WASM: Minerals, Energy and Chemical Engineering

Measuring the Economic Impact of the Minerals Sector: A Regional Western Australian Perspective

Paulina Alejandra Sepulveda Bravo

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DECLARATION

To the best of my knowledge and belief this thesis contains no material previously publish by any other person except where due acknowledgement has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Paulina Alejandra Sepulveda Bravo

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This is the end of a long and complicated road. During these many years I have had to battle with numerous health problems, including a serious accident where I broke my kneecap, surgery to fuse my neck, diagnosis of fibromyalgia, my eternal problem with depression, among others. In addition, we have had to live in these times of pandemic, and in my case it has kept me away, for almost 3 years, from my loved ones in Chile and other parts of the world. But everything comes to an end ...

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Abstract

In Economics the use of Input-Output analysis, General Computable Equilibrium Models and Econometric Models are "traditional" tools used to measure impact. Those techniques have proved to be very helpful in the past to the point that they have become the de-facto standard for such analyses. Underlying those techniques, there are common assumptions regarding the conditions under which the economic agents interact, namely: costless symmetric and perfect information; additionally economic agents are assumed to be rational decision makers, and in the traditional modelling the use of a representative agent is common. As labour force is changing on its composition and dynamics in the Australian resources industries, new mobility patterns appear, new working arrangements are in place, and these novel elements induce a different dynamical behaviour on the economic agents that are not easy - if not impossible - to model using traditional techniques which are of a more static nature. In particular, the richness and diversity of the multiple ongoing processes which characterise human interactions have little or no chance of being properly modelled using those traditional tools.

Classical tools being based on strong assumptions of the agents' behaviour naturally induce thinking that the outcomes might not be realistic. In this thesis, an agent based model is proposed as an alternative to the classical modelling tools with the objective of using these novel models motivated by the need to characterise the contribution of mining activities in a regional context.

The model has been implemented and after testing and validating the implementation a case study was undertaken. The case study compared two possible strategies for distributing an ad valorem tax such as “Royalties for Regions” in order to see if they were equivalent or not with the aid of the computational tool. The results obtained indicate that there is a significant difference in impact, as evidenced by the evolution of local multipliers for the different distribution mechanism used. The sectorial mining multiplier remains very stable for the case study considered and this could suggests that eventually the royalty mechanism may not be appropriate as it undermines the multiplier effect of mining without being compensated by any of the distribution mechanisms considered. It is concluded that these results are encouraging as they open up discussion avenues and future explorations in the area, eventually with better equipped computational models, that could inform policy development on one side, and on the other provide the opportunity to put the focus on a different way of measuring and assessing impact.

The miracle of existing, the instinct to search, the fortune to find, the pleasure of knowing (Joan Manuel Serrat)

Contents

List of Figures

List of Tables

List of Algorithms

List of Listings

Chapter 1

Introduction

The introductory chapter outlines the problem, the rationale for attempting to solve it, a high-level description of the chosen approach to solve it and a summary of the main results contained in the thesis. Additionally, it contains a brief description of the work contained in the remaining chapters.

1.1 Preliminaries

Simplistically, an economic impact can be defined as the effect experienced by the level of economic activity produced by changes in the current state of things. It has to be noted that impact is not only economic in nature and other forms of impact exist, including environmental, social and governance. What differentiates an economic impact from other types of impacts is its focus on economic activity levels, including business output, wealth, jobs, value added, personal income or others. An impact should not be considered economic unless it affects economic activity. This implies that even in the case of impacts that can be assigned a monetary value, they are not necessarily considered to be economic in nature unless they affect the levels of economic activity.

Since there is no generalised method used to measure impact (Bessette, 2003), ad-hoc measurements are usually employed for different areas of interest, thus generating multiple possible ways to asses that impact. For example, the use of contingent valuation techniques is widely used to measure the economic impact of the art sector (Pung et al., 2004; Plaza, 2006; Bosco et al., 2019). We can also observe that the Return on Investment (ROI) formula is frequently used to report the economic impact of libraries by measuring the value their services provide to the community and the impact of their resource availability on it (Kaufman, 2008; Tenopir, 2010; Kelly et al., 2012). In addition, the socio-economic impacts of mines on small communities or towns are often measured using surveys that include employment rates,training opportunities, health status and housing (Tonts et al., 2012; Rodon and Lévesque, 2015).

Before starting to produce numerical results, it is necessary to note some basic considerations that impose constraints on the models and methodologies used to produce those measurements:

- As mining is not an isolated activity (Aroca, 2001; Jenkins, 2004), decisions taken by mining companies have an implicit and/or explicit impact on other goods and services due to complementarity or substitution, and the tools to be used to measure this impact need to relate mining activities to other productive sectors.

- The tools/models to be used need to incorporate an understanding of the effects that mining activities generate on the rest of the economy through monetary transfers between different economic agents, such as homes, government departments, exports and imports.

- Due to the social impacts of mining (Rolfe et al., 2007; Lockie et al., 2008; Solomon et al., 2008; Petkova et al., 2009; Sharma, 2010), it is important to observe and quantify the effects of the mining activities on the social fabric and evolution of society in general. For example, the use of a "fly-in fly-out" work modality by some companies, whilst being studied in terms of regional impact (Johnson, 2009), seems to be a phenomenon not yet properly characterised at the human scale, particularly in terms of dynamics and mobility patterns considering associated economic flows (Haslam McKenzie, 2010).

1.2 Basic Background

According to the Cambridge Dictionary for English Language, **impact** is defined as *the strong effect or influence that something has on a situation or a person* (Walter, 2004). The key words of the definition are *strong effect* and *influence*. Certainly, in everyday use we can observe some variability in the common accepted form of their meaning, which points to potential methodological problems. In this thesis, the first question motivating the work relates to the existence of mechanisms to measure the economic impact of mining and the potential enduring benefits that communities can obtain from this activity. Is such measurement available? What is required to implement it? As far as known, there is no real clarity as to what constitutes impact and that seems to be the starting point to attempt an answer to this question. Consequently, aspects of the present work are devoted to reviewing some of the existing definitions and to take a point of view towards a practical implementation of the same.

In the context of limited economic, human and natural resources, it is logical to consider those projects or activities that generate greater benefits, be that for society in general or for a sector of society, having potential for improvement. The betterment of humanity is the basic tenet of Economic Science, however, the problems of allocating scarce resources are plagued with considerations of a moral and practical nature. All these considerations could be thought as linking back to a definition of impact and how to measure it. However, from a practical standpoint, it is only possible to incorporate some of these considerations if they can be measured or quantified, which is usually difficult if not impossible to achieve, for those with a strong moral component. As a consequence and in the context of the present work this avenue will not be pursued. This is why governments, public institutions, companies and other entities typically conduct studies that assist in deciding which projects, policies, laws or other initiatives will produce the best outcome for themselves, for a particular sector or society in general (Crompton, 2006).

It is important to note that economic impact, even if attempted to be measured on a given area of knowledge/study, can end up being measured in different ways, as the emphasis of the researcher will be guided by the research questions he/she may have. For example, one researcher could be interested in social impact, another one in economic impact, both of them working on a different angle of impact measurement of mining activities. To be more precise, the type of study often depends on:

- The question that one tries to answer, where the question is typically established before the study starts.
- The sector that could receive those benefits.
- The characteristics of the country or region where the potential initiative is going to be put in place.
- The person in charge of the study.

A plethora of possible combinations of these and other factors lead to an endless list of possible methods which are difficult to classify in a taxonomic way.

Traditionally, the mining sector has concentrated its efforts to measure socio-economic impacts in three dimensions: economic, social and environmental (Kotey and Rolfe, 2014; Gueye et al., 2021). From an economic point of view, one of the main problems is to decide how and in what capacity natural resources are used, the costs of regeneration in cases where they are renewable, or the loss of wealth due to depletion. The social aspect focuses on how the population will benefit, if it does, and what does the project contribute to the region. Finally, the environmental aspect concentrates its endeavours into how ecosystems and the surrounding populations, including impacts on flora and fauna, will be affected by mining operations.

As mentioned before, there are a multitude of techniques and/or metrics to measure impact (West, 1995; Pung et al., 2004; Mancini and Sala, 2018). These include:

- Simple/Aggregated Approaches: National Accounts, Cost/Benefit Analysis, Employment, Taxes.
- Tangibles v/s Intangibles Considerations: Infrastructure, Health, Education.

This is to illustrate the nature of the methodological problem relating to choosing an approach to measure economic impact.

1.2.1 The Problem

The research in this thesis is investigating if the impact of mining related activities in a regional location can be approximated by focusing on the economic agents, operating in the region, by means of using heterogeneous descriptions. A second research question that this work could be putting its attention to can be stated as to whether it is possible to achieve enduring community value from mining activities or not. Obviously, a research question formulated in this way is a very broad one, and answering it depends on several factors. For example:

- Is community value measurable?
- How could community value be measured in practical terms?
- What is the appropriate definition of community value?
- Which characteristics of a particular community make it prone to be affected by mining?

- What are the limits of the mining activity?
- Is it possible to predict the level of enduring benefits from mining if any is going to be obtained?
- What is the precise meaning of enduring?
- What is the temporal extent of these concepts?

There are no clear definitions, policies or strategies when it comes to communities, as each community is unique from a historical and cultural point of view.

Answering the previous questions would require a deep knowledge of how a community has been formed, its history, traditions, culture, values, and other relevant characteristics, a non trivial task as it involves people, each with a different need, motivation and outlook on life. However, this understanding is elusive with the current instruments used to measure characteristics of the population. Furthermore, before looking at the enduring community value coming from mining, it is first necessary to be able to measure the impact of the activity as without it there is little chance to see if anything endures.

The thesis of this research work is, that it is possible to approximate the impact of mining related activities in a regional location, by focusing on the economic agents operating in that region, by means of using a heterogeneous description for each one of them. This line of research is motivated by the identified shortcomings of the classical analytical methods, which reduce all the analyses to aggregated facts without considering the individuality and micro-motives of the humans behind the economic agents. It is believed that the foundations laid by the proposed research avenue could pave the road for other forms of enquiry as to what constitutes enduring benefit, and in particular to start to understand that the pursuit of happiness and/or satisfaction has a human side usually ignored by crude data collection exercises.

To achieve the objective of proving/disproving the thesis statement[1], a framework, providing room for flexibility, was developed to perform numerical experimentation. The approach of the thesis, from a methodological point of view, is to implement a computational model, validate it and use it in a case study inspired by reality. It must be noted that the implementation focused on expressing the research ideas into software. However, it is acknowledged that this is one representation of the research ideas, and there is room for improvement both in terms of coding style and tooling to achieve the stated objective.

1.2.2 High-Level Overview of Solution

One possible approach to answering the research question of the thesis is to try to approximate the impact of mining activities from an economic point of view using existing models and approaches. However, the existing models are usually based on assumptions of homogeneity and the pervasive use of representative economic agents. There is a classic theory elaborated by Vasily Leontief (Leontief, 1955, 1986), who by means of matrices containing information of national accounts derived the concept of a multiplier, which is a

[1] "That it is possible to approximate the impact of mining related activities in a regional location, by focusing on the economic agents operating in that region, by means of using an heterogeneous description for each one of them".

measure of how many times money circulates in the economy and reflects the interdependence of the different economic sectors. These models have been further developed to consider structural models of the evolution of the system, but still retaining the classical underlying assumptions of homogeneity of agents.

A novel alternative to the previous characteristic of classical economic models is to model and use heterogeneous economic agents, such as Agent Based Models (ABM) (Tesfatsion, 2002; Heckbert et al., 2010; Hokamp and Pickhardt, 2010; Stiglitz and Gallegati, 2011; Huang et al., 2014; Cogliano and Jiang, 2016; Kremmydas et al., 2018; Stiglitz, 2018; Karimi and Mohammadi, 2019). The ABM technique essentially tries to perform a modelling from bottom up, i.e., modelling economic systems from constitutive elements in all their heterogeneous behaviour, and then observe the outcome of letting the computational implementation evolve to contemplate the emergent behaviour associated with the interaction of these agents. The following works utilising ABM provide insight into the general characteristics of the modelling technique. An example originating in *ecological economics* (Heckbert et al., 2010) mentions that "the ability of ABM to explicitly represent adaptive decision making and interactions provides an opportunity to explore issues in ecological economics which are defined by heterogeneity, feedbacks through interactions, and adaptation". Authors such as Zheng et al. (2013) argued that "The agent based model facilitates an improved understanding of how farmer behaviour and associated environmental consequences change according to the heterogeneity of and interactions among farmers". Finally, Scott et al. (2018) consider two ABM models for the spread of hepatitis consisting of "an agent-based model (ABM) parametrized to local surveillance and behavioural data ("ABM1"); and an ABM with a more heterogeneous population ("ABM2") to determine the influence of extreme variations in sexual risk behaviour", clearly in an attempt to produce a comparison between stylised facts and the observed "reality" due to heterogeneous interacting populations.

One aspect that is not considered within the scope of the thesis, but deemed important in the dynamic evolution of the economic system, relates to the behaviour of human agents. There are several levels of interaction when human agents are added to the model, for example:

- Interaction between human agents.
- Interaction of human agents with economic agents (by means of work and consumption).
- Modelling agents' mobility (particularly important for FIFO operations).

It is believed that the proper modelling of the human aspects is what could give these models a power that the classical tools do not possess: Agent-based models allow the representation of individual preferences and move away from the classical "representative" economic agent, making them potentially more useful to represent reality. Future work should focus on improving the modelling of this heterogeneity, from the initial seed planted in this work, to observe the emerging behaviour of the modelled economic system. It is important to note that this modelling option requires appropriate demographic and geo-located information which has not been incorporated in this work[2].

In general, economic analysis has focused on the use of quantitative information. This bias is due to the fact that classical economic models are of an analytical/numerical nature. ABM, due to its flexibility, could potentially enable the incorporation of qualitative factors in the modelling. However, incorporating the "feelings" of the "in-silico" human beings involved in the simulation model seems unrealistic at present, but it is believed that it could become a very interesting future approach to the problem. On the other hand, the inherent heterogeneity of ABMs makes them attractive as they support modelling agents with bounded rationality and with variations in attributes between them.

1.2.3 Scope of Work

Recall from subsection 1.2.1 that a possible overarching research question for this work could be the possibility (or not) of achieving enduring community value. However, as it was illustrated, there are a number of definitions required to have a more precisely defined research question. Due to this multitude of possibilities, it was decided that this thesis will focus on approximating the economic impact of mining related activities in a regional location. Additionally, it adopts the methodology of using agent based models to provide

[2]The information that can be obtained from the Australian Bureau of Statistics (ABS) is census information, presented at an aggregated level on the ABS website which is not amenable for the level of detail desired. If there are other sources, they are posterior to the time when the code was implemented and currently unknown to the author. When consulted about the same, the Goldfields-Esperance Development Comission (GEDC) just provided a form of annual report with aggregated statistics but with none of the detailed information that could have better informed this aspect.

representation of heterogeneity for the economic agents involved.

The focus of study in this thesis lends itself to a multitude of methodological possibilities, each one with its pros and cons. For this reason, the series of decisions that delineate the scope of the work are listed here.

It is believed that having a clear view of the methodological approach is important when answering the question posed, as this illustrates the complexity of the situation being addressed, and questions faced when proposing and implementing the model. For this reason, it has been decided that the computational implementation be put in appendices and additional complementary information considered/contributing to the development of the model being located in the same place.

This thesis is attempting a methodological approach to the measurement of local economic benefits derived from mining. In this sense, the work, by means of a proof of concept implementation, shows that the chosen methodology can be used to perform comparisons between alternatives, in a simplified case study, that informs conclusions and provides a potential explanation for the phenomenon.

1.2.3.1 Use of Programming as a Tool

The adoption of the agent-based modelling approach requires a computational implementation in a programming language, with a view to enable the customisation that the proposed modelling approach demands. `Python`, the programming language used in this thesis, was selected for reasons of simplicity.

`Python` speeds up the implementation process of software components, but it is important to understand that the proposed implementation is just a tool, and by no means a key milestone in the work. In other words, the implementation is necessary in attempting to answer the research question.

Hence, performing any sensitivity analyses with respect to some of the parameters used, or the determination of estimates of computational complexity, whilst appealing in the context of the proposed work, is considered to be out of scope in this thesis, which leans more towards the social sciences.

1.2.3.2 Modelling Decisions

There are certain modelling decisions adopted in this work which could be sensitised, but whose sensitivity is not deemed central to the main argumentative line of the work developed in the rest of the thesis. For example, the type of production function used as the rational mechanism (heterogeneous) given to each `firm` agent admits several possible options, including:

- Linear Homogeneous
- Cobb-Douglas
- Constant Elasticity of Substitution
- Variable Elasticity of Substitution

In order to conform to usual practices in economic modelling, it has been decided that Cobb-Douglas functions will be applied. Their pervasiveness in the economic/econometric literature is of note.

1.2.3.3 Local Benefits

The thesis starts by considering the classical way of measuring impact of mining activities in an economy: the input-output multipliers. It then discusses an example that shows that very "trivial" agents can replicate those multipliers, thus paving the way for more complex use of the same. Within this context, it becomes a methodological necessity to be able to measure just the local impact, in the understanding that local is limited to a particular location. Here the adoption of a particular key performance indicator (KPI)[3] is chosen as the appropriate metric. Additionally, the definition of this metric is modified to be able to measure the local impact of a particular economic sector. It is believed that this simplification is the starting point for future developments: from local, e.g. municipal, the concepts can be aggregated to a larger level, e.g., county, and even further aggregation to larger levels, e.g., regional, state, nationwide, continental or global.

The ability of measuring local impacts derived from mining activities, the focus of this work, is a keystone that supports the structure of future developments in the area. However, the decision taken of adopting the KPI to measure local benefits requires data that is not readily available, hence, data

[3]The Local Multiplier 3 (LM3), inspired by a previous work applied in a different context (Sacks, 2002) to measure the local impact, which will be explained in Chapter 5.

have been assumed for illustrative purposes.

1.2.3.4 Public Information Use

In this thesis only data available from public sources such as the Australian Bureau of Statistics (ABS) have been used.

1.2.3.5 Reality Inspired Case Study

The main case study considered in this thesis requires the creation of the city of *Fakegoorlie*, an imaginary location that is inspired by the city of *Kalgoorlie*, but is not exactly equal to it. In particular, this decision influences the treatment of the publicly available census data used for the construction of the case studies, and when information was not available via public channels, assumptions were made as noted in the text.

1.2.3.6 Final Comments

It can be observed that the problem studied in this thesis, i.e., to measure local impact derived from mining activities, is not new and there have been previous attempts to solve it. However, most of the existing work is of a qualitative nature, as the enduring impact of mining operations usually have a big social component, and when quantitative techniques are used, the point of view adopted in those studies is predominantly top-down, i.e., the local behaviour is explained from global assumptions. The technique chosen to solve the problem here, being bottom-up, or as some authors identify as "from the ground-up" does not reach an equilibrium. Hence, comparisons with more classical options that typically converge to a "solution" are incomparable with an out of equilibrium technique that is producing simulations. This does not detract from the contribution that can be obtained from having a highly customisable model that can be used to numerically test different scenarios, thus enabling comparisons that otherwise could not be possible.

This thesis does not pretend to increase the knowledge in data-science or software engineering, as these are merely the tools used to answer the research question, however, these tools are needed to implement the propose model as there are no standard tools for the task. Furthermore, despite the potential opportunity of obtaining the most efficient implementation, or case studies using the biggest and most complex datasets available, this thesis first and foremost attempts to illustrate the methodology used to answer the research question, using the tool in simple – based on reality – case studies, which show conclusions that make sense, thus providing a step forward in the analytical treatment of mining impacts in the context of a regional location.

1.2.4 Thesis Main Contributions

This thesis presents a methodological approach to approximate the impact of mining related activities in a regional location. It does so by focusing on the economic agents operating in that region using a heterogeneous description for each one of them. The proposed solution is of a theoretical nature, however, it has been applied to case studies inspired by real data and statistics to make a step forward in economic impact modeling through the use of individuals to explain local behaviour.

The model has proved to being operational and allows for the calculation of a local multiplier for both the region and specific industries within the region. It does so by adopting a specific metric LM3, which is inspired in a previous work applied in a different context (Sacks, 2002).

The results show that a scaled-down version of the case study is able to demonstrate emergent behaviour for heterogeneous agents interacting with each other. It was also observed that the information created using this computational tool can be used to measure local multipliers that are compatible with those observed in reality. It should be noted that some computational limitations due to the complexity of the main simulation algorithm for the agent based system were experienced, leading to the development of the scaled-down version of the case of study.

Finally, the specific case study takes a neutral approach to decide a potential course of action for the use of "Royalties for Regions"[4] funds in two different ways, thus promoting a decision about the most beneficial way of using the funds with a view to increasing the local multiplier effect derived from mining activities as a proxy for enduring benefits for the region.

1.3 Description of Thesis Contents

Chapter 1 has provided a high-level overview behind the motivation for the problem being studied and highlights some of the difficulties and shortcom-

[4]**Royalties for Regions Act 2009**: Available on `http://www9.austlii.edu.au/cgi-bin/download.cgi/au/legis/wa/consol_act/rfra2009252`

ings that emerge when measuring the economic impact of the mining sector on local regions/communities. At the same time it provides an overview of the recommended solution approach and the contributions of the work to knowledge and understanding of the problem.

Chapter 2 provides a technical and historical description and comparison of classical tools (Input-Ouput Analysis, Computable General Equilibrium) used for modelling economic impact, as well as a brief introduction to Agent Based Models (ABM) that will be the starting point for developing the proposed computational model.

Chapter 3 starts by presenting an ABM that follows a simplistic rule. The chapter shows that running this ABM can closely replicate classic results. Subsequently, a more complicated model is presented incorporating some heterogeneity in its behaviour. Both models are then implemented using aggregated information from the Goldfields-Esperance region.

