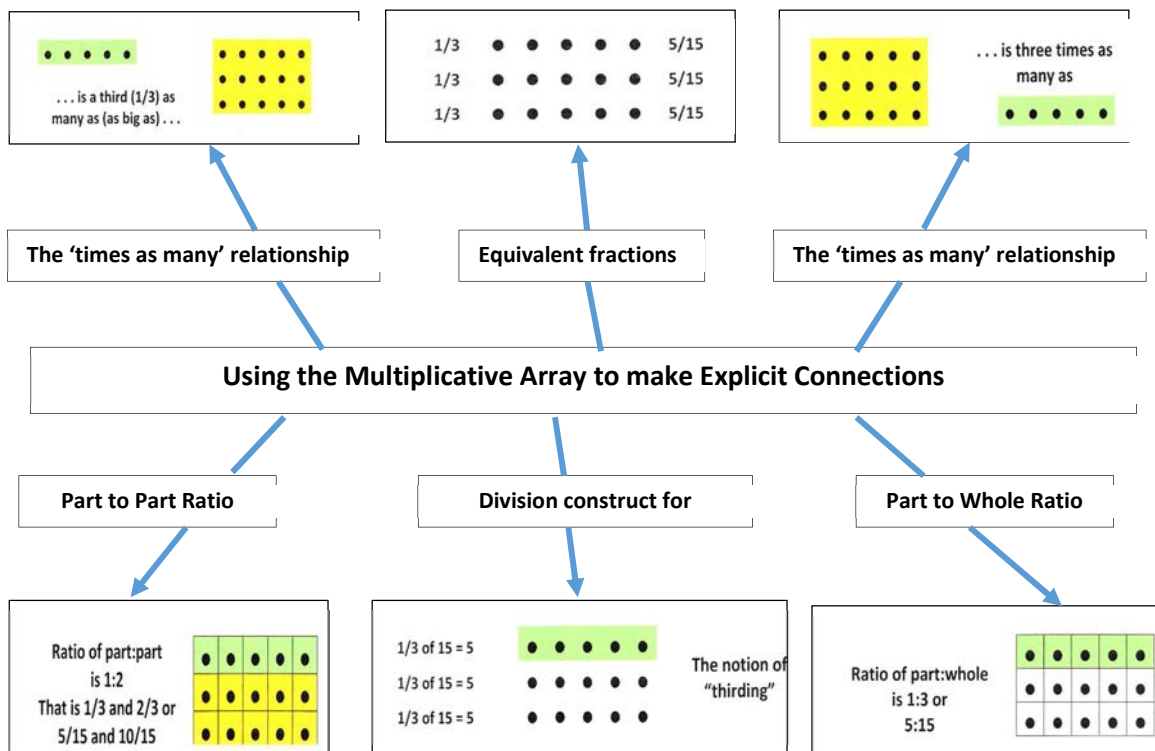


Figure 4: Connected ideas based on the multiplicative array representation

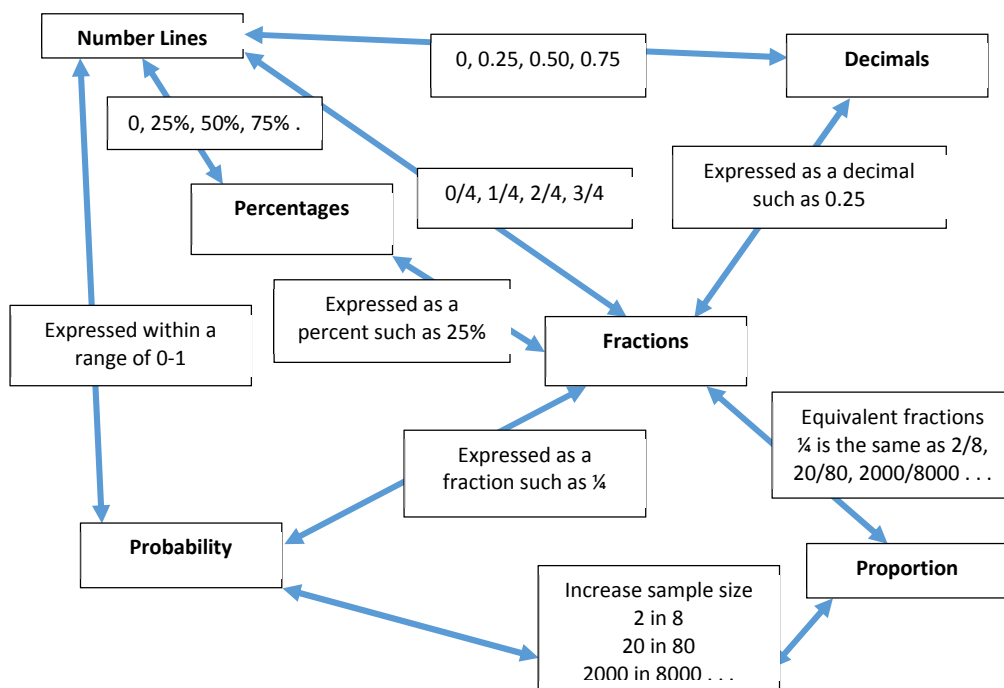


Figure 5: Example of connection of ideas after Ma's 'knowledge package'

And so to return to the problem of how to represent ‘big ideas’. Perhaps curriculum documents such as the revised *Australian Curriculum: Mathematics* could contain a series of examples of concept knots and knowledge packages such as those shown in Figures 4 and 5. They might also include a graphic similar to that shown in Figure 3 depicting the underpinning relationships that exist between the ‘big ideas’. Rather than continue to break up the content as in the current layout, it could be shown as belonging to ‘ideas’ rather than solely being linked to year levels. The development of specific aspects of such ideas or ‘micro content’, the links between them, and the way in which some ideas underpin others needs to be emphasized. Even if the splitting of content by year levels must remain, at least it could be supplemented by the inclusion of a conceptual approach as described here. The following example illustrates how this might be incorporated into curriculum documents such as the *Australian Curriculum: Mathematics*.

A key understanding about the base ten number system is that *There is a pattern that helps us read, write and say numbers into and beyond the thousands and it is cyclical in nature* (Department of Education of Western Australia, 2013). For example, a child can read 307 090 as ‘three hundred and seven thousand and ninety’ and can write ‘four hundred and sixty thousand, five hundred and twenty three’ as 460 523. Being able to understand that and do it correctly would depend upon a range of other understandings and experiences such as those in Figure 6. A teacher needs to understand the connections between the new knowledge and the underpinning knowledge. To facilitate teachers in broadening their own appreciation of the connections and links within and between ‘big ideas’ of mathematics, curriculum developers could include some explicit examples of what such thinking might look like in practice. Figure 6 represents an attempt to do that. The underpinning knowledge and understandings develop in the early primary/elementary years and to an extent, are hierarchical as shown by the arrow. If children lack an understanding of key content in early levels of Figure 6, they are likely to experience difficulty later on.