Chapter 4 presents the essential elements of the model that are used throughout this thesis, with a view to incorporating a more elaborate behavioural model for the different agents. A description and explanation of each particular agent and its characteristics is provided. The discussion includes a mix of mathematical and computational models that are necessary for undertaking the task at hand. Additionally, some discussion regarding the use of data coming from input-output tables is conducted, as well as a presentation of a proposal for fitting both utility and production functions together with some basic numerical experiments. The chapter also presents details pertaining to the numerical and logical validation of the proposed model. As the software piece required to implement the ideas discussed in this chapter has a number of "moving" parts, it is necessary to check individually that they work as intended (unit testing). Also, some crude and basic validation of the operation of the model in its entirety is provided. This chapter is presented in a computational nature to provide the rationale and some of the decisions taken when implementing the model.

Chapter 5 provides the implementation of the baseline case study inspired by Goldfields-Esperance data; some of it coming from the input-output matrix for the region, others from census data. Checks are performed to guarantee that the money flows are correctly implemented, followed by tests and scaled up runs. It is noted in this chapter that with minor modifications the ABM can be used in the rest of the thesis. One of the main contributions of this chapter is the adoption of a criteria for measuring impact, the Linear Multiplier 3 (LM3), found in the literature (Sacks, 2002), which allows practical implementation with minor modifications in the context of the model.

In Chapter 6 the main question of measuring local impacts derived from mining activities is studied. This is done by means of a case study which essentially consists of comparing two possible options for the distribution of a "Royalty for Regions" tax: subsidise companies or offer cash to people. The metric used for the comparison is tied to the LM3 multiplier defined and implemented in the previous chapter. After performing both simulation runs, a comparison is made between both case studies and the baseline case. Some interesting and provocative counterintuitive theory emanates from this comparison that is believed to not be obtainable using classical tools.

Finally, Chapter 7 presents the conclusions of the analysis. The original question is revisited in light of the results obtained throughout the thesis. Limitations of this work are identified in a section devoted to future refinement of the analysis using/editing the model, which have been documented at the time of writing.

1.4 Chapter Concluding Remarks

The purpose of this chapter is to motivate and introduce the research question being addressed in this thesis, and then provide an overview of the solution approach adopted. It does this in a concise manner, providing an outline of the work attempted. The chapter also summarises some of the main results obtained and provides an overview of the organisation of the rest of the document.

Chapter 2

Background to the Work

This chapter contains background information for both the problem and some of the things done by previous researchers. It is not the idea of this chapter to provide a full literature review as this task is only daunting due to the immense number of different approaches and application areas (as mentioned in the introduction). The interested reader is encouraged to deepen his/her knowledge if any of the works cited here pique his/her interest.

2.1 Introduction

Almost no economic activity is performed in isolation and the resources industry's activities are not an exception. It is not difficult to understand that when a mining project opens in a given area, some of the benefits derived from the productive activity will remain in that area; such benefits are derived from factors such as local employment which induces money flow into the local economy (through salaries), and indirect money flow due to consumption of some local services/goods (such as restaurants, hotel, transportation, and so on). Jenkins (2004) points out that some mining companies frame themselves as central components of the communities in which they operate, however, this thesis questions the validity of such a claim as it is not necessarily true that the people affected by the mining project really perceive the same.

Measuring people's feelings, perceptions and any personal point of view, is difficult. However, some attempts have been made to develop some techniques and indicators that try to capture an approximation of happiness (e.g. the Bhutanese definition and pursuit of happiness (Ura et al., 2012)). In the context of mining companies operating within a certain district, the contribution of that company to the local community and the whole economy is usually measured through monetary flows. However, there are other indirect benefits derived from the presence of a mining operation that are not easily linked to monetary transactions, for example better health services, increased police presence, better schooling, just to name some. This moves some companies to "feel" that the community is better off in both the short and long term due to their presence (Jenkins, 2004). On the other side of things, not "everything that shines is gold". Some authors (Petkova et al., 2009; Carrington et al., 2012; The Parliament of the Commonwealth of Australia, 2013) identify some unwanted consequences including increased domestic violence, alcoholism and drugs, and rising housing and living costs to mention just a few, that have an economic impact but are not directly measurable, or are difficult to assess. Carrington et al. (2012) mention that the resources industry and governments have been largely ignoring the devastation being wreaked on rural communities by mining projects, and at the time of their writing, it was expected to become worse as there was a pipeline of new projects worth $116 billion. There are clear differences between the point of view of a company and the communities in which they operate. The development of new measurement techniques to assess the effective contribution of mining operations in the communities in which they operate could reduce those differences.

In order to understand the real impact of mining sector activities to other sectors of the economy, it is necessary to rely on tools that in turn depend on proper definitions of what impact is, and how it can be measured. This last point should not be overlooked because what is being measured will heavily influence the conclusions obtainable from the information generated; for example, if a scientist is trying to measure the size of fish in a lake and is using a net made by squares of 10 by 10 centimetres, his conclusion will be that in that lake there is no small fish (the kind of ones you will put in a home aquarium). Most of the available literature provides ad-hoc definitions with no generalised consensus as to what impact is, and how it is measured. For example, the excellent work of Mancini and Sala (2018) reviews several articles that try to measure impact of mining, with a focus on social impact, and they present a table to summarise their findings where it can be observed that some proposed metrics are local, others are national, other metrics have the focus on employment and education, some consider demographics, to illustrate some of the categories of metrics considered. Their review is abundant, considering in the order of 50 such works, and it is apparent that the range of possible metrics is wide. For completeness, we will list a partial selection of some of the impacts considered by several authors referred to in this work:

- Thefts and accidents in the mining community (Kitula, 2006)
- Reduced water supply or water contamination, competition and other uses (e.g. agriculture) and increased water scarcity (Oyarzún and Oyarzún, 2011; Patrick and Bharadwaj, 2016)
- Poor working conditions, low wages (Worldwide, 2010; für internationalen Frieden und Rechtsstaatlichkeit and Spohr, 2016)
- Positive impacts due to demographic change and population growth (Kotey and Rolfe, 2014; Ticci and Escobal, 2015)
- Increased employment (direct and indirect in local community) (Jul-Larsen et al., 2006; Ejdemo and Söderholm, 2011; Franks, 2012)
- Bribery (to obtain licenses and permits or to sway judicial decision) and corruption (due to bad management of mineral wealth) (Azapagic, 2004; Martin et al., 2007; Franks, 2012)
- Human right abuses (Kumah, 2006; Franks, 2012; für internationalen Frieden und Rechtsstaatlichkeit and Spohr, 2016)
- Business and employment opportunities in other sectors due to revitalised economy and markets (Aroca, 2001; Kitula, 2006; Damigos and Kaliampakos, 2006; Petkova et al., 2009; Ivanova and Rolfe, 2011)
- Social impacts related to boom-bust cycles (e.g. increases in pregnancies, sexually transmitted infections, during bust times, mental health issues such as depression and anxiety; overarching community health issues prominent during both boom and bust periods include burden to health

and social services, family stress, violence towards woman, etc.) (Martin and Newell, 2008; Shandro et al., 2011; Weldegiorgis and Ali, 2016)

There are definitions for economic impact, social impact, environmental impact, and other types of impact. No matter which impact definition is used, it ultimately boils down to quantitative measurements; for example:"happiness", "satisfaction", "peace", "lifestyle", are essential qualities affected by mining projects that cannot be represented easily with monetary or other quantitative measures, though in some cases they can be approximated using proxies (Stone et al., 2018).

From a classical point of view, economists and institutions have proposed some models used to characterise the nature of the interactions between productive sectors. Methodologies such as input-output (IO) models (widely used by government agencies) (Lombardo and Ravenna, 2012; Gretton, 2013; Meng et al., 2013; Rueda-Cantuche et al., 2016) and general computable equilibrium models (GCM) (Devarajan et al., 1986; Dixon and Jorgenson, 2012; Lovo et al., 2018) are the most popular. However, one of the biggest problems of classical economic models is the implicit assumption of rationality in the behaviour of economic agents (Farmer and Foley, 2009; Stiglitz, 2011; Stiglitz and Gallegati, 2011). Some researchers have tried to further develop economic modelling by allowing modified forms of rationality, such as bounded rationality (Gigerenzer and Goldstein, 1996; Conlisk, 1996), but the main dependency on some sort of rationality still remains. Furthermore, the 2008 economic crisis has proved that economic science needs new models due to its inability to predict or even explain the problems observed in the recent past. In fact, some renowned economists are increasingly becoming more critical of the role that economic science has played in the problems observed, and have proposed alternative solutions to the current mainstream economic thinking (Stiglitz, 2011).

Adding to the previous basic problems of economic theory, it is Stiglitz' opinion that the measurement of the impact of the resources industry's activities is still an elusive task (Stiglitz et al., 2009). It can be reasonably argued that the inability of current tools to capture the "not so rational" behaviour of economic agents, just provides models, in some cases of elaborate formulation and complexity, that only allow the attainment of conclusions that might not be representative of reality.

Finally, compounding on all the previous problems, the new reality of the resources industry, with a changing labour force, dynamic in nature, that is not yet well understood, makes the whole task a "titanic" one. Classical tools are essentially static and do not provide good modelling options for a dynamic situation (Joyce and MacFarlane, 2001). New tools that could help in incorporating dynamic aspects of the workforce should certainly have more influence on the measurement of impact. Furthermore, tools with capabilities to incorporate qualitative behaviour (Allin, 2014) could allow for a better understanding of the underlying human processes, that result in emerging patterns, which cannot be predicted by current models.

The importance of the measurement of the impact of the resource industry cannot be overstated. It is not only important due to methodological advances, but the finite nature of the resource and the necessity of charging the adequate tax, pay for negative externalities and the contribution to sustainable development are all very compelling reasons to look for new modelling options (Stiglitz et al., 2009; Stiglitz, 2011; Allin, 2014).

Traditional *input-output* (IO) analysis was the first tool introduced to provide a measurement of the impact of a productive sector into others and the whole economy (Leontief, 1955). Wassily Leontief, the author of the methodology, received a Nobel prize for his model in 1973, which is an indicator of the level of acceptance by economists and industry of this technique. IO models are essentially demand propagation models that aid in the calculation of an exogenous variation of the final demand, under the hypothesis of Leontief, where production functions are of fixed proportions, and where the coefficients are calculated using an input-output table (Lemelin, 2008). It has to be noted that input-output tables were introduced by Leontief based on the mathematical model of General Equilibrium due to Walras (Davar, 2000). Later, a new class of models were developed to deal with the limitations of IO analysis, collectively termed *computable general equilibrium models* (CGEM). They are essentially an empirical application of Walras' equlibrium model. These models are free of the rigid hypotheses of strict proportionality and perfectly elastic supply that are present in IO models (Lemelin, 2008; Sue Wing, 2011). Some other extensions to these models have been presented in the literature, mainly addressing the stochastic nature of the problem (Buckley, 1989; Beynon and Munday, 2006, 2007; Diaz and Morillas, 2011; Linda and Manic, 2012). Other research avenues have been related to better estimation of the coefficients in IO tables (Oosterhaven and Escobedo-Cardeñoso, 2011) or the sensitivity associated with these coefficients (Wilting, 2012).

The models currently in use are not specifically tailored to the particularities of the mining industry, as they typically derive a static snapshot at a given point in time, but do not allow the study of the dynamic processes involved in mining. Just to mention an example, it is generally agreed that mining is a dynamic industry, which faces complex decisions in changing environments, and therefore great levels of uncertainty. It is not difficult to observe that commodities prices exhibits a lot of changes in relatively short periods when compared to the usual time-scale of mining operations (which in mining are in the order of a couple of decades). Prices are evolving stochastic processes that play a major role in the definition of the reserve, thus making any static assumption/equilibrium based on relative pricing useless as the reserve concept is by its very nature dynamic. These characteristics allow us to consider the resource sector as a special case, and therefore, an opportunity exists to develop new methodologies to study its impacts, in particular with a view to incorporate and comprehend the dynamic aspects of it.

It is believed that moving away from traditional economic theory could give new insight and modelling capabilities that may allow for more of the particularities of the mining industry to be captured. In particular, the use of methodologies such as agent-based models (ABM) (Tesfatsion, 2003; Xu et al., 2008; Oliva et al., 2010), which can be concisely defined as the construction of economies from the bottom-up (Tesfatsion, 2002), could be utilized in modelling the dynamic aspects and human aspects of the phenomenon that are missing in classical methodologies.

This chapter provides the required background to understand the model that will be later proposed. In Section 2.2 a historical/technical description of the Input-Output technique is provided. Section 2.3 will discuss (in some level of detail) the approach used by Computable General Equilibrium (CGE) models. It will present the basics of this technique and will illustrate its use via an example. Section 2.4 will introduce the reader into the novel world of Agent-Based Modelling. Section 2.5 will briefly mention the system in place in Bhutan to measure happiness, mainly to point out the potential this could have in future versions of the proposed model. The chapter continues with a comparison between the different modelling tools in Section 2.6. Section 2.7 briefly introduction to Dynamic Sthocastics General Equilibrium models. Finally, the last section of the chapter (Section 2.8) presents a brief mention of utility and productions functions and an overview of the different auction mechanisms, because it relates to the problem under consideration, even though the development of this work to a slightly different path.

2.2 Input-Output Models

2.2.1 A Little bit of History

In 1758, François Quesnay (1694-1774) a medical surgeon and physician published his most famous work "Tableau Économique" which would became the precursor of a series of models and publications.

Quesnay, is considered the father of the economic school of thought known as Physiocracy (government or rule of nature) (Bauer, 1895; Phillips, 1955; Baudin, 1958; París, 2007). The phyciocrats or *"les économistes"* as they called themselves, conceived the economy as a circle, which is the basis of what is now known as the *circular flow of the economy* (París, 2007).

Quesnay and his disciples explained the movements of economic activity through the interaction between the "productive class", dedicated to agricultural activity, which provided what they called "net product" (surplus over costs); the "sterile class", engaged in non-agricultural activities, such as manufacturing or trade; and the "proprietary class" (landowners, the King and the clergy) who received part of the "net product" by the way of rents, taxes and tithes.

Thus, with the interaction of these three classes, their income and expenditures within a one year period, Quesnay began to shape the "Tableau Èconomique" and to establish the idea that the strategic variable "net product" was the determinant factor of the magnitude of the economic activity, and therefore the government should guide and focus its policies on increasing the said "net product".

His original work have been considered the first attempt at a dynamic theory of development. Quesnay's work would inspire economists as diverse as Karl Marx , Wassily Leontief, and Joseph Schumpeter among others to develop their own research work (Hishiyama, 1960; Cartelier, 1984; Moss, 1996; Biddle et al., 2001). The "Tableau" proposed by Quesnay was successively refined until 1764. Quesnay and his principal collaborator Victor de Requeti, Marquis de Mirabeau (Charles, 2003) explained that "He was providing all the propositions needed to form an exact and complete theory of the working of economies" (Eltis, 1975).

By 1764, the refined model was capable of giving two multipliers, explaining the dynamic effects of changes on taxations and productivity, and calculate the propensities to consume food and manufactured goods. This refined model is the predecessor and inspiration for Input-Output Models, which are the object of interest in this section. Despite the rudimentary nature of the model, this work earned Quesnay the nomination as one of the greatest economists of all times by Schumpeter in 1935 (Eltis, 1975)

Another work is the "General Equilibrium Theory" developed by Léon Walras. In 1874 and 1877 Walras published "Éléments d'économie politique pure" (1899, 4th ed.; 1926, éd. définitive) which was the first work to put together a mathematical equilibrium model without an empirical background. Walras was trying to disprove the notion raised by Antoine Augustin Cournot that, "although it could be demonstrated that there was a price that could equate supply and demand to clear an individual market, it was impossible that an equilibrium existed for all markets simultaneously". Leontief, some decades later, created the first empirical input-output table, based in real economic life and supported by Walras' model as the theoretical base.

2.2.2 Input-Output Model

The input-output model was introduced by Leontief in 1936 (Rose and Miernyk, 1989), receiving a Nobel Prize in Economics for his contributions in 1973. Economists have extensively used input-output models to characterize local, county, state, national, regional and international economies (Johnson, 1983).

According to Johnson (1983), the input-output model assumes that:

- Each industry produces according to linear, continuous, reversible functions of inputs;
- Inputs must be combined in constant proportions;
- These functions are constant over time; and
- There are no constraints on the level of any variable.

Assume that we have an economy with n sectors, and that each sector produces a single, homogeneous good, x_i. Denote the number of units required from sector j to produce one unit for sector i as a_{ij}. Also, each sector sells some of its output to other sectors and to consumers (final demand, denoted by d_i). It can be seen that conservation occurs as indicated in Equation (2.1).

$$x_i = \sum_{j=1}^{n} (a_{ij} x_j) + d_i \tag{2.1}$$

Equation (2.1) can be written in matrix form as in Equation (2.2)

$$\vec{x} = A\vec{x} + \vec{d} \tag{2.2}$$

with $\vec{x}$ being the n-dimensional production vector, A the matrix containing the coefficients a_{ij}, and $\vec{d}$ the final demand vector. If the terms are regrouped, Equation (2.2) can be expressed as:

$$\vec{x} - A\vec{x} = \vec{d} \iff (I - A)\vec{x} = \vec{d} \tag{2.3}$$

This last system admits solution if $(I - A)$ is invertible. In practical terms, if an economy has too many sectors, the calculation of the previous inverse becomes an important numerical problem. It has to be noted that certain conditions are required of the input-output matrix in order for it to be invertible. One of the most utilised conditions is the existence of sectors external to the economy, which is translated (after normalisation of the columns) that the sum on each column must be less than one.

It is the opinion of some authors (Oosterhaven, 1988; Duchin et al., 2007; Okuyama and Santos, 2014) that input-output models have some characteristics that make them attractive: they are mathematically simple, easy to use, they facilitate different levels of aggregation of information, and it is easy to find tools to perform the calculations.

Leontief (1955) has identified some practical problems that need to be faced when applying input-output analysis beyond the numerical problems naturally associated with the calculation of the inverse of the matrix $(I - A)$. For the most part, the problems are related to the fact that the technique is data intensive, and that data quality is not necessarily the best, or the estimations required could affect the results. Also, note that the technological coefficients are not fixed over time, as technology, prices and demand changes add extra burden in the application of the technique. Additionally, it should be noted that as input-output analyses assumes a linear relationship between demand and outputs, it will not properly model increasing/decreasing returns to scale, which could prove a limitation. Some criticism of the applicability of the technique in certain contexts has been published by Grady and Muller (1988):

the models are static, are linear and do not incorporate economic feedbacks. Lemelin (2008) observes that the root of the criticism for input-output models is due to the strict proportionality between inputs and outputs in the Leontief production functions and that the model has a Keynesian closure, i.e., the supply of factors (work and capital) are perfectly elastic and their prices exogenous which is barely found in the real world. Johnson (2003) adds to the list of potential problems by stating that in the context of regional impact studies, the difficulty in obtaining suitable regional data becomes an obstacle for the application of the methodology, and that the cost of developing survey-based tables for regional impact studies transform those studies in exceptions rather than the rule.

To illustrate the technique, the model has been implemented in a very simple example of an economy with three sectors defined by Table 2.1, which, after normalisation, translates to matrix A (Equation (2.4)).

Table 2.1: Input-Output table for a very simple example

	Agriculture	Manufacture	Services	External Sector
Agriculture	34.69	4.92	5.62	39.24
Manufacture	5.28	61.28	22.99	60.02
Services	10.45	25.95	42.03	130.65
Net Output	84.56	163.43	219.03	

$$A = \begin{bmatrix} 0.410241249 & 0.030104632 & 0.025658586 \\ 0.06244087 & 0.374961757 & 0.10496279 \\ 0.123580889 & 0.158783577 & 0.191891522 \end{bmatrix} \tag{2.4}$$

If the Leontief inverse of matrix A is calculated ($U = (I - A)^{-1}$), we obtain the matrix of Equation (2.5).

$$U = \begin{bmatrix} 1.720369112 & 0.100038161 & 0.067617807 \\ 0.223416179 & 1.667485704 & 0.223678503 \\ 0.306987949 & 0.342939292 & 1.291748286 \end{bmatrix} \tag{2.5}$$

and from here the production income multiplier can be obtained as the sum of every row to give $(1.88802508, 2.114580386, 1.941675528)$, and also for the external demand vector $\vec{d}^T = (39.24, 60.02, 130.65)$ we can obtain the final production due to the initial demand (calculated as $U \cdot \vec{d}$):

$$(U \cdot \vec{d})^T = (82.34572137, 138.0728242, 201.3962449)$$

2.3 Computable General Equilibrium

In this subsection CGE models are introduced. Simply stated, CGE models are similar to IO models. The main differences are in the assumptions of the model, where the CGE models are more restricted than IO analysis, in the sense that the first typically assume optimizing behaviour and that the economy is an equilibrium (Rose, 1995; Koks et al., 2016), but at the same time provide more flexibility because permit non-linearities, as well as a bigger emphasis on prices and elasticities. Sue Wing (2011) defines a CGE model as an algebraic representation of the abstract Arrow-Debreu general equilibrium structure. This algebraic representation needs to be calibrated on economic data which gives rise to a numerical problem of finding the equilibrium of the model. The solution of the problem is characterised by supplies, demand and prices supporting equilibrium across a specified set of markets ranging from a single sub-national region to multiple groups of countries interacting within the global economic context. It is observed that despite the wide range of markets to which the modelling could be applied, every economy represented in the models typically possesses the same structure characterised by producers, consumers and governments that "interact" in the markets by means of commodities and factors, as well as taxes, subsidies, among others externalities.

There are several possible ways to express this model, some examples can be found in Johnson (2003) and in Sue Wing (2011). For practical purposes it should be noted that the information required for the CGE models is similar to that required for the IO model. However, the mathematical formulation is much more complex, which contributes to CGE models being seen as "black boxes" by some practitioners (Sue Wing, 2011).

The conceptual core of every CGE model is the circular model of the economy. The flows stem from market interactions between three sectors: households, firms and governments. CGE models are usually expressed using a system of equations that involves a large number of equations. Concisely expressed, a CGE model can be represented by a system of N non-linear equations (Horridge and Pearson, 2011) as follows:

$$F(\vec{z}) = \vec{0} \tag{2.6}$$

The variable $\vec{z}$ can be divided into N endogenous variables $\vec{y}$, which are determined by the system, with the remaining exogenous variables represented by $\vec{x}$. Equation (2.6) would then take the form:

$$F(\vec{y}, \vec{x}) = \vec{0}$$

In CGE practice, it is assumed (Horridge and Pearson, 2011) that an initial solution $(\vec{y}^0, \vec{x}^0)$ is known (i.e. $(\vec{y}^0, \vec{x}^0)$ is such that the following identity holds $F(\vec{y}^0, \vec{x}^0) = \vec{0}$). A different solution is sought with a different setting of the exogenous variables, $\vec{x}^1$, which leads to solve the system and obtain a solution for $\vec{y}^1$:

$$F(\vec{y}, \vec{x}^1) = \vec{0}$$

where percentage differences between $\vec{y}^1$ and $\vec{y}^0$ are reported.

Some or all of the equations in the CGE model can be expressed using *complementarities* (Horridge and Pearson, 2011). Each complementarity equation, F_i, has an associated variable, Z_i, which is bounded between L_i and U_i. As a result, *just one* of the following relationships holds:

$$F_i(\vec{z}) = 0 \quad \text{and} \quad L_i \leqslant Z_i \leqslant U_i$$

$$F_i(\vec{z}) > 0 \quad \text{and} \quad Z_i = L_i$$

$$F_i(\vec{z}) < 0 \quad \text{and} \quad Z_i = U_i$$

This type of model is of a more advanced nature than input-output models and attempts to alleviate some of its inadequacies, like its lacks of dynamism, fixed technology, etc. (West, 1995; Koks et al., 2016). It is certainly of a dynamic nature and facilitates the adding of more detail into the modelling which allows for the modelling of economic interactions at different levels. However, according to (Lemelin, 2008), the great flexibility in functional forms allowed by CGE modelling implies a big number of parameters which is usually bigger than the number of independent equations in the model. The usual solution for this problem (Wigle, 1986; Abdelkhalek and Dufour, 1998; Lemelin and Savard, 2017) is to take values from the literature or to create other models to estimate those parameters. Hertel et al. (2007) mention that CGE models have been widely criticized for performing poorly and having weak econometric foundations. Cochrane (2004) adds to the previous criticism that relative price changes are conspicuously absent from CGE models and he believes that this robs CGE models of one of their chief advantages. He mentions too that CGE models contain a number of ad hoc assumptions shaping product substitutions that could make the inner workings of the model opaque and the result difficult to interpret.

2.4 Agent-Based Models

During the last twenty years a new movement in economics has appeared (Arthur, 2006). This new movement has introduced tools that enable the examination of how the economy behaves when it is out of equilibrium and not in a steady state. Most of this new movement is based on the use of computational techniques that allow for complex systems to be modelled.

According to Tesfatsion (2002) decentralized market economies are complex adaptive systems. They consist of large numbers of agents, of an adaptive nature, that engage in local parallel interactions that give rise to macroeconomic regularities that in turn feed back into the local interactions. The obvious result is a complicated system, dynamic in nature, which provides a rich description of individual behaviours and social outcomes.

This emphasis on the dynamic aspects and nature of market interactions, together with causality relationships are a beneficial departure from the classical models which it is believed, may enable a better characterisation (Smith and Conrey, 2007; Stiglitz et al., 2009; Stiglitz and Gallegati, 2011) of the impact of the minerals sector when compared to other sectors of the economy. Traditional quantitative economic models have been characterised by their top-down construction (Tesfatsion, 2003), whilst the new paradigm proposes the opposite. The main idea behind artificial economics' techniques is that interactive groups of agents can produce a collective behaviour, sometimes unexpected, and usually characterized by emergent features that are lawful in their own right (Batten, 2000).

To better understand the mechanisms used by *Agent-Based Models* (ABM), let us consider a classical example identified by Schelling (1969) called the segregation model. The model proposed by this author divides the space into cells (or houses) and each cell can be occupied by an agent that has one of two possible skin colours (in our implemented example the skin colours are green and red). The happiness of an individual is defined by the percentage of neighbours that have the same skin colour as him, and this is a parameter of the model. If an agent is unhappy, he will move to a different neighbourhood by changing cells (or moving to a different house). Using the open source platform NetLogo (Wilensky, 1999), and starting at a random configuration (Figure (2.1) - left), the algorithm applies this local rule repeatedly until reaching a stopping criteria (Figure (2.1) - right), showing that the agents have grouped together based on their skin colours.

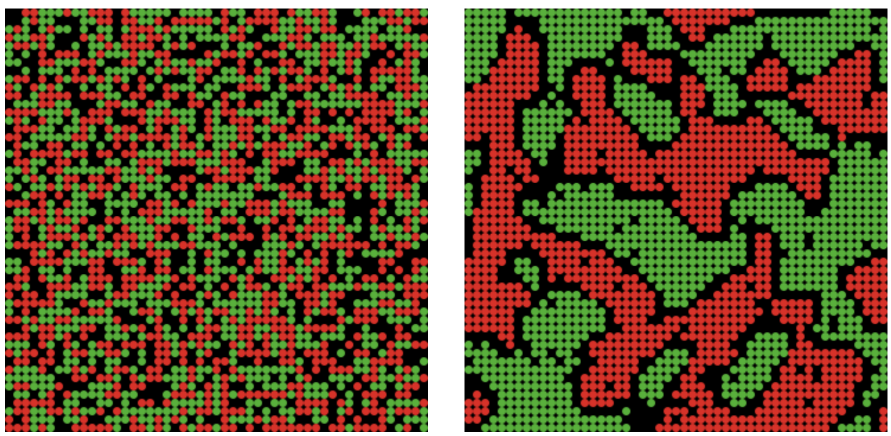


Figure 2.1: Left: Initial Configuration, Right: Segregation After Running the Algorithm, produced by the author

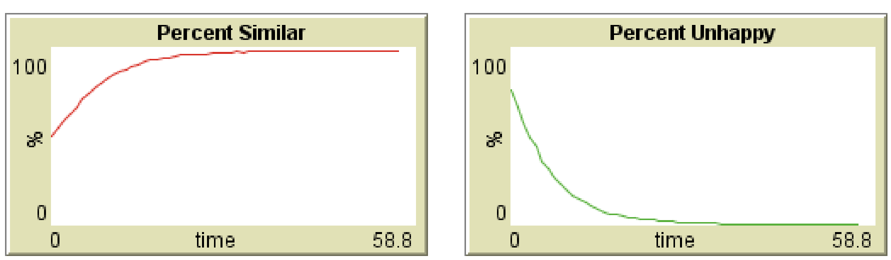


Figure 2.2: Left: Evolution of Similarity, Right: Evolution of Unhappiness, produced by the author

In an essay that appeared in Nature (Farmer and Foley, 2009) a strong case for the use of agent-based models in economics was made. Simply stated, these are computational models in which a large number of interacting agents are endowed with behavioural rules. Such models generate complex dynamics as the interactions between agents can give rise to emergent properties that could not be deduced by closer examination of the rules themselves. ABM have been used in a number of research areas. Some examples of these areas within the context of economics are (Tesfatsion, 2002):

- Evolution of behavioural norms
- Bottom-up modelling of market processes
- Formation of economic networks
- Modelling of organizations
- Design of computational agents for automated markets, and
- Parallel experiments with real and computational agents.

2.4.1 Agent-Based Models in Economics

The objective of this section is to give the reader an overview of how ABM has been applied specifically in the field of economics. But first, it is important to remember some basic elements that are founded in the "Neoclassical Approach of Economics", and how those assumptions are a stumbling block to create a representative model of reality.

The neoclassical theory assumes that markets are in an equilibrium state, and in turn, that equilibrium is based on a typical set of hypothesis (Mankiw, 2012; de Jesús and Campo, 2017), which are:

(i) Agents are rational

(ii) There are perfect information

(iii) Agents are utility maximisers given a price vector

(iv) No agent has enough power to influence the market

(v) Everything else is constant (known as *ceteris paribus*)

Under this framework, it is easy to see that the models created are one-time or static models (Gowdy and Olsen, 1994; Kaufman, 2007), and leads to the thought that they are unrealistic in reality.

In the opinion of the author, to think that all assumptions are met in a world that is intrinsically dynamic, it is to suppose that every agent is omnipotent and they can have knowledge of every price past, current and future, that there is no cost to obtain information, etc. which is hardly credible (Hosseini, 1990; Davis, 2006; Ivarola, 2014). And in front of this "real" reality one is compelled to seek a different paradigm to represent it and that is where ABM modelling is leading the steps due to its flexibility (Bianchi et al., 2007; Bazghandi, 2012; Cogliano and Jiang, 2016).

2.4.2 Examples and their principal characteristics

As mentioned before, ABM models are becoming more and more popular in different fields of study. In Tables (2.2) and (2.3) a non-exhaustive summary table of recent ABM models on the economic field will be presented in conjunction with a short explanation of each one.

Table 2.2: Summary of Novel ABM Economics Models (Part 1)

Author	Title	Summary
1. Bacelar and Antunes (2019)	• Generational Politic Dynamics of Retirement Pensions Systems: An Agent Based Model	• This work tries to represent the conditions to maintain a healthy public pension system, allowing heterogeneous behaviours of the agents (either at individual or collective level) in a "created" society
2. Shoukat (2019)	• Cost-Effectiveness Analysis Using Agent-Based Modelling: A General Framework With Case Studies	• This research utilises ABM models to explain epidemics dynamics and a cost-effectiveness analysis of potential vaccine candidates

2.5 The Gross National Happiness Study

Due to its importance in terms of proposing a way to measure happiness, and due to the importance it could have in the modelling using an ABM approach, it is potentially interesting to introduce the Bhutanese Gross National Happiness measurement system (Ura and Galay, 2004; Burns, 2011). The interested reader should consider perusal of the work presented in (Ura et al., 2012). This work explains the measurement mechanism used in the actual survey that Bhutan has implemented to approximate the population's happiness. This need arose when the King of Bhutan declared that every citizen's pursuit of happiness should be the main goal. For this purpose a Ministry of Happiness was created and the initiative has been at least adopted by the United

Table 2.3: Summary of Novel ABM Economics Models (Part 2)

Author	Title	Summary
3. Woźniak et al. (2020)	• Virtualising Space - New Directions for Applications of Agent-Based Modelling in Spatial Economics	• Combination of ABM model with geographical data. This approach allows for more realistic socio-economics simulations
4. Karimi and Mohammadi (2019)	• An Introduction to Agent Based Modelling and Agent Based Computational Economics; A Simple Model for Markets Where Consumers are Imperfectly Informed	• On the same lines of other works, this study take advantage of ABM models to relax classical assumptions, in this case imperfect information
5. Beklaryan and Akopov (2019)	• Modelling the Efficiency of the Use of Production and Investment Resources at the Regional Level: the Case of Russia	• The authors describe the behaviour of enterprises, investors and regions by using ABM, thus allowing a more realistic model
6. Mignot and Vignes (2019)	• The Many Faces of Agent-Based Computational Economics: Ecology of Agents, Bottom-up Approaches and Technical Breakthrough	• This paper signal how ABM models can improve the understanding of social interactions, as well as they can incorporate heterogeneous agents and interactions between different levels of agents

Arab Emirates. In the future, the proposed ABM model could consider these types of measures to generate different KPI's. However, due to the envisioned complexity of the model it has been decided that going in this direction would deviate attention from the main focus, which is that it is possible to measure the impact of mining activities within a regional location (Western Australia in this case).

2.6 Comparison of Approaches

Input-output analysis appear to be a well-established technique despite its limitations (Grady and Muller, 1988; Lemelin, 2008). Nevertheless, the complex nature of the economic interactions between sectors, especially when looking at the mining industry, cannot properly be captured by the use of the classical tools (Stiglitz et al., 2009; Farmer and Foley, 2009). Furthermore, new tools that have the ability to not only reproduce the classical results, but also be extended to incorporate new features, could add to the existing set of tools, and thus constitute a contribution. Few applications of agent based models into IO or CGE analyses have been suggested in the literature with differing levels of development (Parris, 2005; Taghawi-Nejad, 2012). As they are not maturely developed, further research in this area will result in a positive contribution to the knowledge base.

Next, a summary table taken from (West, 1995) with some additions is presented on Table 2.4 [1]. On this table, the ABM column has been added to expand on the existing comparison between IO and GCE models. Additionally, Table 2.5 presents a very high level comparison based on the literature review performed for this study. Though this table is not exhaustive, the idea

[1]The IO model discussed here relates to type two multipliers.

is to give the main differentiating elements to the reader.

Table 2.4: Summary of the Principal Characteristics of the Different Modelling Approaches compiled by the author based on literature review

IO	CGE	ABM
1. Static	1. Can be dynamic	1. Can be dynamic
2. Linear functions	2. Can be non-linear	2. Can be non-linear
3. Demand driven	3. Demand and supply driven	3. Demand and supply driven
4. No price effect	4. Full response to price	4. Dynamic Price Mechanism
5. Partial equilibrium	5. General equilibrium	5. Out of Equilibrium
6. Leontief primary factor demands	6. CES or Cobb-Douglas primary factor demands	6. Allows for any function
7. Leontief intermediate factor demands	7. Leontief intermediate factor demands	7. Allows for any function
8. Partial optimization	8. Optimization Model	8. Optimization Model

2.7 Dynamic Stochastic General Equilibrium Models

In this section we talk succinctly and tangentially about the Dynamic Stochastic General Equilibrium Models (DSGE). DSGE are an alternative method that economists have mainly used in policy making, with a clearly defined rationale to it and that could be considered to belong or more aligned with the “classical” alternatives for the problem under study.

One of the first things that are important to note is that DSGE models are mainly used in macroeconomic analysis and incorporate monetary pol-

Table 2.5: Comparison between the different modelling alternatives

Technique	Pros	Cons
IO	Simple, well understood	Linear, unrealistic, too simplistic, static analysis
CGE	More representative, more flexibility	More involved modelling, potential lack of data, rationality assumptions
ABM	Very flexible, potential for uncertainty modelling, can be made dynamic, potential to include qualitative behaviour	Untested, complicated, usually requires programming, extra data requirements

icy, which although influencing to the problem under consideration, is at the present time excluded from the scope of this work, as its incorporation would lead to a loss of focus on the regional aspects of the modelling by having to consider monetary policy, inflation, etc. There is vast literature for the topic available for the benefit of the interested reader, for example we can mention Fernández-Villaverde et al. (2016); Costa (2018) among many others. But in order to show the main characteristics of the methodology, we will briefly summarise some of the works discussing this modelling approach.

DSGE came to life in the 80's, as a macroeconomic tool with strong microeconomics foundations, when Kydland and Prescott (1982), and Long and Plosser (1983), proposed their models based on the concept of Real Business Cycle Models (RBC) (Slanicay, 2014; Boum Baha, 2019). RBC models assume that individuals are rational agents that seek to maximise their utility, not an uncommon assumption in most economic models. However, under this modelling paradigm, a key characteristic of individuals is that they seek to invest in capital whilst the price of labour is determined by market forces, and under a set of pre-specified conditions, it can be shown that investment, work and output are converging to a consistent rate (i.e., they form an equilibrium). Other authors commonly appearing on this topic are (Gertler and Bernanke, 1989), (Moore and Kiyotaki, 1997), and several Greenwald and Stigliz' models (Greenwald et al., 1984; Greenwald and Stiglitz, 1990; Stiglitz and Greenwald, 2003), who resurrected the thinking of Irving Fisher and are known as neo-keynesian models (Slanicay, 2014; Stiglitz, 2018).

DSGE are macroeconomics models that take their foundations on five elements (Sergi, 2015):

- The study of the economic cycle, understood as the co-movement of aggregates series (Lucas Jr, 1977). Methodologically, they are inter-temporal general equilibrium models "à la Walras".
- The agents are homogeneous and rational. The goal of the agents is utility maximisation, and they possess rational expectations "à la Muth" (Muth, 1961).
- The dynamic is generated by stochastic components that influence the evolution of monetary and real variables, or the so called "shocks".
- Prices and wages are semi-rigid (the changes are onerous and not immediate). The rigidities can be explained by the presence of monopolistic competition (Dixit and Stiglitz, 1977). Calvo's model offers additional explanations (Calvo, 1983).

- The model describe that monetary policy plays an active role in the determination of the equilibrium and on the fluctuations.

Mathematically, a basic DSGE model can be represented by three key equations, with the first one being:

$$y_t - y_t^n = \mathbb{E}_t(y_{t+1} - y_{t+1}^n) - [r_t - \mathbb{E}(\pi_t)] \tag{2.7}$$

Equation 2.7, called Euler's equation, is a consequence of the utility maximisation of the rational agent, and it allows to define the equilibrium on the goods market, under the shape of an output gap $(y_t - y_t^n)$, with y_t^n being the level of production of equilibrium and with completely flexible prices, only depending on the expected inflation $(\mathbb{E}(\pi_t))$ and the nominal interest rate (r_t).

The second equation is:

$$\pi_t = \beta\mathbb{E}_t(\pi_{t+1}) + \phi(y_t - y_t^n) + \epsilon_t \tag{2.8}$$

In Equation 2.8, (π_t) denotes the level of inflation on period t and is defined by a neo-keynesian Phillips curve, that contains rational expectations and the level of prices rigidities expressed using ϕ, representing a relation between the output gap and global marginal cost, and finally the term ϵ_t represents an stochastic component.

The third equation is stated as:

$$r_t = \rho r_{t-1} + \sigma\pi_t + \tau(y_t - y_t^n) + \eta_t \tag{2.9}$$

Equation 2.9 represents a monetary policy rule "à la Taylor" (Taylor, 1993), where r_t is fixed by the monetary authority and is dependant of the present inflation rate π_t, the output gap $(y_t - y_t^n)$, and a stochastic residual term η_t.

As can be seen, the DSGE model presents itself as a multi-level model approach that attempts to link different aspects ranging from rational agents decisions in the micro-economic level (Equation 2.7) to characterisations representing monetary policy at the macroeconomic level (Equation 2.9).

DSGE models, despite the good intentions of the modelling approach and standard assumptions, are not free from criticism. One of the most important critique is that good policy making requires and understanding of the underlying determinants of behaviour. And in this particular point is were the problems are manifested. Several authors (Garcia, 2011; Napoletano et al., 2012; Stiglitz, 2018; Korinek, 2018; Storm, 2021) agree that the assumptions on where these models are build, are to say the least shaky. For example, the standard models consider that the agents are rational, homogeneous and have completely access of information; and that are arguments that behavioural economics, game theory, the economics of information have more or less debunked.

Some opponents to DSGE models are particularly acid in their critics. Consider for example Joseph Stiglitz, a Nobel prize winner, has published critiques such as the following: *"Although, DSGE have played an important role in modern discussions of macroeconomics, they fail to serve the purpose which a well-designed macroeconomics model should perform"* and *"They have made the wrong modelling choices"*, following the 2008 crisis Stiglitz (2018)[2].

[2] In his paper, Korinek also provides a critique from a statistical point of view (see

However, it is important to note that an ABM "version" of a DSGE model could be interesting to explore, in order to lift some of the assumptions that DSGE models possess and that are usually used to critique it. However, that requires adding macroeconomic policy considerations into the modelling, which in turn requires to study the macroeconomic agents in order to characterise their rationale and behaviour, and to model them in such a way that the heterogeneity that ABM promotes can be embedded into these agents' representation.

2.8 Additional Literature Remarks

2.8.1 Utility Functions

It is important to mention that utility and production functions are an important tool in the development of this work, as they empower the modelling process with mechanisms to add some rationality into the agents, although collectively heterogeneous but with a common functional form/language to express their heterogeneity. They are used both in the construction of the Household agent and the Firm agent. Despite being a common and widely taught topic, Appendix A is provided to the reader for completeness.

2.8.2 Auctions

One important aspect that the modelling technique adopted in the present work, relates to the interaction between economics agents, at the transactional

(Korinek, 2018))

level. The agent based model paradigm enable the specification of the mechanisms used by economic agents to interact. The resulting observable quantity that results from the interaction of producers and consumers is reflected on the prices of goods. A price mechanism is a system by which the allocation of resources and distribution of goods and services are performed. These allocation and distribution decisions happen under the guidance of relative market prices.

Competitive markets are usually considered to be an ideal market model, but it is so pervasive in economic training, that we will perform a basic review here, with a view to delineate the price formation mechanism. Competitive markets are characterised for possessing the following features:

- There are no barriers to entry, i.e., there are no financial or physical barriers for new companies to enter the market. This feature is usually considered to be theoretical because there will always be some barriers to enter or leave a market.
- The number of buyers and sellers is large, which implies that no single consumer or seller can affect the price of the goods, i.e., they are price takers.
- Homogeneity of the goods and services, i.e., product and services are homogeneous in price, availability and quality. In reality though, real-world markets differ from the competitive paradigm due to differences in production, marketing and selling.

The way competitive markets operate could be better understood by using a thought experiment. Let us suppose that buyers and sellers of a given good are put together in a very large sport arena. In this shared space, sellers publish the price at which they are willing to sell their product/services, and at the same time, consumers list the price they are willing to offer to acquire goods. If a match is produced, then a transaction happens. The resulting market price will reflect all these transactions happening on a given day.