Read, write, and say numbers into and beyond the thousands.	
<i>Underpinning knowledge in terms of ‘micro content’</i>	<i>Typical experiences to develop the underpinning knowledge</i>
The additive property – the quantity represented by a whole numeral is the sum of the values represented by its individual digits.	Use of ‘Arrow Cards’ and ‘Number Expanders’ to illustrate the partitioning and combining of number parts according to expanded values.
The positional property – the quantities represented by individual digits are determined by the places they hold within the numeral.	Partitioning numbers according to the standard place value partition and doing so in association with MABs, and bundling sticks.
The 1-9 sequence continues after ten, in the decade sequence, & within each decade. The decades & 1-9 sequences continue through & within each hundred.	Patterns in number charts such as the 1-100 chart, 101-200 chart, 1-200 chart etc. Skip counting by 10, 100 etc.
Zero can hold a place – it signifies that there are none of a particular value in a place or column. It is first encountered when we write a number bigger than nine.	Building numbers with ten frames, bundling sticks & MABs and writing them on place value charts
The ten group is a special entity and we can consider a ‘ten group’ as ‘ten ones’, ‘ten tens’, ‘ten hundreds’, and so on.	Bundling sticks, MABs – building numbers from 10 to 100 and beyond. Trading up and down. Writing the numbers on place value charts.
There can be no more than nine in any place or column in the number system.	Ten frames – building from single digit to two digit numbers & writing numbers on two column chart.
Part-part-whole relationship	Use connecting cubes such as Unifix to represent single digit numbers as combinations of smaller numbers.
Counting principles Subitizing – recognizing numbers as flexible combinations of different groups & smaller numbers.	Use dot cards, counters to see smaller groups within a given number.

Figure 6: *Linking and connecting existing knowledge to new understandings through ‘big ideas’*

In essence, this is an example of what is termed ‘horizon content knowledge’ (Ball, Thames & Phelps, 2008). Teachers with well-developed ‘horizon content knowledge’ (HCK) are able to look both forwards and backwards from a particular point of mathematical understanding and consider how to help a child to develop new knowledge or to see what understanding might be lacking in order to correct a misconception. If curriculum writers could focus on connections and links within and between ‘big ideas’ of mathematics, perhaps teachers would be better enabled to develop their HCK and be better able to identify specific mathematics needed to help children in particular situations.

The *Common Core State Standards* document (NGA Center, 2010) provides one good example of how teachers can assist their students to see connections between ideas. The following statement about Grade Three content is linked to the multiplicative array used in Figure 4 in that it shows how other ideas can be specifically linked to one other through the use of the array. As such, it is indicative of what an emphasis on ‘big ideas’ could look like in practice. It is hoped that the reviewed *Australian Curriculum: Mathematics* might include many such statements based on the connections within the ‘big ideas’ of mathematics.

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle. (NGA Centre, 2010, p. 21)

This is encouraging as it also makes links to principles of measurement which in itself is another ‘big idea’. In this case, ‘area’ is seen as the conduit for linking to other ideas in keeping with the point that different people view the connections in different ways. However, this content statement, whilst encouraging, is the exception rather than the rule in both the Australian and United States curriculum documents. Too often there are statements (probably well intentioned) such as “recognizing the connection between the order of unit fractions and their denominators” (ACARA, 2012a, p. 37), and “connect fractions, decimals and percentages and carry out simple conversions” (ACARA, 2012a, p. 47). This paper is not disputing that those ideas and connections are important but rather that the way in which they are presented in the curriculum generally doesn’t go far enough in expressing HOW the connections are to be developed. Consequently there is a danger that the traditional procedural view of teaching mathematics will continue to prevail. It is in the ‘big ideas’ way of looking at connections that a richer conceptual view of mathematics can be developed.

Another view of curriculum content and organization was presented by Schmidt, Houang & Cogan (2002, p. 9) who defined a ‘coherent curriculum’ as being “articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives”. They also noted that teaching should reflect the “key ideas that determine how knowledge is organized and generated” and that content “must evolve from particulars to deeper structures inherent in the discipline. This deeper structure serves as a means for connecting the particulars” (Schmidt, Houang & Cogan, 2002, p. 9). This ‘deeper structure’ is very much akin to what Charles and Siemon et al. described as ‘big ideas’.

‘Big process ideas’ of Mathematics

It seems to follow that an emphasis on seeing and understanding the links and connections within and between mathematical ideas should be achieved through appropriate vehicles and conduits. Such vehicles are already in place in both the *Australian Curriculum: Mathematics* (ACARA) and the *Common Core State Standards* (NGA Center). However, the four statements about the *Proficiencies* (ACARA) contain some very laudable ideas but they are simply not sufficiently integrated with the content that follows. It is almost as if they are an ‘add on’ rather than being the centre of practice which is what they should be in order for the ‘big ideas’ and their associated connections can be genuinely developed. Unfortunately the *Australian Curriculum: Mathematics* has not elaborated how that could occur.