The previous example, illustrate the basics of market operation. In practical terms, markets were originally physical places were traders and sellers interacted. Today, these transactions happen in virtual spaces at volumes that were not achievable in physical markets. The most common form of market for competitive bids and offers is an auction market. In auction markets, the price at which a stock trades represents the pairing of matching bids and offers which are subsequently executed.

The use of auction mechanisms was considered in the early stages of the research work. However, this layer of complexity was considered to be an unnecessary distraction in the modelling efforts. It was decided that a market clearance mechanism, based on the use of a mathematical programming model with dynamic pricing considerations was enough for the purposes of the present research. Nothing prevents changing this mechanism in the future. For completeness, included in the appendix is a small literature review of the basics of auctions (See Appendix B).

2.9 Chapter Concluding Remarks

This chapter provided the theoretical foundations of the main techniques that are used in the development of the model. Some additional information could have been added, as it was surveyed during the initial stage of conceptual development (e.g., auctions), however, this material has been put in a separate appendix to benefit the interested reader but didn't make it into the main body of the text as it could introduce some confusion.

Further information, for specific tools/techniques, is added in subsequent chapters as needed. It is believed that a couple of references in isolation do not fare well within the context of the more generalistic information provided in this chapter.

Chapter 3

First Example

This Chapter provides a practical introduction to input-output analysis and agent based modelling. It does so by looking at a particular example, doing the calculations in the classical way using Leontief's methodology (described in (Leontief, 1955, 1986)) and then proposing, implementing and testing an agent based model which are shown to replicate the classical multipliers. Furthermore, the test is expanded to add some uncertainty to test the stability of the approach with respect to the multiplier.

3.1 An Agent-Based model to Approximate Impact

This section introduces, with increasing levels of complexity, models intended to replicate the multipliers obtainable using Leontief's input-output model. The models are specified by means of agent-based models with simplistic behaviour for the agents. A first model already proposed in the literature (de Andrade et al., 2010) will be discussed in sufficient detail to introduce the type of modelling involved. The two sub-sections that follow will introduce extensions to this basic model to illustrate the capabilities of the modelling.

3.1.1 A Preliminary Model for Input-Output Analysis

The first model that comes to mind when trying to model the behaviour of the economy is one that uses one agent per economic sector, such model has already been presented in de Andrade et al. (2010). The model assumes a homogeneous behaviour from the agents and a reactive behavioural rule: once the agent (sector in this case) receives income, it distributes that income according to the input-output matrix. As the agent will continue to receive income from other agents, in the next turn the process is repeated with the new income received; the whole process is repeated until the convergence criteria is met.

The previous model has been implemented in the Python programming language[1]. Each agent i is initialised to the following set of parameters:

- A *Technology Vector* (denoted by $\vec{\tau}_i$), which is essentially a normalised version of the corresponding column in the input-output matrix
- A property that accounts for the accumulated money up to time t (denoted by α_i^t), initially set to the external demand figure
- A property that accounts for the income received at iteration t (turn) of the algorithm (denoted by κ_i^t), originally set to the external demand figure

On each iteration of the algorithm (turn), the sequence of steps is as follows:

- For each economic agent in the agent list:

[1] The interested reader can go to http://www.python.org/ to find more details about the language

- Distribute the income received during the last iteration (κ_i^{t-1}) following the proportions defined by the technology vector $\vec{\tau}_i$
- Determine the income received during the current iteration of the algorithm and add that value to α_i^t and assign it to κ_i^t

Each iteration is run until a certain criteria is met (i.e. a number of iterations, a tolerance, etc.). In the particular case of the code that has been implemented, a tolerance parameter (ε) has been chosen. If the maximum difference between each component of the previous scores vector and the current scores vector are greater than the tolerance, then the algorithm will continue its execution. The previous description is summarised in Algorithm 1.

The model has been implemented (Details in appendix D.3) in a very simple example of an economy with three sectors defined by Table 3.1, which, after normalisation, translates to matrix A (Equation (3.1)).

$$A = \begin{bmatrix} 0.410241249 & 0.030104632 & 0.025658586 \\ 0.06244087 & 0.374961757 & 0.10496279 \\ 0.123580889 & 0.158783577 & 0.191891522 \end{bmatrix} \tag{3.1}$$

If the Leontief inverse of matrix A is calculated ($U = (I - A)^{-1}$), we obtain the matrix of Equation (3.2).

$$U = \begin{bmatrix} 1.720369112 & 0.100038161 & 0.067617807 \\ 0.223416179 & 1.667485704 & 0.223678503 \\ 0.306987949 & 0.342939292 & 1.291748286 \end{bmatrix} \tag{3.2}$$

Algorithm 1: A basic ABM for Input-Output Analysis ((de Andrade et al., 2010))

Data:
A: normalised input-output matrix
$\{d_i\}_{i\in I}$: external demand for each sector i
Result:
$\{\alpha_i^{t^*}\}_{i\in I}$, with t^* the stopping time of the algorithm defined by the tolerance criterion ε

```
begin
    Initialisation
    for i ∈ I do
        α_i^0 ⟵ d_i;
        κ_i^0 ⟵ d_i;
        τ⃗_i ⟵ A_{•i};
        t ⟵ 1;
    while max{|α_i^{t−1} − α_i^t|}_{i∈I} ⩾ ε do
        Distribution of money between sectors
        for i ∈ I do
            for j ∈ I do
                κ_j^t ⟵ {τ⃗_i}_j · κ_i^{t−1};
        Sector updating
        for i ∈ I do
            α_i^t ⟵ α_i^{t−1} + κ_i^t;
```

Table 3.1: Input-Output table for a very simple example

	Agriculture	Manufacture	Services	External Sector
Agriculture	34.69	4.92	5.62	39.24
Manufacture	5.28	61.28	22.99	60.02
Services	10.45	25.95	42.03	130.65
Net Output	84.56	163.43	219.03	

From here the production income multiplier can be obtained as the sum of every row to give $(1.88802508, 2.114580386, 1.941675528)$, and also for the external demand vector $\vec{d}^T = (39.24, 60.02, 130.65)$ we can obtain the final production due to the initial demand (calculated as $U \cdot \vec{d}$):

$$(U \cdot \vec{d})^T = (82.34572137, 138.0728242, 201.3962449)$$

When the program is run, the following output can be observed:

```
Final Production due to Initial Demand: [82.34584086789937,
138.0729392850388, 201.39633704752853]

[[39.24, 60.02, 130.65],
 [55.33786661076, 84.97538439394, 170.10013172520001],
 [67.10104978893358, 109.05125248272194, 183.62217141129003],
 [73.69030000934377, 122.95408156595087, 191.4934932763437],
 [77.46523381681266, 129.99786031462193, 196.0257795415172],
.
.
.
 [82.34584086672511, 138.07293928321738, 201.3963370462991],
 [82.34584086748507, 138.07293928439617, 201.39633704709476],
 [82.34584086789937, 138.0729392850388, 201.39633704752853]]
```

To determine the production income multiplier, the same procedure is repeated but using an initial demand of $(1, 1, 1)$. In that case the output of the model is:

```
Production Income Multiplier: [1.888025079209744,
2.1145803853214633, 1.9416755270574233]

[[1, 1, 1],
 [1.410241249, 1.437402627, 1.474255988],
 [1.63430234938108, 1.731990495033782, 1.6854120233150809],
 [1.75155803822951, 1.9062194813382518, 1.8003964617137975],
 [1.8139475831254177, 2.001033761979459, 1.8646162645351465],
.
.
.
 [1.8880250780269598, 2.1145803834868153, 1.94167552581908],
 [1.888025078792431, 2.114580384674158, 1.941675526620508],
 [1.888025079209744, 2.1145803853214633, 1.9416755270574233]]
```

The whole simulation took just `8.92138 ms`[2]. Finally, in Figure 3.1 the evolution of the iterations converging to the multipliers together with the path that final production levels exhibit are shown.

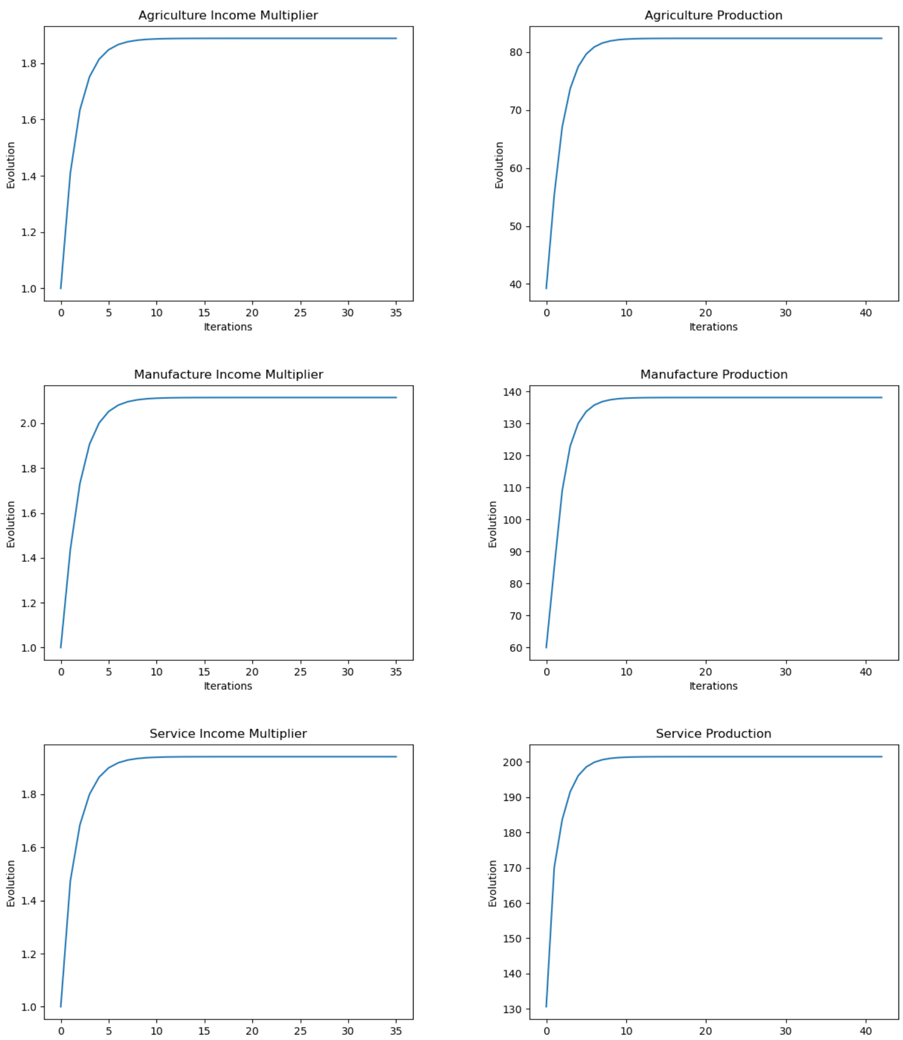


Figure 3.1: Evolution of Multipliers and Production

To summarise, Table 3.2 shows both the results that can be obtained by

[2]This test was run on an 11th Gen Intel(R) i7-1165G7 processor with 8 cores running 2.80Ghz

calculating the Leontief inverse (or classical method) and the results that can be obtained using the ABM (or novel method).

Table 3.2: Comparison of both methods (Classical versus Agent-Based Model)

	Production Income Multiplier		
Method	**Sector 1**	**Sector 2**	**Sector 3**
Leontief	1.88802508	2.114580386	1.941675528
ABM	1.88802507920974	2.1145803853214633	1.9416755270574233

As it can be seen, the ABM model at this basic level, with agents that are provided no intelligence at all, but just forcing them to follow the simplest of rules, is capable of replicating the multiplier determined by Leontief's technique (and implicitly calculating the pseudo-inverse used by that method). In fact, the differences are minimal and in this particular example, determined by the tolerance parameter (ε) used in the algorithm. In order to understand better the time required to run the algorithm, an experiment was performed consisting on 5,000 repetitions to measure the execution time. Out of this 5,000 repetitions, the majority of them did not record time at all (0 ms), of the ones that this record a positive time (784 runs), the average time is 3.078693106 ms with a standard deviation of 2.92401598 ms.

This result, by means of a very simplistic ABM model, validates the approach as it is able to perfectly replicate the classical results. However, it is hypothesised that the expressiveness that is allowed in ABM modelling, will allow the modelling approach to extend into the domain of heterogeneous economic decision makers, which is deemed more realistic. This finding cannot be overstated enough, as it is the necessary starting point to incorporate additional complexity into the modelling. From here, models that focus on additional aspects, not considered by the traditional Nobel-winning theory, can be formulated and analysed.

3.1.2 A More Elaborate Agent-Based Model for Input-Output Analysis

The model proposed in this subsection is an extension of the model presented in the previous section. It is similar to the previous one in the sense that it has sectors as agents, but introduces agents within each sector thus providing different levels. Each agent within a sector will utilise resources according to a technology vector (defined in a manner similar to the previous model) and the overall multiplier of the sector will be calculated based on the individual transactions of the agents within the sector. This model, from a practical point of view, implements an ABM that appears to possess a bi-level structure of interaction: on one hand we have the sectors for which we know historical data that informs us about the flow of money, and on the other hand we have the companies that belong to a sector, companies that can be characterised individually. However, the structuring of the code is just trying to approximate the problem and to decompose it into manageable pieces. In reality the interactions of individual company agents will produce as an output the flows on money observed at the aggregated sector level, i.e., a sector really is characterised in terms of the observed emergent behaviour, but from a practical point of view, starting from the sector level technology we can approximately reproduce this technology on the agents that belong to a sector, with heterogeneity allowed for the individual company agents that belong to the sector, but that at the aggregated level should honour the observed emergent behaviour of the sector.

In this new model, company agents are created within a sector each one consisting of heterogeneous sizes. They can be input as data or generated based on some probability distribution. For the purposes of the current study it has been decided to use a power law distribution for the size of the companies, but by no means does this choice limit the applicability of the approach, which can be generalised to any distribution of company sizes.

The idea behind this new model is that by allowing sectors to have individual heterogeneous agents, modellers can incorporate non-homogeneous behaviour.

The idea is to extend the functionality of the basic ABM model presented before, which we have already shown that perfectly replicates the classical theory, so we can start to understand the impact of deviations from this basic behaviour in the outcomes of the modelling. It is not the purpose to introduce too much variations, just enough to see how far things go when deviating a little bit from the very basic classical model output. The proposed ABM is described in Algorithm (2).

At a basic level, the first obvious test to perform is to check that the new implementation is able to reproduce a trivial case, or a system test. The model is thus run with the same dataset as before and with the technology rule maintained. Just to be more explicit with the mechanism used for technology at the company level, for the current test, the rule used for each company's consump-

Algorithm 2: An extended ABM for Input-Output Analysis

Data:
$\{d_i\}_{i \in I}$: external demand for each sector i
$\vec{\tau}_{i,j}$: normalised technology vector for agent j beloning to economic sector i, $i \in I, j \in J(i)$
$\{w_{i,j}\}_{j \in J(i)}$: size of agent j belonging to sector i, $i \in I, j \in J(i)$
Result:
$\{\alpha_i^{t^*}\}_{i \in I}$, with t^* the stopping time of the algorithm defined by the tolerance criterion ε

```
begin
    Initialisation
    for i ∈ I do
        α_i^0 ⟵ d_i;
        κ_i^0 ⟵ d_i;
        t ⟵ 1;
    while max{|α_i^{t-1} − α_i^t|}_{i∈I} ⩾ ε do
        Distribution of money between sectors via agents
        for i ∈ I do
            for j ∈ J(i) do
                for k ∈ I do
                    κ_k^t ⟵ {τ⃗_{i,j}}_k · κ_i^{t-1} · w_{i,j};
        Sector updating
        for i ∈ I do
            α_i^t ⟵ α_i^{t-1} + κ_i^t;
```

tion of a sector is the same defined for the sector as a whole. Not surprisingly, the results come out the same as before, which confirms the correctness of the implementation of the extended model.

For illustration purposes of the evolution path observed, when the program is run on this example, the following output is generated:

```
Final Production due to Initial Demand: [82.34584086797258,
138.0729392851031, 201.39633704750335]

[[39.24, 60.02, 130.65],
 [60.497040884300006, 98.68877290744001, 170.10013172520001],
 [71.39389966257173, 118.65618795603348, 186.4372076354843],
 [76.88453856720825, 128.53839939576997, 194.08929507692386],
.
.
.
 [82.3458408666573, 138.07293928291014, 201.39633704580976],
 [82.34584086753799, 138.0729392843785, 201.39633704694376],
 [82.34584086797258, 138.0729392851031, 201.39633704750335]]
```

For the production income multiplier we obtain:

```
Production Income Multiplier: [1.8880250792300866,
2.114580385297296, 1.9416755269632615]

[[1, 1, 1],
 [1.466004467, 1.542365417, 1.4742559880000001],
 [1.6856751709240838, 1.8246086627124105, 1.7089696586171217],
 [1.790312404820172, 1.968791717611765, 1.8259719151225484],
.
.
.
 [1.8880250777393657, 2.1145803828119663, 1.9416755250438724],
 [1.8880250787375261, 2.1145803844760906, 1.9416755263290564],
 [1.8880250792300866, 2.114580385297296, 1.9416755269632615]]
```

Clearly, we can see that the results observed do not exhibit dependence on the size distributions of companies. The small differences observed between this result and the previous are consistent with the ε used in the algorithms runs ($\varepsilon = 0.000000001$). For example, the multiplier obtained for the first sector in the first run is `1.888025079209744` and for the second run is `1.8880250792300866`, their difference (in absolute value) is `0.0000000000203426`, which is clearly smaller than the tolerance specified. Similar checks can be performed for the other multipliers. To illustrate the dependence on ε, a new test was run using $\varepsilon = 0.0001$, for this test the first multiplier obtained is `1.8879861924785377`, and the difference (in absolute value) with the first multiplier obtained on the first test is `0.0000388867312063` which is clearly larger than the difference obtained when using a smaller ε.

As it was done in the previous subsection, an experiment was performed consisting on 5,000 repetitions to measure the execution time. The average time is 3.719280958 ms with a standard deviation of 0.521307784 ms (all experiments measured a strictly positive time). Also, the evolution of multipliers and product throughout the iterations of the algorithm is exhibited in Figure 3.2.

In the next subsection the impact of a different use of technology will be analysed. In particular, this will be reflected on each agent having a different technology vector that will be initially modelled as a random variation of the sector technology vector.

3.1.3 An ABM with Heterogeneous Agents

In this sub-section a new ABM model based on the ABM models described in the previous sub-sections is presented. This new ABM model illustrates the potential of the modelling technique by incorporating novel elements not present in the classical IO model, by allowing the modeller to deviate from the assumptions that the classical input-output model introduce, which were presented already in the previous chapter. This will allow agents, i.e., the com-

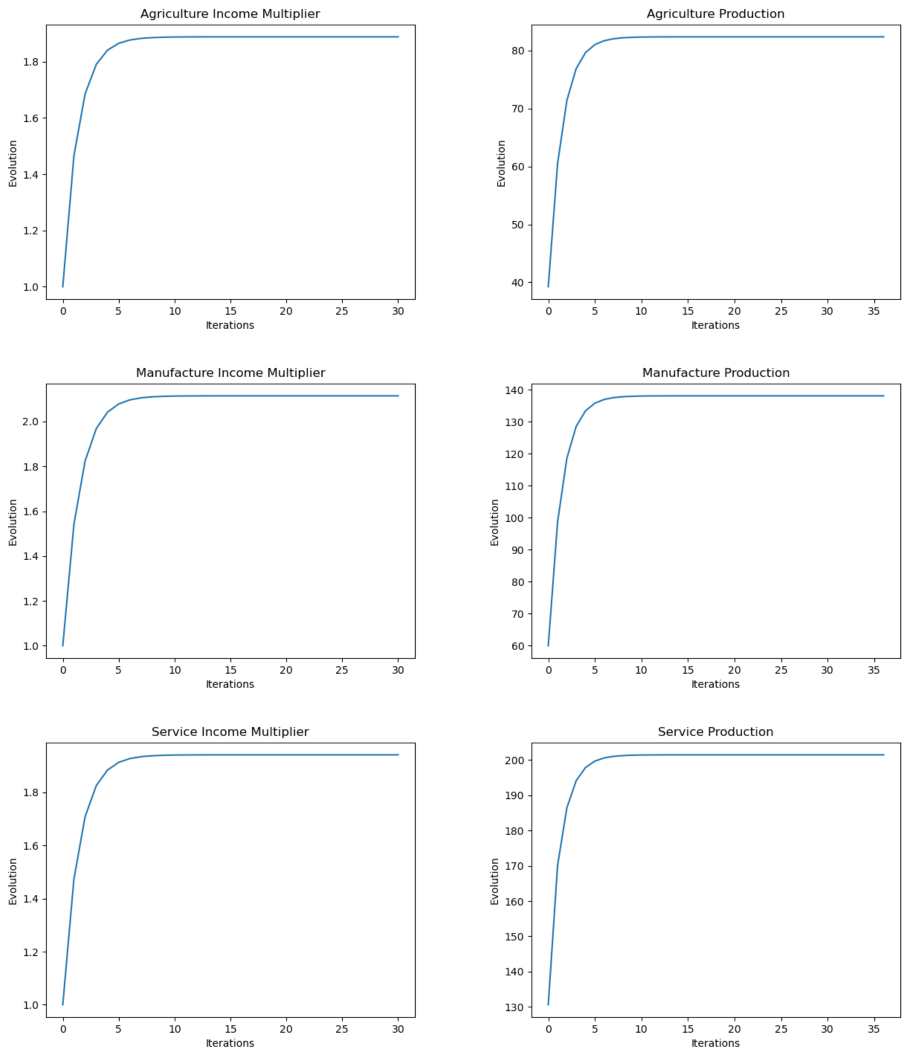


Figure 3.2: Evolution of Multipliers and Production

panies that compose a sector, to exhibit heterogeneous behaviour. The two previous models exactly reproduce the multipliers calculated by the classical IO model. To some extent, this is a validation of the agent-based approach in the sense that it is reasonable to assume that any future model built on such foundations will have consistency with IO results, but at this point in time it is just a conjecture which might require a rigorous proof to transform the conjecture to a fact. However, the inherent flexibility of ABMs provide the modeller with a tool that allows for a richer description of the economic agent's behaviour, which is expected to lead to alternative explanations of the nature of the impact and its measurement.

The new model will be one in which the technology vector will be randomised for each company within the sector. The idea behind this new rule is to see how the multiplier is affected by underlying uncertainty in the technology vector. In order to achieve this objective each agent will no longer follow the technology vector obtained using the IO table, but they will be assigned one that slightly differs from agent to agent.

In the particular example presented on the following sub-subsections, the random differences were modelled as a normal distribution with zero mean and a small standard deviation of δ, with everything else being held constant as in the previous two examples. The model was run 400 times for each value of δ. In the following sub-subsections the resulting multipliers are summarised in terms of their distributional characteristics.

Code similar to that exhibited in appendix D.3 was used, mainly by adding an extra loop and collecting the results at the end of each simulation run.

3.1.3.1 $\delta = 0.00001$

The screen output is given by:

```
INFO:timing:main took 702.5868892669677343750 ms
Agriculture Production, avg. 1.8880253940409104 - stdev.
5.333173893692658e-05
Manufacture Production, avg. 2.114577809527682 - stdev.
4.764213355047026e-05
Service Production, avg. 1.9416738686053463 - stdev.
3.846917354273457e-05
INFO:numexpr.utils:NumExpr defaulting to 8 threads.
```

The histograms for each multiplier is shown in Figure 3.3

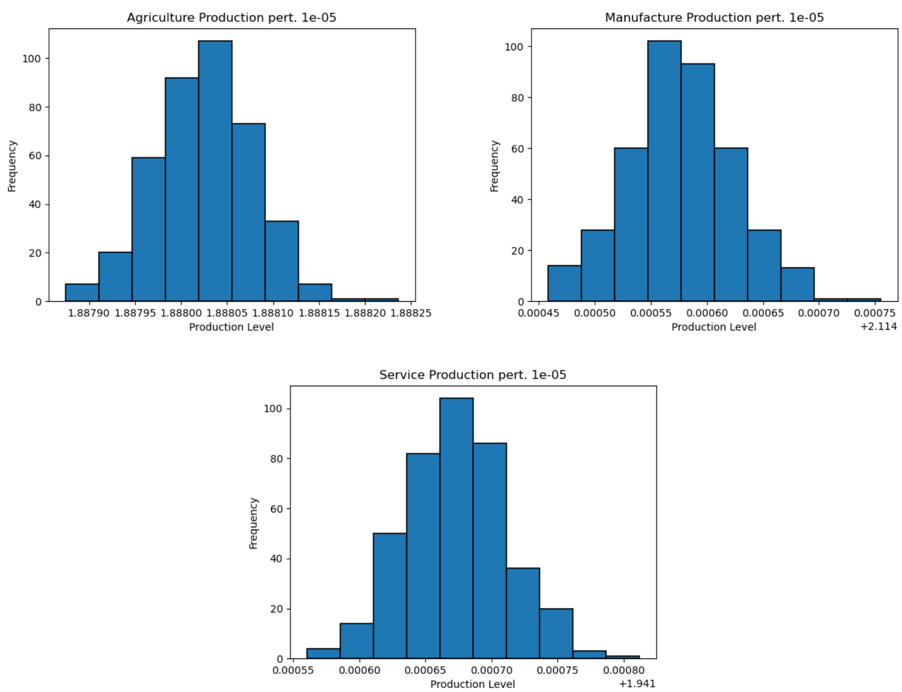


Figure 3.3: Histograms of Multipliers, 400 repeats, $\delta = 0.00001$

3.1.3.2 $\delta = 0.0001$

The screen output is given by:

```
INFO:timing:main took 712.887048721313476562500 ms
Agriculture Production, avg. 1.8880340052598 - stdev. 0.0005116232101791
Manufacture Production, avg. 2.1145578628623 - stdev. 0.0004950125287979
Service Production, avg. 1.94165963316975 - stdev. 0.00042480835056682O5
INFO:numexpr.utils:NumExpr defaulting to 8 threads.
```

The histograms for each multiplier is shown in Figure 3.4

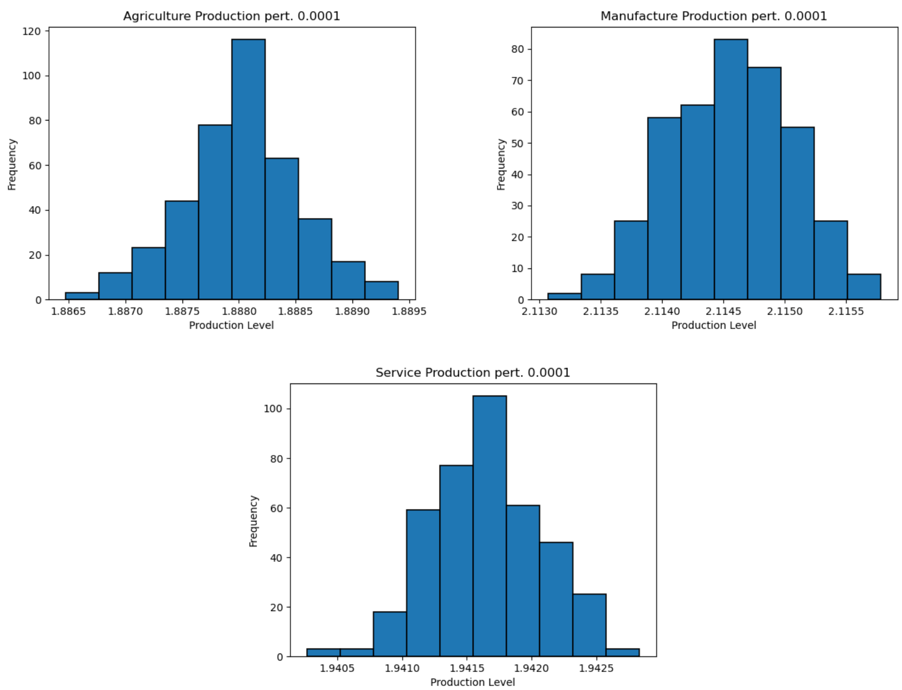


Figure 3.4: Histograms of Multipliers, 400 repeats, $\delta = 0.0001$

3.1.3.3 $\delta = 0.001$

The screen output is given by:

```
INFO:timing:main took 711.313724517822265625000 ms
Agriculture Production, avg. 1.88856572296414 - stdev. 0.00488174227615654
Manufacture Production, avg. 2.114509191213181 - stdev. 0.00449468581756069
Service Production, avg. 1.9415024939116652 - stdev. 0.0037356658672554974
INFO:numexpr.utils:NumExpr defaulting to 8 threads.
```

The histograms for each multiplier is shown in Figure 3.5

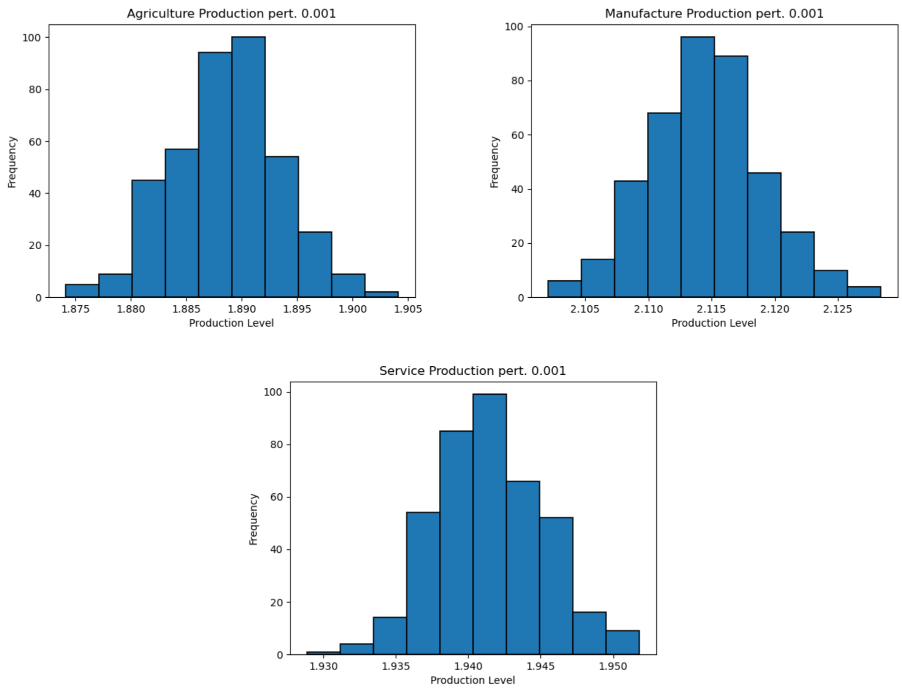


Figure 3.5: Histograms of Multipliers, 400 repeats, $\delta = 0.001$

3.1.3.4 $\delta = 0.01$

The screen output is given by:

```
INFO:timing:main took 709.23948287963867187500 ms
Agriculture Production, avg. 1.884861249647641 - stdev. 0.0485594493775495
Manufacture Production, avg. 2.115226734299216 - stdev. 0.04734767647331945
Service Production, avg. 1.9405253248120726 - stdev. 0.039649963780544466
INFO:numexpr.utils:NumExpr defaulting to 8 threads.
```

The histograms for each multiplier is shown in Figure 3.6

3.1.3.5 $\delta = 0.1$

The screen output is given by:

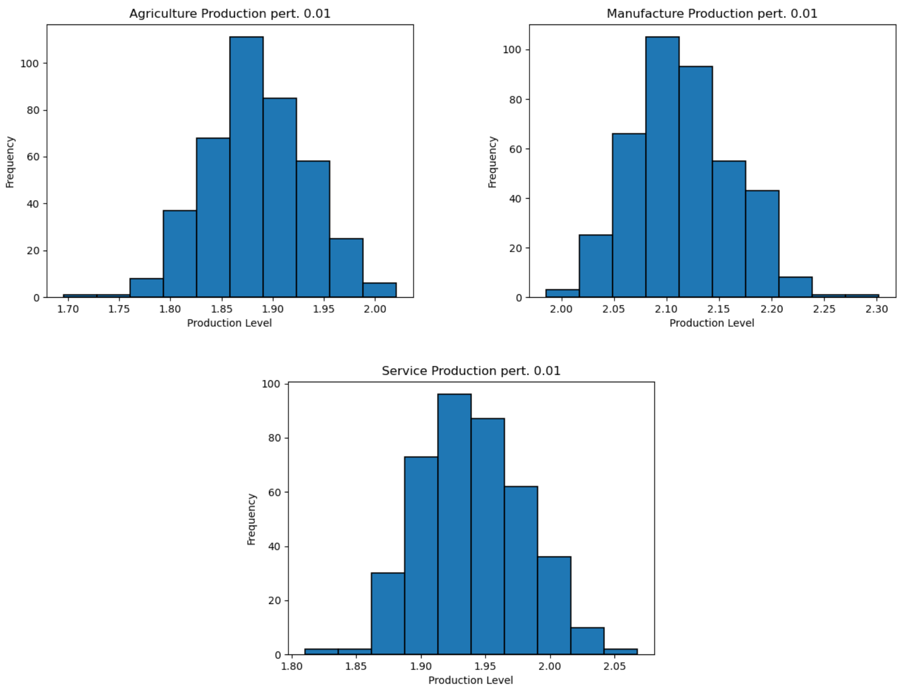


Figure 3.6: Histograms of Multipliers, 400 repeats, $\delta = 0.01$

```
INFO:timing:main took 733.35409164428710937500 ms
Agriculture Production, avg. 1.9965146621970928 - stdev. 0.5578239991008627
Manufacture Production, avg. 2.1823153107935958 - stdev. 0.5631082709116358
Service Production, avg. 1.9953738727597456 - stdev. 0.47273979210240236
INFO:numexpr.utils:NumExpr defaulting to 8 threads.
```

The histograms for each multiplier is shown in Figure 3.7

3.1.3.6 Some Closing Comments

The spread for the multipliers seems to transfer linearly from the technological perturbation to the histograms. Actually, the standard deviation of the histograms is of the same order of the standard deviation used for the perturbation (δ).

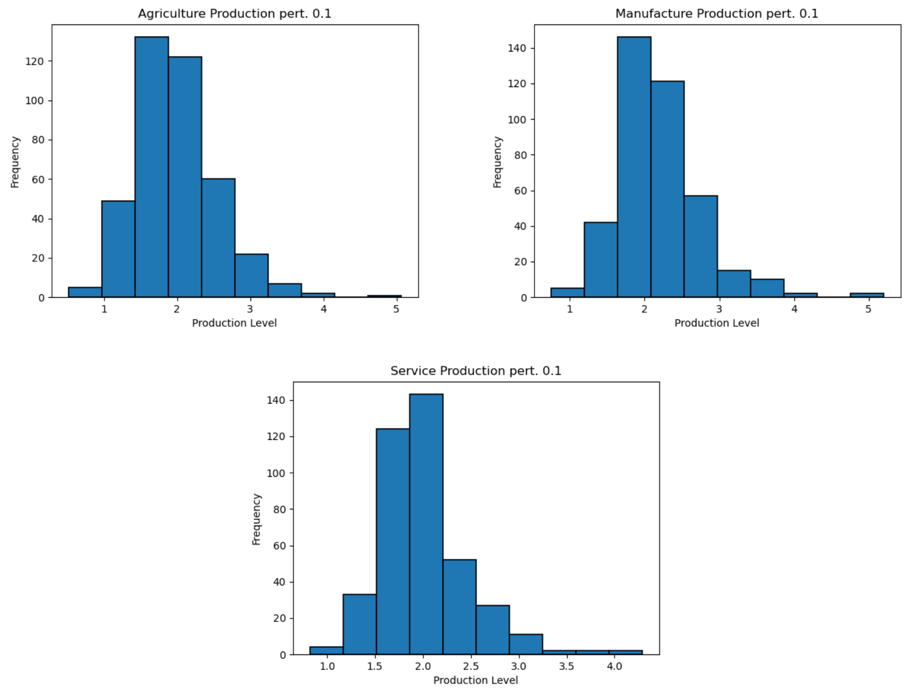


Figure 3.7: Histograms of Multipliers, 400 repeats, $\delta = 0.1$

3.2 Application of the ABM in a Real Case: The Goldfields-Esperance Region

On this section the results obtained with the application of the ABM model are presented in a case with real data. The Goldfields-Esperance region is located in the south-eastern corner of Western Australia and incorporates nine local government areas: the City of Kalgoorlie-Boulder and the Shires of Coolgardie, Dundas, Esperance, Laverton, Leonora, Menzies, Ngaanyatjarraku, and Ravensthorpe. With a land area of 771,276 square kilometres, the region is over three times the size of the state of Victoria and just under a third of Western Australia's total land mass. The region has a population of just under 60,000 people and is comprised of three sub-regions: the Goldfields sub-region, the Esperance sub-region and the Northern Goldfields sub-region (see Figures 3.8

and 3.9).

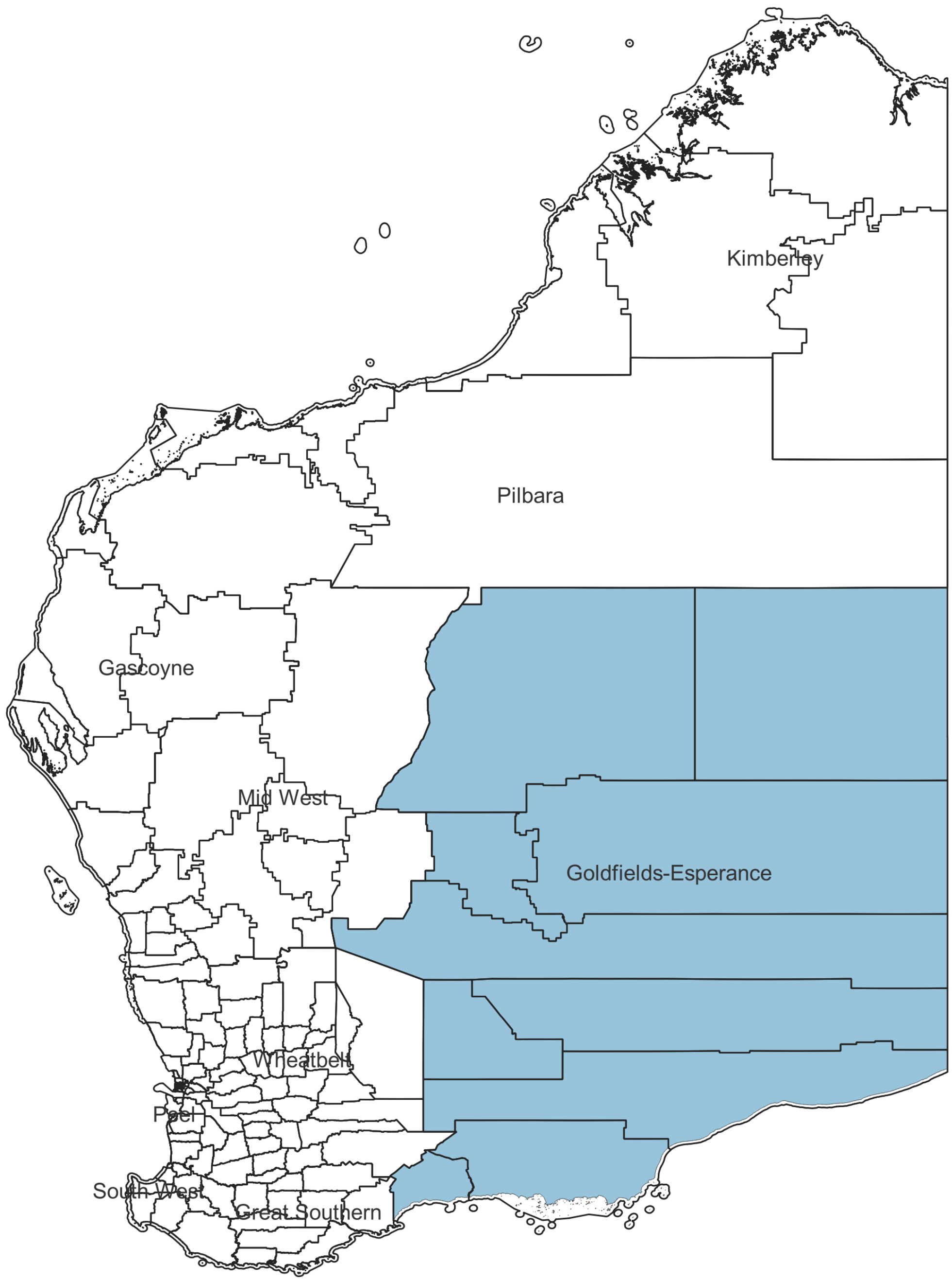


Figure 3.8: The Goldfields-Esperance Region within Western Australia

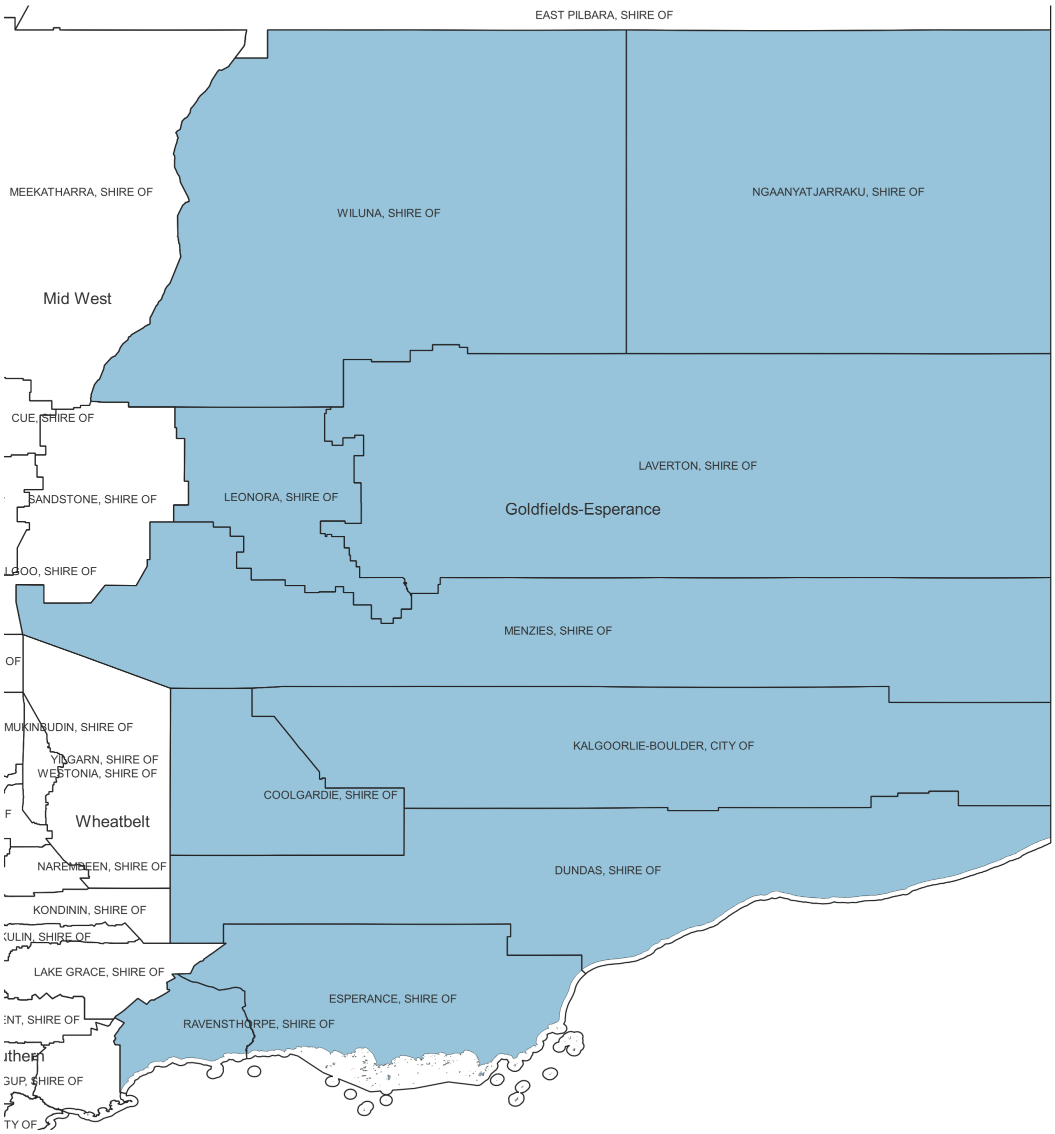


Figure 3.9: The Goldfields-Esperance Region, Shires

Initially a dataset for the region comprising 111 economic sectors was obtained. This dataset was initially used to test the speed of the algorithm and it was found to provide the multipliers after little time (approximately 2 seconds in a machine with an Intel(R) Core(TM) i7-3517U CPU @1.90 GHz 2.40 GHz with 8 GB of RAM memory).