In the *Australian Curriculum: Mathematics*, there are statements that supposedly link the *Proficiencies* to the content at each year level but the statements are quite vague and give no real guidance to teachers as to HOW to go about using the *Proficiencies*. For example at Year Three level, the statement about problem solving says “Problem Solving includes formulating and modelling authentic situations involving planning methods of data collection and representation, making models of three-dimensional objects and using number properties to continue number patterns” (ACARA, 2012a, p. 26). This statement is very similar to that for Year Four but somewhat more substantial than that for Year Five which says “Problem Solving includes formulating and solving authentic problems using whole numbers and measurements and creating financial plans” (ACARA, 2012a, p. 35). The *Australian Curriculum* does contain a very important section about *General Capabilities* that contains, among other things a section about critical and creative thinking that highlights four ideas – Inquiring (identifying, exploring and clarifying information); Generating innovative ideas and possibilities, Reflecting on thinking, actions and processes, and Analyzing, synthesizing and evaluating information (ACARA, 2012a, p. 155). Again, these are very important ‘big process ideas’ but they are contained in a section of the curriculum that is separate from the content.

In the UK, a similar situation existed. Jones (2003) had previously noted the ‘ambivalence’ of the *National Numeracy Strategy* (UK) by the placement of problem solving as the third of five strands in *the Framework for Teaching Mathematics from Reception to Year 6* (DfEE, 1999). The associated examples provided in the curriculum document suggested that the only decision making required of children was about which operation and calculation method to use, thus reducing problem solving to little more than word problems. Jones suggested that this was a far cry from ‘real problem solving’ where children should “decide how to tackle the problem, how to gather and organize the data, and how to represent and communicate their findings” (Jones, 2003, p. 87). The current curriculum document under scrutiny here (*Australian Curriculum: Mathematics*) seems to have repeated the same mistake over a decade later. Problem solving skills and other ‘big process ideas’ need to be at the heart of teaching and learning and therefore must be embedded as central elements of the curriculum, not as ‘add-ons’.

The depth to which the *Australian Curriculum: Mathematics* describes the ‘big process ideas’ is generally reflected in their coverage of context/s. This needs to be reviewed given the importance of ‘contextual knowledge’ as one of the three elements of the numeracy model (Kemp & Hogan, 2000; Willis, 1998) even though some reasonable statements are made. The joint abilities to ‘decontextualize’ and ‘contextualize’ (NGA Center, 2010) are clearly vital elements of numeracy yet they are not discussed in the *Australian Curriculum: Mathematics* and are only given brief recognition in the *Common Core State Standards*. Similarly, the AC: M gives fleeting recognition to the notion of context and again, reference to context/s in the content strands is very general and is found in statements like ‘in everyday contexts’, ‘meaningful contexts’, and ‘in familiar contexts’. Other points contained in the ‘General Capabilities’ section of the curriculum (which is placed away from the content strands) state things like “Students have opportunities to transfer their mathematical knowledge and skills to contexts outside the mathematics classroom” (ACARA, 2012a, p. 24) and “Apply knowledge gained from one context to another unrelated context” (ACARA, 2012a, p. 58). These are important ideas but again, no indication is provided of the actual skills needed by children to attain such outcomes.