In practical terms it is not easy to understand and visualise the whole dataset as it originally contains too many sectors, for this reason the sectors were aggregated into 35 sectors resulting in Table 3.3.

Table 3.3 just lists the final definition of sectors to avoid an overwhelming number of them. This is done with a view to simplify the analysis process and calculations. For the purposes of the aggregation performed, it was done by considering proximity of economic activities. For instance, the original input-output table contains 111 economic sectors identified within. To illustrate the aggregation process and the rationale behind it, consider for example *Agriculture & Aquaculture* from Table 3.3, this sector has been defined aggregating the following sub-sectors:

1. Sheep, Grains, Beef and Dairy Cattle
2. Poultry and Other Livestock
3. Other Agriculture
4. Aquaculture

There are certain imperfections to this method due to the inability to differentiate some particular items. For example, the input-output matrix contains the item *Agriculture, Forestry and Fishing Support Services* which contains some portion connected to the new *Agriculture & Aquaculture* item, however being unable to exactly determine the proportion it has been decided to leave on its entirety within *Forestry & Fishing*. The information coming from the input-output table is used later to derive a "proxy" production function for

Table 3.3: 35 Sectors Defined for the Case Study

Sector
Agriculture & Aquaculture
Forestry & fishing
Mining
Meat and dairy products
Other food products
Beverages and tobacco products
Textiles
Clothing and footwear
Wood and wood products
Paper and paper products; printing and publishing
Petroleum and coal products
Chemicals
Rubber and plastic products
Non-metallic mineral products
Basic metals and products
Fabricated metal products
Transport equipment
Other machinery and equipment
Miscellaneous manufacturing
Electricity, gas and water
Construction
Wholesale trade
Retail trade
Accommodation, cafes and restaurants
Transport and storage
Communication services
Finance and insurance
Ownership of dwellings
Property and business services
Government administration and defence
Education
Health and community services
Cultural and recreational services
Repairs
Personal and other services

each particular sector, and as we will see, unfortunately this problem admits multiple local solutions if just input-output tables' information is used. However, it has been decided that this particular aspect whilst certainly improvable, should not be an obstacle in the development of the methodology. This particular issue is later approached by choosing a specific starting point for the optimisation process by means of a reproducible process, thus eliminating multiple local optima. In general, one possible way of dealing with this issue is to have a collection of observations of production levels with the intensity of use of sectors required to achieve that production, and "fit" a model (e.g., Econometric approach or similar), but data being unavailable to perform this, we have opted for a simplified approach.

3.2.1 ABM with Homogeneous Agents

A normalised input-output matrix is used for the example, it can be found in appendix D.4. The model was run, using the implementation listed in Algorithm 2, but using a set of homogeneous agents first. Results coming out of the execution of the algorithm were compared with those that can be obtained using the Leontief inverse, as per the usual IO method, they can be seen to be very similar. The numerical comparison is exhibited in Table 3.4.

It is worth to note that "Property and business services" are the biggest multiplier of the list and that "Mining" ranks second closely followed by "Communication services".

Table 3.4: Multipliers obtained using ABM with homogeneous agents

Sector	**Multiplier**
Agriculture & Aquaculture	1.0155528438
Forestry & fishing	1.29474431265
Mining	2.25203669466
Meat and dairy products	1.15759143359
Other food products	1.0654768901
Beverages and tobacco products	1.06805411865
Textiles	1.05422819529
Clothing and footwear	1.02178663129
Wood and wood products	1.11482293631
Paper and paper products; printing and publishing	1.07751270837
Petroleum and coal products	1.16837108137
Chemicals	1.60866022527
Rubber and plastic products	1.26821807085
Non-metallic mineral products	1.26236846259
Basic metals and products	1.1842140247
Fabricated metal products	1.47567774581
Transport equipment	1.1045833924
Other machinery and equipment	1.21615208265
Miscellaneous manufacturing	1.01774050446
Electricity, gas and water	1.62022709113
Construction	1.90871015949
Wholesale trade	2.09300631285
Retail trade	1.45290943332
Accommodation, cafes and restaurants	1.28528685773
Transport and storage	1.77486956549
Communication services	2.18890699531
Finance and insurance	1.97094417559
Ownership of dwellings	1.6759303137
Property and business services	3.53247980631
Government administration and defence	1.23616437395
Education	1.11889307094
Health and community services	1.02421779118
Cultural and recreational services	1.09671151729
Repairs	1.43260390695
Personal and other services	1.03863907645

3.2.2 ABM with Heterogeneous Agents

The same dataset for the Goldfield-Esperance region was used for the ABM, the algorithm used being Algorithm 2 as before, but now allowing for heterogeneous agents (variations of technology vector) for 400 instances of some uncertain data. The results obtained are summarised in Table 3.5.

Recall that the heterogeneity in the agents relates to the technology function used, i.e., random variations of a base technology are considered and assigned to each company[3]. Also, the sizes of the different companies that belong to a sector are different. The idea behind this test is to show, in a very simple way, that the classical results can be extended and thus justify the construction in detail of such a model.

[3]Information regarding technology used by individual companies has not been found so far. It is always possible that such a repository exists, but unfortunately, if it exists, access to it has proved elusive.

Table 3.5: Multipliers obtained using ABM with heterogeneous agents

Sector	**Average Multiplier**	**Standard Deviation**
Agriculture & Aquaculture	1.01555590751	7.64303692631e-05
Forestry & fishing	1.29474113063	7.94547570265e-05
Mining	2.25203220354	8.75679079227e-05
Meat and dairy products	1.1575907762	7.52342742518e-05
Other food products	1.06547674186	7.93493852641e-05
Beverages and tobacco products	1.06805511421	8.10255762992e-05
Textiles	1.0542218692	7.57586403333e-05
Clothing and footwear	1.02179044421	7.4766225669e-05
Wood and wood products	1.11481567139	8.04587952772e-05
Paper and paper products; printing and publishing	1.07750998013	7.50524870433e-05
Petroleum and coal products	1.16837096219	7.91394008769e-05
Chemicals	1.60865514382	9.38626275011e-05
Rubber and plastic products	1.26821612261	8.02014219379e-05
Non-metallic mineral products	1.26236925192	8.32366131364e-05
Basic metals and products	1.18421444308	7.17733520229e-05
Fabricated metal products	1.47567600142	8.64325895361e-05
Transport equipment	1.10458809241	8.30390696937e-05
Other machinery and equipment	1.21615888747	7.93273865888e-05
Miscellaneous manufacturing	1.01773233209	7.22553064834e-05
Electricity, gas and water	1.62022681921	8.28619541955e-05
Construction	1.90871535698	9.01760881461e-05
Wholesale trade	2.09300422871	7.75706554217e-05
Retail trade	1.45290268047	7.73718865345e-05

Table 3.6: Multipliers obtained using ABM with heterogeneous agents (continued)

Sector	**Average Multiplier**	**Standard Deviation**
Accommodation, cafes and restaurants	1.2852923915	8.06948586311e-05
Transport and storage	1.77487064117	7.94753535348e-05
Communication services	2.18891599821	8.48364380996e-05
Finance and insurance	1.97094299431	8.33216352097e-05
Ownership of dwellings	1.67593956175	9.07684192042e-05
Property and business services	3.53248294472	0.000100298143415
Government administration and defence	1.23616005632	8.12260565615e-05
Education	1.11889619043	7.66589508017e-05
Health and community services	1.0242146233	7.79333532798e-05
Cultural and recreational services	1.09671516361	7.74770074584e-05
Repairs	1.4326032865	7.81341695008e-05
Personal and other services	1.03864150208	7.56294853778e-05

For the purposes of illustrating the flexibility that the modelling tool provides, company 1 of sector 2 was chosen, some repetitions were run (20), its multipliers were calculated and their evolution plotted as it can be seen in Figure 3.10 (perturbation parameter used $\delta = 0.0009$).

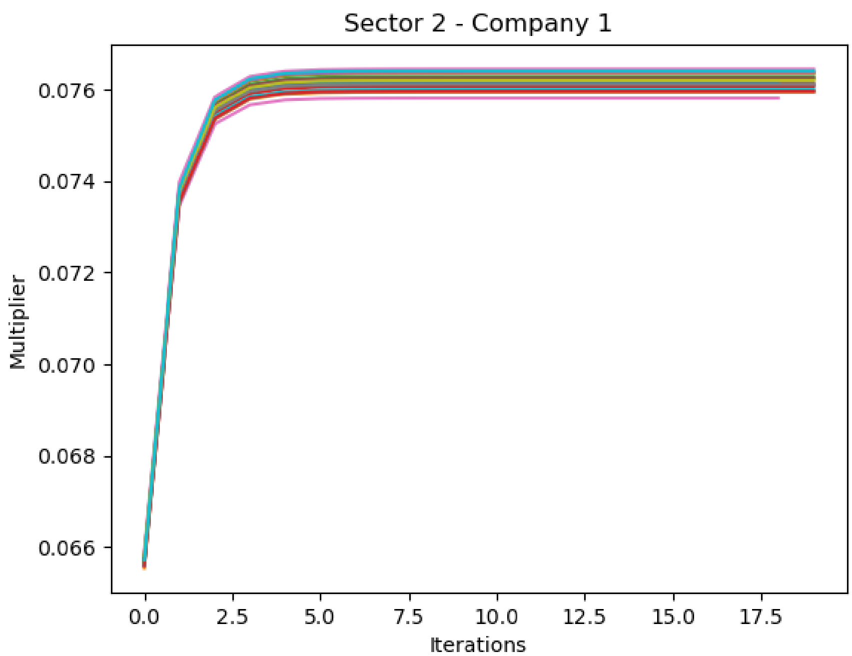


Figure 3.10: Evolution Multiplier Company 1, Sector 2

It can be seen that as the algorithm progresses the multiplier calculated converges to a value up to the point where no further changes are observed for each repetition, however, the final level depends on the iteration being run due to the technological perturbation. Being only one company of the whole sector, it is natural that the multiplier is just a fraction of the overall multiplier for the sector. On the other hand, another less relevant agent (smaller in size) was also selected to see how the multiplier evolves in this case. As the iterations progress, the evolution of the multipliers exhibit similar behaviour to company 1 chosen previously, i.e., exhibiting convergence, but with obvious differences in scale relating to their size. The evolution on the calculation of this multiplier is shown in Figure 3.11.

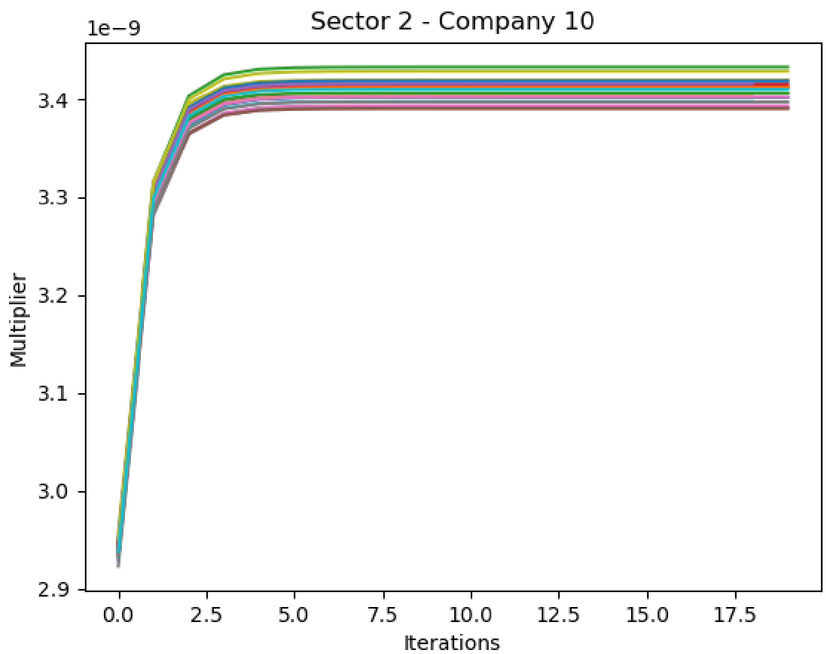


Figure 3.11: Evolution Multiplier Company 10, Sector 2

3.3 Chapter Concluding Remarks

The chapter has presented a critique of the current tools used for measuring impact. It first focused on a descriptive analysis of the tools currently in use to make the argument of the need of new tools to accomplish the same objective. A novel methodology was proposed that was shown to not only replicate classical results, but is able to extend them by allowing flexibility in the agent's behaviour.

The ABM model presented so far is simple but by no means useless. It is capable of replicating the multipliers that classical theory predicts but with an extra value: they can extend the classical theory to incorporate other behaviours of the agents. It also illustrates the benefits that new modelling such as ABM brings into the modeller's toolbox. For example, the fact that ABM models allow for finer level of detail in the modelling with no substantial extra cost (except that of getting the information) is something that needs to be highlighted. The proposed modelling technique moves away from classical paradigms and has the potential to add to the existing corpus of techniques.

As shown in the examples provided, the fact that a particular company can be "followed" allows to measure the real impact of that particular company in the economy. In times where major mining players are facing closure of operations or will in the near future, it is important to have a methodology that allows prediction of the consequences. That way the local governments can react to the negative consequences of those closures.

One of the main identified benefits of the use of CGE models is that of breaking the rigid nature of Input-Output models. The ABM model presented in this chapter has the potential to accomplish a similar function as CGE models and more. For example, it is not difficult to envision that the usual dynamics introduced by CGE models can be modelled in a discretised fashion within the framework of ABM. However, the additional ability of ABM to introduce local models of interactions between agents and the environment, and to introduce heterogeneity in their behaviour, certainly has the potential to further extend CGE. This chapter has shown that an ABM can replicate the classical multipliers approach, and also can extend it. On the basis of their designed flexibility and the fact that the point of view for ABM is that of building an economy from the bottom-up, as opposed to the still prevalent top-down view of CGE models, encourages and justifies the development of an ABM to attempt to improve on the classical tools.

One possible addition to the model is to incorporate some rationality in the decisions of the agents. In the model shown in previous sections, the agents are assumed to be described by a technological vector that details which inputs from every sector are required to produce units of products/services. However, that mix depends on the relative prices of inputs and the current modelling does not reflect this. It is believed that differences in relative prices, in some cases substitution effects, and ultimately changes in preferences, will lead to a different description of the evolution of impact over time.

As the modelling introduced in this chapter has shown, it is possible to model uncertainty into the agents' behaviour. The attempt introduced in this chapter modelled uncertainty in the technology vector used by each agent, but it is believed that things can go further as the technology vector should be a consequence of available technology and relative prices which are essentially uncertain. Related to this point, it is interesting to note that innovation processes could be modelled using agents as individual companies within sectors, as done in this chapter. There are also some agent-based models that model spread of ideas, and it is believed that similar modelling techniques could be used to model technology adoption within sectors based on individual companies. This effect will impact the technology vector used and consequently add dynamism to the evolution of the agents in the model.

As seen, the use of ABM in Input-Output analyses provides a tool that not only replicates what current IO analysis achieves but that has the potential to do much more. It is believed that increases in computational power coupled with the flexibility that the ABM paradigm provides, offer this technique as a welcomed addition to the toolbox of every researcher interested in measuring economic impact. In particular it is believed that due to the dynamic nature of mining, these techniques are particularly suited to answer questions relevant to the sector and the state economy.

Chapter 4

Proposed Model, Implementation and Validation

This chapter introduces the elements that characterise each of the agents involved in the model introduced in a very simplistic manner in Chapter 3, further extending that simple model to a more complex overall modelling effort. The present chapter is a mix of mathematical models and also provides pointers to the code implementation (which can be found in the appendix). It discusses the rationale behind the proposal of the behaviour of the agents in the proposed model and also their implementation. In the final section of the current chapter the proposed implementation is validated and the correctness of the implementation checked. Also, contained in the same section some stress testing is performed to assess the limits of the implementation.

The proposed model is comprised mainly of three classes of agents: `Households`, `Firms` and `Market`. Every agent behaves following its own local logic relative to the state of the system at discrete points in time. This last modularisation has been adopted to use existing simulation libraries, thus avoiding the complexity of implementing a specific world-view for the simulation process. The atomic treatment by means of software components of the agents, provides a layer of abstraction that enables certain independence in the treatment of them. The agents certainly vary in their complexity, with the `Market` agent being by far the one that encapsulates the most complex behaviour. The `Market` agent acts as coordinator for all other agents and is in charge of "deciding" who buys from whom and how the money and goods flow between agents.

It is important to point out that where possible, classical microeconomic models have been adopted with no or little variations for implementing some of the behavioural aspects of the agents. To illustrate the nature relating to the previous modelling decision, we can observe that on each iteration of the algorithm a `Household` agent must:

- Decide what to buy from every economic sector
  - Maximising an utility function (e.g., Cobb-Douglas)
- Proceed to "pass-on" messages to actuate the transactions
- Update available resources accordingly

Similar decisions were taken for the `Firm` agent with some small variation with respect to the classical profit maximisation problem that every producing agent faces, the most important one being the addition of a budget constraint. In the case of the `Market` model, the solution proposed utilises classical assignment models in a series of rounds to simulate the way market transactions are performed but within the constraints of a discrete event simulation model. These individual decision elements can be naturally changed for others. However, in this thesis, the focus is the overall methodology of using agent-based models for the problem at hand rather than spending too much time on particular functional forms for these decision elements.

This chapter is essential to the understanding of the proposed model, the way it was implemented and how it attempts to integrate all the heterogeneous elements within a single simulation framework. This chapter links with the next in line that will look at validating and checking the consistency of the implementation, after which the model is deemed usable for performing numerical experiments.

4.1 Household Agent

A Household represents a group of people living under the same roof. Individual household compositions will provide different preferences for products and services. One way to represent those preferences is through the use of a utility function. There are different types of utility functions: Cobb-Douglas, Leontief, Linear, Additive, and many others. A more extensive description of utility functions can be found in Appendix A. In this work, and in accordance with the way economic subjects are usually thought, we assume that that the utility functions are all of the Cobb-Douglas type, i.e., the utility function for household i can be described by:

$$u_i(\vec{x}_i) = \prod_{k=1}^{K} x_{i,k}^{\beta_k} \tag{4.1}$$

Note that the chosen functional form for the utility function changes the specifics of the calculations, nevertheless, the use of utility functions remains unchanged.

In Equation 4.1, the preferences are expressed for any of the K products, with β_k being the coefficient that represents this preference and $x_{i,k}$ the consumption level of good k for household i. An additional constraint given by:

$$\sum_{k=1}^{K} \beta_k = 1$$

can be considered, which essentially normalises the values. The following is observed:

- $\beta_k \geqslant 0 \ \ \forall k \in \{1, \ldots, K\}$
- If for some k^* we have that $\beta_{k^*} = 0$ then consumption level for that item can be zero as zero to the power of zero is defined to be 1. Thus, the smaller the β_k the less important the good is (i.e., it provides less utility)
- Conversely, if for some k^* we have that $\beta_{k^*} = 1$, then due to the constraint $\sum_{k=1}^{K} \beta_k = 1$ we know that $\beta_k = 0 \ \forall k \in \{1, \ldots, K\} \setminus \{k^*\}$, and this will imply that only one item is interesting for the household

The existence of a utility function for a household naturally calls for the provision of a rationale to assign a constrained budget on every period. This is classically done for consumers (households are consumers) by solving an optimisation problem, of the form described in Equations (4.2a - 4.2c), for household i:

$$(P_i) \quad \max_{\{x_{i,k}\}_{k=1}^{K}} \quad \prod_{k=1}^{K} x_{i,k}^{\beta_k} \tag{4.2a}$$

s.t.

$$\sum_{k=1}^{K} p_k \cdot x_{i,k} \leqslant B_i \tag{4.2b}$$

$$x_{i,k} \geqslant 0 \ \ \forall k \in \{1, \dots, K\} \tag{4.2c}$$

Where:

- i is used to represent the i-th household
- P_i is a label denoting the optimisation problem associated to household i
- K is a variable indicating the number of sectors, with k represting a specific sector ($k \in \{1, \dots, K\}$)
- p_k is the price of the good sold in sector ($k \in \{1, \dots, K\}$)
- B_i is the available budget for household i

Equation (4.2a) represents the objective function to be maximised, which corresponds to the Cobb-Douglas utility function. Equation (4.2b) expresses that a household cannot spend more money than available. Depending on the economic sectors considered, it is always possible to include savings and credits as part of the goods that a household can consume, being p_k the price of good k. The left hand side of Equation 4.2b represents the total expenditure, thus effectively representing the record-keeping of expenditures and ensuring that it is not bigger than the household budget (B_i). Finally Equation (4.2c) represents the nature of the decision variables from a theoretical point of view; positive with no upper bounds. However, additional upper bounds could be introduced to ensure that demand is within existing supply. Nevertheless, this option has not been considered as the solution of model (P_i) will be used later as an input of the optimal/desired consumption level of each household but after an allocation model is solved, it will be finally determined how much of each good each household receives.

The budget at iteration t, B_i^t is affected by two basic operations:

1. Payment of bills: This operation basically affects budget in the following way $B_i^t \leftarrow B_i^{t-1} - b^{t-1}$, with b^{t-1} the bills that need payment in period $t-1$.

2. Obtaining salary: This operation basically affects budget in the following way $B_i^t \leftarrow B_i^{t-1} + s^{t-1}$, with s^{t-1} the salary of the household in period $t-1$.

Thus, the household agent needs to have at least those two functionalities implemented. We observe that these two methods actually act in conjunction to provide the balance (or budget) for any given period t:

$$B_i^t \leftarrow B_i^{t-1} - b^{t-1} + s^t$$

Additionally, a household agent could be initialised to have a starting capital s_0, but this can be optional and essentially will depend on how the simulation proceeds in the early stages before reaching some level of stationarity.

All the previous discussion can be better visualised in the diagram contained in Figure (4.1).

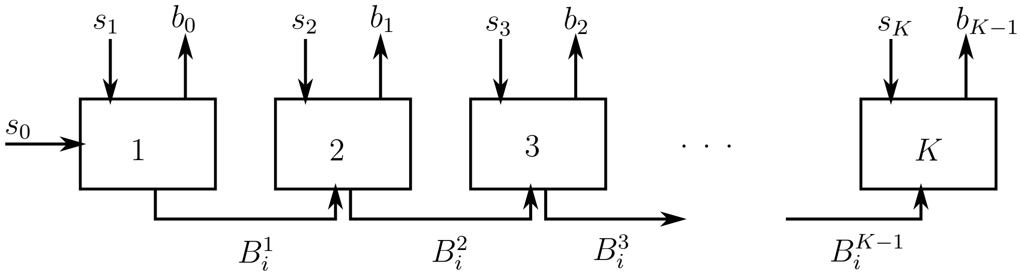


Figure 4.1: Pictorial Representation of the Money Flows from a Household Point of View

4.2 Firm Agent

A firm can be classically described as possessing a production function that describes the quantity of a product to be produced, based on availability of production factors (raw materials, capital, labour, and so forth). There are several types of production functions: Cobb-Douglas, Leontief, CES, along with others. Production functions follow a rationale similar to that followed by utility functions, and in fact their functional form looks very similar. However, there are some small differences due to the nature of the different problems that consumers and producers solve. A more extensive description of production functions can be found in any economics textbook (Samuelson and Nordhaus, 1998; Krugman and Wells, 2005; Varian, 2010). For the purposes of consistency with commonly taught economics, we will assume in the successive that the production functions are all of the Cobb-Douglas type, i.e., the production function for firm j can be described by:

$$q_j(\vec{y}) = \alpha_0 \cdot \prod_{k=1}^{K} y_{j,k}^{\alpha_k} \tag{4.3}$$

In Equation 4.3, the efficiencies in the use of productivity factors is expressed for any of the K sectors, which for the modelling being undertaken in this thesis will be assumed as one productivity factor per sector (i.e., there are $|K|$ productivity factors, also called “raw materials”), with α_k being the coefficient that represents this efficiency, α_0 a parameter called total factor productivity and $y_{j,k}$ the consumption level of productivity factor k. Sometimes an additional constraint is used, that essentially normalises the values:

$$\sum_{k=1}^{K} \alpha_k = 1$$

However, there is no imposition to do so. Similar observations to those made in Subsection 4.1 regarding the size of α_k can be made.

The existence of a production function for a firm naturally calls for the provision of a rationale to define how much product the firm should produce given its efficiency in the use of factors. This is classically done for firms by solving an optimisation problem over its utility (with the problem labelled π_j); as can be seen in Equations (4.4a, 4.4b) for firm j:

$$(\pi_j) \quad \max_{\{y_{j,k}\}_{k=1}^{K}} \quad p \cdot \alpha_0 \cdot \prod_{k=1}^{K} y_k^{\alpha_k} - \sum_{k=1}^{K} p_k \cdot y_{j,k} \tag{4.4a}$$

s.t.

$$y_{j,k} \geqslant 0 \ \ \forall k \in \{1, \dots, K\} \tag{4.4b}$$

where p represents the final good price and p_k the prices of productivity factors. Equation (4.4a) represents the objective function (profit maximisation) and Equation (4.4b) represents the nature of the decision variables.

This problem is considered to be the ideal problem to solve, because there are no budgetary constraints associated with the profit maximisation objective. However, it has been deemed than in the context of repeated interactions of the agents, the lack of a budget constraint seems unrealistic, hence, in the context of the proposed modelling, an additional constraint will be added to the problem, thus transforming the original model formulation (π_j) to that contained in Equations (4.5a - 4.5c) $(\tilde{\pi}_j)$:

$$(\tilde{\pi}_j) \quad \max_{\{y_{j,k}\}_{k=1}^{K}} \quad p \cdot \alpha_0 \cdot \prod_{k=1}^{K} y_{j,k}^{\alpha_k} - \sum_{k=1}^{K} p_k \cdot y_{j,k} \tag{4.5a}$$

s.t.

$$\sum_{k=1}^{K} p_k \cdot y_{j,k} \leqslant M_j \tag{4.5b}$$

$$y_{j,k} \geqslant 0 \;\; \forall k \in \{1, \ldots, K\} \tag{4.5c}$$

The budget at iteration t, M_j^t is affected by two basic operations:

1. Payment for inputs: This operation basically affects budget in the following way $M_j^t \leftarrow M_j^{t-1} - b^{t-1}$, with b^{t-1} the bills that need payment in period $t-1$.

2. Obtaining income from sales: This operation basically affects budget in the following way $M_j^t \leftarrow M_j^{t-1} + s^{t-1}$, with s^{t-1} the sales of firm j in period $t-1$.

Thus, the firm agent needs to have at least these two update mechanisms implemented. Observe that these two methods actually act in conjunction to provide the balance (or budget) for any given period t:

$$M_j^t \leftarrow M_j^{t-1} - b^{t-1} + s^t$$

Similar to the household agent, a firm agent could be initialised to have a starting capital s_0, but this is optional.

4.3 Economic Sector and Economy Agents

In order to simplify the presentation, and to not complicate things unnecessarily, it will be assumed that each economic sector produces only two goods; a “retail” good that is bought by consumers and a “raw” good. This means that the firms that operate in the sector only produce those two goods in different quantities. The pricing will be done in a way that simplifies things and essentially will leave every producer fixing their own retail prices with the raw price being a percentage of the average retail price. This percentage will be an attribute of every economic sector. Also, it is important to note that to further simplify the problem, the raw material/producer services market will be assumed unbounded in the sense that we will assume that it will be able to respond to any demand for good/services from producers to produce their final goods.

In the Market Agent model (see Subsection 4.4), a mechanism using repeated assignments will be used and will implicitly help to determine a price evolution.

4.4 Market Agent

The market agent plays a central role in the proposed model. By using the market, different agents can exchange money for different types of goods. A market has essentially two main participants: offerers and buyers. There are different possible ways that markets use to operate, some of them reviewed in Appendix B, which serve to illustrate the type of considerations and difficulties found when implementing a market. Despite the different mechanisms involved, it has been decided that a centralised market will be used to assign goods from producers to consumers. It is known that markets are rarely centralised, however, using this one single mechanism simplifies the implementation of the model, and for this reason is the choice selected for this work. If the impact of market mechanisms into the economy were the focus of this thesis, some reasonable rationale for a market behaviour should be a must. However, the proposed mechanism is considered the effective compromise between level of detail and complexity for this thesis.

Figure (4.2) presents a diagrammatic representation of the problem that needs to be solved at the market level. We can see that the market primarily needs to link households to firms. A secondary model of a similar nature links firms to firms in a different market (called the raw materials market). In this figure, we can identify different players such as households (or consumers) and firms (or producers).

Also, each consumer is characterised by an available budget in every period (B_i^t) and a desired consumption vector $\vec{d_i^t} \in I\!R^K$, where every component k expresses how many units of product from sector k the household desires (this is performed by solving the utility maximisation problem for average market prices). On the other side of the diagram, each firm is characterised by a supply vector $\vec{s_j^t} \in I\!R^2$ (first coordinate for retail good supply and second

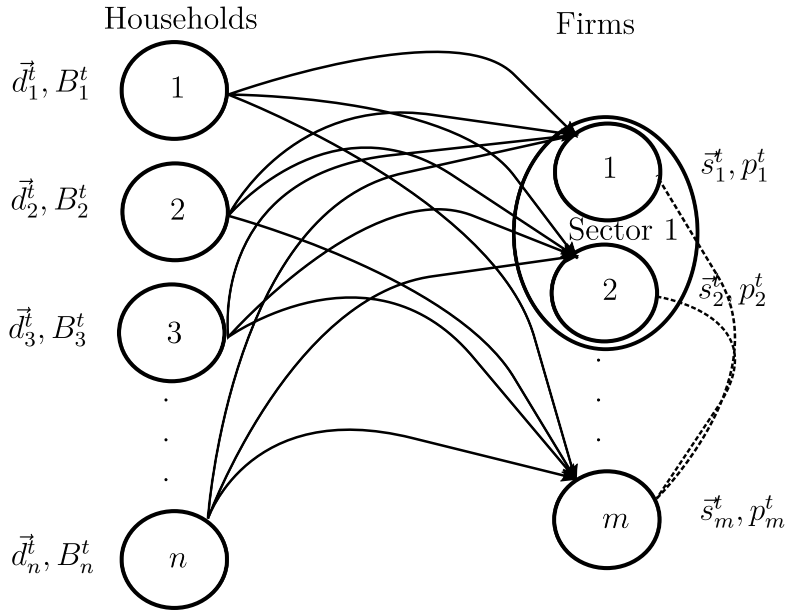


Figure 4.2: Market Model Diagram

coordinate for raw good supply) and a retail price p_j^t with the additional simplifying assumption that the raw price will be a percentage of the retail price. As discussed in Subsection (4.3), this percentage will change from sector to sector and is a property of the economic sector.

The first assignation model that can be proposed relates to the consumer assignation model, where an assignment is pursued in such a way as to maximise a notion of collective utility for the agents (in this case households). If we denote by $X_{i,j,k}^t$ the variable that defines how much of good k is bought from producer j by consumer i at time t, defining also the following sets:

- I: The set of households (possibly the set $\{1, \dots, n\}$)
- J: The set of firms (possibly the set $\{1, \dots, m\}$)
- $J(k)$: The subset of firms restricted to sector k (i.e., all the firms that

belong to sector k)

- K: The set of sectors (possibly the set $\{1, \ldots, r\}$)

Then the first model that comes to mind to produce an assignation for period t has the following form:

$$P(t) \quad \max_{\{X^t_{i,j,k}\}_{i\in I, j\in J, k\in K}} \quad \mathcal{U}(\{X^t_{i,j,k}\}_{i\in I, j\in J, k\in K}) \tag{4.6a}$$

s.t.

$$\sum_{j\in J(k)} X^t_{i,j,k} \leqslant \vec{d}^t_i|_k \;\; \forall i \in I, k \in K \tag{4.6b}$$

$$\sum_{j\in J} \sum_{k\in K} p^t_j \cdot X^t_{i,j,k} \leqslant B^t_i \;\; \forall i \in I \tag{4.6c}$$

$$\sum_{i\in I} \sum_{k\in K} X^t_{i,j,k} \leqslant \vec{s}^t_i|_1 \;\; \forall j \in J \tag{4.6d}$$

$$X^t_{i,j,k} \geqslant 0 \;\; \forall i \in I, j \in J, k \in K \tag{4.6e}$$

For this problem, Equation (4.6a) represents the objective function of an unclear form that motivates some further discussion. Equation (4.6b) indicates that demand for a good k cannot be exceeded for household i. In Equations (4.6b) and (4.6d), the projection $|_k$ or $|_1$ has been used to indicate component k or component 1 of the respective vectors to which the operation has been applied. Equation (4.6c) indicates that the budget of consumer i cannot be exceeded. Equation (4.6d) is used to impose a limitation on the amount of goods a producer can sell (limiting it to its declared supply for the period) and Equation (4.6e) is the nature of the variables where implicitly it is assumed that the goods are infinitely divisible. In the case where they are not, then the problem becomes a Mixed Integer Program which takes longer to solve.

In the previous formulation, in Equation (4.6a), the use of a global utility function $\mathcal{U}(\cdot)$ has not been made explicit, which prompts the question: Which function form is this? What can be said about it? Individual utility functions for every agent are non-linear, as all of them have the form $u_i(\vec{x}) = \prod_{k=1}^{K} x_k^{\beta_k}$. One possible temptation here is to linearise using logarithms, i.e., transform the utility function to $\ln(u_i(\vec{x}))$ given by:

$$\ln(u_i(\vec{x})) = \sum_{k=1}^{K} \beta_k \ln(x_k)$$

But unfortunately, this does not seem a good path as the constraints are all in other variables ($X_{i,j,k}^t$), thus producing a mismatch between the objective functions and constraints.

Instead of using the utility function as it comes, a new idea has come to light consisting the use of the newly introduced concept of "relative gain" of buying from producer j with respect to the publicly informed average market price. As far as known, this concept has not been used in the context of such modelling. However, the rationale for it comes from the usual analysis of financial time series, where instead of focusing on the overall level of a share price, the focus is usually centred on the return, or most commonly on the relative return. To illustrate the concept, consider the hypothetical case where the price of a given good as offered by agent j at time t (p_j^t) is smaller than the average market price ($\bar{p}_k^t$). It can be observed as the household considers the average market price for the good as the "available" information, then the opportunity of buying from producer j at a cheaper price will introduce savings of ($\bar{p}_k^t - p_j^t$) with respect to the "publicly" available price at time t. The amount of goods the consumer can buy is limited by either his own demand or by the quantity offered by the producer. In any case, by buying as much as needed/possible from this producer will introduce savings with respect the original price expectations (the average market price). This illustrates a case of positive relative gain[1].

Now, in case that $\bar{p}_k^t < p_j^t$, the following can be said:

1. The original resource allocation destined for buying good k by household i is $\vec{d}_i^t|_k \cdot \bar{p}_k^t$

2. At the "higher" price of p_j^t

   $$\frac{\vec{d}_i^t|_k \cdot \bar{p}_k^t}{p_j^t}$$

   units can be bought (as the budget remains the same for the current price less units can be bought)

3. The difference in utilities in this new scenario can be calculated as:

   $$\begin{aligned}
   \text{(old) } u_i(\vec{d}_i^t) &= \prod_{\substack{r \in K \\ r \neq k}} (\vec{d}_i^t|_r)^{\beta_r^i} \cdot (\vec{d}_i^t|_k)^{\beta_k^i} \\
   \text{(new) } \underline{u}_i(\vec{d}_i^t) &= \prod_{\substack{r \in K \\ r \neq k}} (\vec{d}_i^t|_r)^{\beta_r^i} \cdot (\frac{\vec{d}_i^t|_k \cdot \bar{p}_k^t}{p_j^t})^{\beta_k^i}
   \end{aligned}$$

   and by taking the difference $(\underline{u}_i(\vec{d}_i^t) - u_i(\vec{d}_i^t))$ the following is obtained:

$$(\underline{u}_i(\vec{d}_i^t) - u_i(\vec{d}_i^t)) = \underbrace{\prod_{\substack{r \in K \\ r \neq k}} (\vec{d}_i^t|_r)^{\beta_r^i} \cdot (\vec{d}_i^t|_k)^{\beta_k^i}}_{u_i(\vec{d}_i^t)} \overbrace{\left(\left(\frac{\bar{p}_k^t}{p_j^t} \right)^{\beta_k^i} - 1 \right)}^{\gamma_{i,j,k}} \tag{4.7}$$

[1]It might not look that the "relative gain" concept is a great intellectual contribution, but for the particular problem at hand was the key concept that enabled appropriate discrimination of supplier options

4. We observe that supplying good k at the new price p_j^t decreases utility by a factor of:

$$\gamma_{i,j,k} = \left(\left(\frac{\bar{p}_k^t}{p_j}\right)^{\beta_k^i} - 1\right)$$

5. We observe the following:

   - $\gamma_{i,j,k} \in I\!R^-$ if $\bar{p}_k^t < p_j^t$
   - $\gamma_{i,j,k} \in I\!R^+$ if $\bar{p}_k^t > p_j^t$

6. The same logic shown before, which uses the average price of the good offered by sector k at time t, denoted by $\bar{p}_k^t$ can be adjusted to make it relative to an individual household by replacing the term $\bar{p}_k^t$ by $p_{i,k}$ defined as the willingness of household i to pay for the good of sector k

   - This allows the implementation of an iterative scheme that adjusts willingness to pay as the turns progress based on the relative abundance/scarcity of the different goods (to be seen later in the document)

7. By defining a modified $\gamma_{i,j,k}$ version as:

$$\Gamma_{i,j,k}^t = \left(\frac{p_{i,k}^t}{p_j^t}\right)^{\beta_k^i} - 1$$

   where $p_{i,k}^t$ is the willingness to pay for product of sector k by household i and p_j^t is the price that firm j charges for their product at time t.

By adopting $\Gamma_{i,j,k}^t$ as a measure of the appreciation or loss of utility under variations of willingness to pay, as the price that producer j pays for product k, we can now formulate problem $P(t)$ as in Equations (4.8a - 4.8e), which is a linear programming model:

$$P(t) \quad \max_{\{X^t_{i,j,k}\}_{i\in I, j\in J, k\in K}} \quad \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\Gamma^t_{i,j,k}\cdot X^t_{i,j,k} \tag{4.8a}$$

s.t.

$$\sum_{j\in J(k)} X^t_{i,j,k} \leqslant \vec{d}^t_i|_k \;\; \forall i \in I, k \in K \tag{4.8b}$$

$$\sum_{j\in J}\sum_{k\in K} p^t_j \cdot X^t_{i,j,k} \leqslant B^t_i \;\; \forall i \in I \tag{4.8c}$$

$$\sum_{i\in I}\sum_{k\in K} X^t_{i,j,k} \leqslant \vec{s}^t_j|_1 \;\; \forall j \in J \tag{4.8d}$$

$$X^t_{i,j,k} \geqslant 0 \;\; \forall i \in I, j \in J, k \in K \tag{4.8e}$$

The problem formulation contained in Equations (4.8a - 4.8e) needs to be run several times for each period t. If it is run only once, then it does not guarantee that everyone will get assigned goods. For instance, if $\Gamma^t_{i,j,k} < 0$ then the trivial solution for the corresponding variable $X^t_{i,j,k}$ is to be zero.

From here it is observed that the model $P(t)$ will need to be used in a series of rounds, or tournaments, where the best possible assignment is performed and the prices for both the buyers and producers need to be modified to reach a deal. With this idea in mind, we reformulate $P(t)$ to depend on a sub-iteration $P_r(t)$. The reformulation can be found in the Model described by Equations (4.9a - 4.9e), and the corresponding algorithm making use of such model can be found in Algorithm (3).

$$P_r(t) \quad \max_{\{X^t_{i,j,k,r}\}_{i\in I, j\in J, k\in K}} \quad \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\Gamma^t_{i,j,k,r}\cdot X^t_{i,j,k,r} \tag{4.9a}$$

s.t.

$$\sum_{j\in J(k)} X^t_{i,j,k,r} \leqslant \vec{d}^t_{i,r}|_k \ \ \forall i\in I, k\in K \tag{4.9b}$$

$$\sum_{j\in J}\sum_{k\in K} p^t_{j,r}\cdot X^t_{i,j,k,r} \leqslant B^t_{i,r} \ \ \forall i\in I \tag{4.9c}$$

$$\sum_{i\in I}\sum_{k\in K} X^t_{i,j,k,r} \leqslant \vec{s}^t_{j,r}|_1 \ \ \forall j\in J \tag{4.9d}$$

$$X^t_{i,j,k,r} \geqslant 0 \ \ \forall i\in I, j\in J, k\in K \tag{4.9e}$$

where some of the quantities are updated as follows:

- $\Gamma^t_{i,j,k,r}$[2]

  should be calculated as:

  $$\Gamma^t_{i,j,k,r} = \left(\frac{p^t_{i,k,r}}{p^t_{j,r}}\right)^{\beta^i_k} - 1$$

  where $p^t_{i,k,r}$ is the willingness to pay for product of sector k by household i during round r of turn t, and $p^t_{j,r}$ is the price that firm j charges for their product in round r of turn t

- $p^t_{i,k,r}$ can be updated as $p^t_{i,k,r} = p^t_{i,k,r-1} + \delta$, with $\delta > 0$ a small quantity (parameter), another possibility is to use $p^t_{i,k,r} = p^t_{i,k,r-1}\cdot\delta$, with $\delta > 1$

[2] *A historical footnote:* During some stage of development of this model, a unnoticed factorisation error lead to a slightly different definition, $\Gamma^t_{i,j,k,r} = \text{sign}\left(\rho^t_{i,k,r}\right)\cdot\left|\rho^t_{i,k,r}\right|^{\beta^i_k}$. With $\rho^t_{i,k,r} = \left(\frac{p^t_{i,k,r}}{p^t_{j,r}}\right)^{\beta^i_k} - 1$. The "presence" of the extra exponentiation using β^i_k for $\rho^t_{i,k,r}$ opened the possibility of obtaining complex numbers, given that $\rho^t_{i,k,r}$ could be negative, thus forcing the calculation of the length of the complex expression and the introduction of the sign to honour the relative ordering.