Contextual knowledge is clearly important as a component of numeracy and various researchers and mathematics educators have commented about it. Van den Heuvel-Panhuizen (2001) suggested that ideas are best learned in rich and realistic contexts while Morgan (2000) warned of the dangers of using predominantly decontextualized examples to teach content. In discussing quantitative literacy (which can be considered equivalent to numeracy) Steen noted that it was “anchored in the messy contexts of life” (2001, p. 1) while Trafton (1999) wrote of the virtues of ‘making mathematics messy’ by using real examples embedded in real contexts where numbers don’t always divide evenly or easily. Peter-Koop described some of issues experienced by children when working in contexts noting that “Many children have difficulty with real world problems frequently engaging in a rather arbitrary and random operational combination of numbers given in the text” (2004, p. 454). Perhaps the answer lies in the notion that “Contextual knowledge involves and ability to recognise the mathematics that is needed to make sense of a situation” (Hurst, 2007,

p. 25). One's capacity to make sense of a mathematical situation would surely be enhanced by one's ability to see connections and make links between ideas.

In reflecting on the *Common Core* statement about decontextualizing and contextualizing, we need to consider the work of Parkin and Hayes (2006) who described how children can be encouraged to deconstruct context based problems and then reconstruct them to better understand what the context is telling them about the mathematics but also what the mathematics might be telling them about the context. The importance of the 'deconstruct-reconstruct' nexus has been noted by Clarke, Clarke, & Sullivan (2012) and already mentioned earlier in this paper. Part of the process involves teachers and students negotiating to identify possible contexts where problems (based on a concept like volume) would be relevant. The key contextual skill that needs to be clarified in curriculum documents is for children to ask questions such as 'What mathematics is used here?' 'How is it used?' 'What does it tell me about the context?' 'What mathematics do I know that I can use to help me understand this better?' This sort of thinking links nicely to the first of Polya's stages of problem solving, namely, 'understand the problem' (Mutter, n.d.). Again, the capacity to understand must surely be enhanced by an ability to see connections and links within and between mathematical ideas.

'Big process ideas' may be reasonably well documented, or at least identified, there is insufficient detail about specific skills needed to use them in context. In attempting to compile a set of identifiable criteria, one would do well to begin with the *Common Core* statement about 'decontextualizing' and 'contextualizing'. This could be followed by points such as the following:

- Interprets specific data contained in the context and/or poses questions that require the interpretation of specific mathematical data.
- Describes, in one's own words, mathematically related ideas contained in the contextual information.
- Poses questions that suggest connections between different aspects of the data and between them and their causative factors.
- Relates, uses, or applies aspects of the data, or poses questions, to compare ideas in the context to other situations (Hurst, 2007).

Conclusion

There is plenty of scope for curriculum developers to assist teachers as they seek to deal with the commonly held perception of a 'crowded curriculum'. The current version of the *Australian Curriculum: Mathematics* (ACARA, 2012a) has departed very little from earlier curriculum documents in that it continues to present content on the basis of year levels. It contains reasonably clear statements of content, but fails to make explicit the many links and connections within and between concepts. It seems to 'make the right noises' about higher levels of thinking and using proficiencies to develop conceptual understanding of content. However it has not provided teachers with the means to do this. As such, it represents an opportunity lost with the great danger being that it is 'just another curriculum' and that it is 'business as usual' characterized by a continuation of procedural teaching.

The main message in this position paper should be clear – it is about changing the way in which teachers view content knowledge and pedagogies for delivering it. It is no longer adequate to compartmentalize knowledge in the manner of traditional curricula and teachers need to be encouraged to actively seek links and connections within and between concepts and bodies of knowledge and explicitly show children how those links exist and can be used. Children are likely to become more numerate and apply their mathematical knowledge if that knowledge is rich in connections. Teachers can make this task easier by recognizing and making explicit the links between the 'big content ideas' of mathematics and by modeling the 'big process ideas' to enable their students to become contextual and strategic thinkers. However, teachers need support in the form of a better organized curriculum document that emphasizes the myriad connections that exist rather than being focused on the same yearly breakdown of content. The review of the *Australian Curriculum: Mathematics* has provided an opportunity to do this – let us hope that the opportunity is not lost this time.

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