- The rationale is that if there is unsatisfied demand for household i after turn r, then its willingness to pay needs to increase to hopefully secure goods from offering firms
- In the code the first of these options has been implemented

- $p^t_{j,r}$ is calculated for every firm based on stock levels and number of rounds so far at time t, one possibility is to express $p^t_{j,r} = p^t_{j,r-1} - \varepsilon$, with $\varepsilon > 0$ a small quantity (parameter), another possibility is to use $p^t_{j,r} = p^t_{j,r-1} \cdot \varepsilon$, with $\varepsilon < 1$
  - The rationale is that if there is unsold product for firm j after turn r, then its price needs to reduce to hopefully find an interested buyer
  - In the code the first of these options has been implemented

- $\vec{d}^t_{i,r}|_k$ is calculated as:

$$\vec{d}^t_{i,r}|_k = \vec{d}^t_{i,r-1}|_k - \sum_{j \in J(k)} X^{t,*}_{i,j,k,r-1}$$

with $\vec{d}^t_{i,r-1}|_k$ the demand for product k by household i in round $r-1$, $X^{t,*}_{i,j,k,r-1}$ the optimal solution of the model in round $r-1$, and $X^{t,*}_{i,j,k,0} = 0$

- $B^t_{i,r}$ is calculated as:

$$B^t_{i,r} = B^t_{i,r-1} - \sum_{j \in J} \sum_{k \in K} p^t_{j,r-1} \cdot X^{t,*}_{i,j,k,r-1}$$

with $B^t_{i,r-1}$ the available budget for household i in round $r-1$, $X^{t,*}_{i,j,k,r-1}$ the optimal solution of the model in round $r-1$, and $X^{t,*}_{i,j,k,0} = 0$

- $\vec{s}^t_{j,r}|_1$ is calculated as:

$$\vec{s}^t_{j,r}|_1 = \vec{s}^t_{j,r-1}|_1 - \sum_{i \in I} \sum_{k \in K} X^{t,*}_{i,j,k,r-1}$$

  with $\vec{s}^t_{j,r-1}|_1$ the available supply for company j (implicitly some product k) in iteration $r-1$, $X^{t,*}_{i,j,k,r-1}$ the optimal solution of the model in iteration $r-1$, and $X^{t,*}_{i,j,k,0} = 0$

Note that in Algorithm (3) the stopping condition is given by the logical negation of $(\sum_{i \in I} \vec{d}^t_{i,r}|_k > \vec{0}$ and $\sum_{j \in J(k)} \vec{s}^t_{j,r}|_1 > \vec{0})$ $\forall k \in K$. This means the algorithm stops if any of the conditions are not met (i.e., there is no more supply to sell or all households have satisfied their needs for each product $k \in K$).

Algorithm 3: Market Algorithm

Data:
$\left\{\vec{d}^t_{i,0}\right\}_{i \in I}$, $\left\{B^t_{i,0}\right\}_{i \in I}$, $\left\{\vec{s}^t_{j,0}|_1\right\}_{j \in J}$ and $\left\{p^t_{j,r}\right\}_{j \in J}$

Result:
$\left\{X^{t,*}_{i,j,k,r}\right\}_{i \in I, j \in J, k \in K, r \in R}$ a collection of optimal sub-assignations during $|R|$ rounds

begin
- *Initialisation*
- $r \longleftarrow 0$;
- **while** $\sum_{i \in I} \vec{d}^t_{i,r}|_k > \vec{0}$ *and* $\sum_{j \in J(k)} \vec{s}^t_{j,r}|_1 > \vec{0}$ $\forall k \in K$ **do**
  - Calculate $\left\{\Gamma^t_{i,j,k,r}\right\}_{i \in I, j \in J, k \in K}$
  - Solve $P_r(t)$
  - Update $\left\{\vec{d}^t_{i,r+1}\right\}_{i \in I}$, $\left\{B^t_{i,r+1}\right\}_{i \in I}$, $\left\{\vec{s}^t_{j,r+1}|_1\right\}_{j \in J}$, $\left\{p^t_{j,r+1}\right\}_{j \in J}$ using $\left\{X^{t,*}_{i,j,k,r}\right\}_{i \in I, j \in J, k \in K}$
  - $r \longleftarrow r+1$;
- Calculate $\left\{p^{t+1}_{k,0}\right\}_{k \in K}$ and $\left\{p^{t+1}_{j,0}\right\}_{j \in J}$ as the weighted average of transactions that happened in t

4.5 Data Consumption

In order to run the model and to create instances of the objects it is first necessary to input data to the system. One possibility to perform such a task is to create data at random, but then the resulting simulations will have no connection to reality and the results can not be interpreted.

The problem is not a minor one as it requires taking data coming from national accounts and adjusting the coefficients used mainly in the Cobb-Douglas' functions for both households and firms. On the other hand, there is some data available regarding the number of firms and households, but at an aggregated level only via the census reports, that requires some work in order to be incorporated into this work. For the purposes of establishing comparisons between runs, a way to read a predefined set of firms, households and economic sectors is necessary. The strategy going forward is, to generate realistic looking data base on the national accounts matrix (as the data for households and firms composing the economy was available just in aggregated form and not individualised), to save data to create a baseline, to in subsequent experiments, use the same baseline data as a starting point but complementing it with additional information which may be missing or is not derived from the national accounts matrix. The idea is to implement the methodology as much as possible based on existing data, but some missing information will be created based on first principles to show the value of the methodology to study the problem, and at the same time identifying the non-existing information and pointing to the importance of collecting such information in the future.

4.5.1 Fitting Consumption Behaviour

In the ideal world, information regarding each household's consumption would be available. However, the best that could be hoped for is the information summarised in the CPI basket, which is representative of an average household. Unfortunately such information is scarce or plainly unavailable. The input-output matrix shows a summarised view of the movements of goods between different economic sectors on an aggregated basis (i.e. without identifying individual consumers or producers). Furthermore, economic sectors trade goods that are final and intermediate production means between sectors. The absence of information regarding the type of goods transacted but just the total dollar figures moved between sectors make it impossible to identify consumption at the household level, hence the data will need to be simulated.

In Subsection 4.5.1.1, a strategy to calibrate the technology coefficients (Cobb-Douglas) based on actual consumption is presented. Also, due to the non-existence of information at the household level, a method using a particular type of distributions is presented to explain how variations of particular average values satisfying the condition that their sum is equal to one can be produced (Dirichlet distribution).

4.5.1.1 Adjusting Consumers' Coefficients

For the case of consumers, the main idea that comes to mind to determine the Cobb-Douglas' coefficients that characterise a "typical" household is to actually solve an inverse problem, i.e., knowing the composition of the actual "average" consumption of a representative household we know that the consumption of the different goods is a solution of the utility maximisation problem that the consumer "solves".

One interesting property that should be noted about the optimal solution for that problem is that the ratio of the expenditure on a particular good with respect to the total expenditure (or budget) is equal to the Cobb-Douglas coefficient, i.e., the following relationship holds at optimality:

$$\beta_k = \frac{p_k \cdot x_k^*}{\sum_{i=1}^{K} p_i \cdot x_i^*}$$

It is easy to check that this is true. For example, consider the case where we have three sectors providing each a single product/service for which the relative prices are $p_1 = \$12$, $p_2 = \$25$ and $p_3 = \$5$, also consider a budget of \$5,385 and the coefficients given by $\beta_1 = 0.512534818$, $\beta_2 = 0.208913648$ and $\beta_3 = 0.278551534$. In this case it is not difficult to check (by solving the consumer's utility optimisation problem) that the following is the optimal solution:

- $x_1^* = 230$
- $x_2^* = 45$
- $x_3^* = 300$

In the previous case, a simple calculation allow us to check the property that characterises optimal solutions:

- $230 \cdot \$12/\$5,385 = 2,760/5,385 = 0.512534818 = \beta_1$
- $45 \cdot \$25/\$5,385 = 1,125/5,385 = 0.208913648 = \beta_2$

- $300 \cdot \$5/\$5,385 = 1,500/5,385 = 0.278551534 = \beta_3$

Finally, note that we can solve this problem for different groups when reading the actual data coming from statistics or surveys (within some clearly identified cluster of households). By this mechanism it is possible then to fit the coefficients to the data (if individual data is available).

For practical purposes, in absence of survey data, the coefficients can be initialized at random using the following:

```
a = np.random.dirichlet(np.ones(len(sectors)), size=1).round(4)
dictoH = {key:value for key, value in zip(sectors, a.tolist()[0])}
```

Listing 4.1: Assignment of Cobb-Douglas coefficients for `Household` instances

which in particular is using a Dirichlet distribution that creates a set of values $a_1, \ldots, a_K \geq 0$ such that $\sum_{i=1}^{K} a_i = 1$, i.e., produces a random vector whose coefficients are normalised. The Dirichlet distribution of order $K \geq 2$ with parameters $\alpha_1, \ldots, \alpha_K \geq 0$ (also called concentration parameters) is defined by the following pdf:

$$f(x_1, \ldots, x_K, \alpha_1, \ldots, \alpha_K) = \frac{1}{B(\vec{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

the normalising constant is the multivariate Beta function which can be expressed in terms of the Gamma function as:

$$B(\vec{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{K} \alpha_i)}, \quad \vec{\alpha} = (\alpha_1, \ldots, \alpha_K)$$

A classical example of the use of the Dirichlet distribution is the case when one wants to cut strings (each one of initial length 1.0) into K pieces with different lengths, where each piece has a designated average length but allowing variation in the relative size of the pieces, with the variance being inversely proportional to α_i. For example, the following simple code shows the production of 5 repetitions of a draw from a Dirichlet distribution:

```
import numpy as np
a = np.random.dirichlet([100,300,400,200], size=5).round(4)
print(a)
```

Listing 4.2: Dirichlet distribution example

with the following output:

```
[[0.086  0.3106 0.4176 0.1857]
 [0.091  0.3069 0.3979 0.2041]
 [0.0901 0.3083 0.3942 0.2074]
 [0.0814 0.32   0.3901 0.2085]
 [0.0876 0.3009 0.4081 0.2034]]
```

changing the concentration parameters from `[100,300,400,200]` to `[1,3,4,2]` gives:

```
[[0.0574 0.2211 0.5103 0.2112]
 [0.0242 0.3955 0.1184 0.4619]
 [0.1039 0.3092 0.2668 0.3201]
 [0.0156 0.1425 0.6221 0.2198]
 [0.063  0.3707 0.4527 0.1136]]
```

which clearly exhibits more variability than in the previous example.

4.5.2 Fitting Economic Sectors and Producers' Behaviour

It would be natural to think that solving a problem like the following one could fit the coefficients for the production function:

$$P(t) \quad \min_{\alpha_0,\ldots,\alpha_K} \ (Q^* - \alpha_0 \cdot \prod_{i=1}^{K} (x_i^*)^{\alpha_i})^2$$

s.t.

$$\sum_{i=1}^{K} \alpha_i = 1$$
$$\sum_{i=1}^{K} \alpha_i \geq 0 \ \ \forall i \in \{1, \ldots, K\}$$

with Q^* and x_i^* the production level and use of inputs respectively.

Unfortunately, the previous problem does not provide a unique solution as the solution obtained depends on the starting point chosen to numerically solve the optimisation problem (i.e., the optimisation landscape possesses several local minima). To better understand the nature of the problem, let us consider a simple example. In this simple example, for three economic sectors providing each a single product/service, and for a given firm (which can later be extended to an economic sector using the same rationale), we have the following information:

- Quantity of the good to be produce: 5,503
- Consumption of different raw materials:
  - Good 1: 230
  - Good 2: 45

- Good 3: 300

Implementing the optimisation model and solving it numerically for the starting point $\vec{\alpha}^1 = (1, 0.3, 0.3, 0.4)$ provides the following solution:

- α_0: 18.34333
- α_1: 0
- α_2: 1
- α_3: 0

If the same optimisation is now run from the starting point $\vec{\alpha}^1 = (30, 0, 0.6, 0.4)$ gets the (totally different) solution:

- α_0: 28.57907
- α_1: 0.18477
- α_2: 0.23791
- α_3: 0.58731

It is clear that the two solutions cannot be more different, nevertheless, they provide the same output of 5,503 units. On the other side, from a given starting point, one would expect that small changes in the required production level should cause small changes in the coefficients due to the continuous nature of the objective function with respect to $\vec{\alpha}$, however, this is not really the case as the optimisation landscape is wild to say the least. To illustrate the point, for the last example and starting always from the same initial $\vec{\alpha}$, the output required will be changed to different levels to assess which impact it produces on the determination of the coefficients:

- **Base case**: Output level 5,503 producing a vector

$$\vec{\alpha}^* = (28.57907, 0.18477, 0.23791, 0.58731)$$

- **0.1% more output**: Output level 5,508.503 producing a vector

$$\vec{\alpha}^* = (28.7512, 0.04572, 0.26003, 0.70425)$$

- **1% more output**: Output level 5,558.03 producing a vector

$$\vec{\alpha}^* = (28.5779, 0.20212, 0.23022, 0.57766)$$

- **2% more output**: Output level 5,5613.06 producing a vector

$$\vec{\alpha}^* = (29.16287, 0.00022, 0.26398, 0.7458)$$

- **3% more output**: Output level 5,668.09 producing a vector

$$\vec{\alpha}^* = (29.10066, 0.0792, 0.24665, 0.68415)$$

- **0.1% less output**: Output level 5,497.497 producing a vector

$$\vec{\alpha}^* = (28.57402, 0.16201, 0.24154, 0.60645)$$

- **1% less output**: Output level 5,447.97 producing a vector

$$\vec{\alpha}^* = (28.95386, 0.29236, 0.23501, 0.48263)$$

- **2% less output**: Output level 5,392.94 producing a vector

$$\vec{\alpha}^* = (28.93914, 0.84768, 0.16232, 0)$$

- **3% less output**: Output level 5,337.91 producing a vector

$$\vec{\alpha}^* = (29.64294, 0, 0.3016, 0.7084)$$

It can be seen that the non-linear nature of the optimisation process and the shape of the optimisation landscape certainly contribute to solutions that differ in some cases substantially in terms of the coefficients that can be obtained. Based on this example, a way out of this conundrum is needed and it is actually proposed in Subsection 4.5.2.1.

4.5.2.1 Proposal for Adjusting Producer's Production Functions

To appropriately characterise the production function of a given firm, information of different levels of output for certain inputs is used to build regression type models from which the coefficients are obtained. With such level of information it is possible to get an idea of the technology a firm is using. As we have seen in the previous sub-subsection, starting from just a production function does not uniquely determine the technology which is being used by a firm (represented by a production function).

The attempts made at solving this problem proved unfruitful for the data available. However, the idea into the future is to use a model to make comparisons for different case studies against a baseline case. From this point of view, once the production functions are determined or created, they remain constant (this is an assumption), and for the experiments that were performed, this should not play a role in the comparisons to be performed[3]. So, for the

[3]It will obviously affect the actual production level for each company

purposes of being able to move forward with the use of the model, a convention will need to be adopted in order to fit coefficients for production functions solely based on the information available from the national accounts matrix.

The mechanism used was to fix the starting point of the fitting process at the following point:

$$\vec{\alpha}_0 = (\sqrt{\sum_{k=1}^{N} x_k^*}, \frac{x_1^*}{\sum_{k=1}^{N} x_k^*}, \ldots, \frac{x_N^*}{\sum_{k=1}^{N} x_k^*})$$

with N being the number of economic sectors of the economy and x_i^* being the desired firm's consumption level of good i. This arbitrary point is the result of several trials and errors and the only attributable rationale for it is that somehow it looks like an average point for all coordinates except the first. The first coordinate represents the value of the constant in the model and is "proportional" to the length of the desired consumption level vector $\vec{x}^*$.

As the optimal solution of the associated problem of determining the coefficients, to obtain a given production level based on certain consumptions of factors, heavily depends on the starting point, after an extensive process of trial an error, this starting point showed to produce solutions which are at least reproducible.

From that starting point, the problem that is solved is:

$$\tilde{P}(t) \quad \min_{\alpha_0,\ldots,\alpha_K} \; (Q^* - \alpha_0 \cdot \prod_{i=1}^{K} (x_i^*)^{\alpha_i})^2$$

s.t.

$$\sum_{i=1}^{K} \alpha_i \approx 1$$

$$\alpha_i \leq \frac{x_i^*}{\sum_{k=1}^{N} x_k^*} + \rho \;\; \forall i \in \{1, \ldots, K\}$$

$$\alpha_i \geq \frac{x_i^*}{\sum_{k=1}^{N} x_k^*} - \rho \;\; \forall i \in \{1, \ldots, K\}$$

with $\sum_{i=1}^{K} \alpha_i \approx 1$ meaning that $\sum_{i=1}^{K} \alpha_i$ is within a reasonable tolerance from 1, and with ρ a small number.

With this definition, and with the objective of illustrating the type of output that can be obtained, $\rho = 0.05$ is used. A previously presented example determines the following solution for the starting $\vec{\alpha}_0 = (23.97931, 0.4, 0.078261, 0.521739)$ due to consumption levels of $\vec{x} = (230, 45, 300)$:

$$\vec{\alpha}^* = (23.97931, 0.39973, 0.08363, 0.5161)$$

4.6 Validation for the Proposed Model

As the proposed model is going to be applied in a realistic setting, with a lot of complexity and interactions, it could become difficult to identify potential problems with the individual components that comprise the overall software implementation. In order to ensure that some basic behaviour of the different agents is actually implemented and to test the logical correctness of the implementations, it is first necessary to perform some basic testing.

This section focuses on the different implementations of the agents. By means of isolated testing of the individual components, it checks that their implementations do not exhibit logical errors, in particular, for each software component (class or agent), a test is performed to assess whether their respective implementation is able to reproduce minimum outcomes based on a set of assumptions, and that the software component, as individually implemented, operates as expected.

Each upcoming subsection describes tests performed to check the logical validity of the implementations for each separate component. Additionally, a last section will check that the interoperation of the different software components produces correct/reasonable results based on a minimal case study.

4.6.1 `Household` Agent

The main aspect of the `Household` agent that is the focus of the testing is checking the correctness of the optimal solution for the utility maximisation problem using Cobb-Douglas' utility functions. Recall that each agent in our proposed model has its own utility function $u(\vec{x})$ given by:

$$u(\vec{x}) = \prod_{i=1}^{n} x_i^{\beta_i}$$

The utility maximisation can be defined as:

$$\begin{aligned} \max_{\{x_1,\ldots,x_n\}} \quad & \prod_{i=1}^{n} x_i^{\beta_i} \\ s.t. \quad & \\ & \sum_{i=1}^{n} p_i \cdot x_i \leqslant I \\ & x_i \geqslant 0 \;\; \forall i \in \{1,\ldots,n\} \end{aligned}$$

where p_i is the price of good i, I is the available budget and the β_i are the coefficients of the Cobb-Douglas utility function.

In order to solve this optimisation problem, it is standard practice to write the associated Lagrangian function for the problem given by:

$$\mathcal{L}(\vec{x},\lambda) = \prod_{i=1}^{n} x_i^{\beta_i} - \lambda \cdot (I - p_i \cdot x_i)$$

To solve this problem, it is usual to express the first order conditions (to find candidate equilibrium points):

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_i} = 0 \quad & \Rightarrow \quad \beta_i x_i^{\beta_i - 1} \prod_{k=1,k\neq i}^{n} x_k^{\beta_k} - \lambda p_i = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad & \Rightarrow \quad I - \sum_{i=1}^{n} p_i \cdot x_i = 0 \end{aligned}$$

Solving this system yields the optimal solution $\vec{x}^*$. In order to illustrate the process, we will study a particular example.

Let us assume that we have just two goods x_1 and x_2, that the prices of the goods are $p_1 = 20$ and $p_2 = 15$, that the coefficients of the utility function are $\beta_1 = 0.4$ and $\beta_2 = 0.6$ and finally an income I of 4,000. From the above discussion we arrive at the following system:

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0 \quad \Rightarrow \quad 0.4x_1^{-0.6}x_2^{0.6} + 20\lambda = 0 \tag{4.10}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0 \quad \Rightarrow \quad 0.6x_1^{0.4}x_2^{-0.4} + 15\lambda = 0 \tag{4.11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Rightarrow \quad 20x_1 + 15x_2 - 4000 = 0 \tag{4.12}$$

From (4.10) we obtain

$$\lambda = -\frac{0.4x_1^{-0.6}x_2^{0.6}}{20} \tag{4.13}$$

and from (4.11) we obtain

$$\lambda = -\frac{0.6x_1^{0.4}x_2^{-0.4}}{15} \tag{4.14}$$

From (4.13) and (4.14) we obtain:

$$x_2 = 2x_1$$

which put back into Equation (4.12) gives the following equation:

$$20x_1 + 30x_1 = 4000 \Leftrightarrow 50x_1 = 4000 \Leftrightarrow x_1^* = 80 \Rightarrow x_2^* = 160$$

with corresponding utility of 121.25.

A test was performed by implementing this simple example in the manner used by the proposed agent-based model to check the correctness of the implementation used in the code. The code used was:

```
import numpy as np
from scipy.optimize import minimize

relative_prices = [20,15]
cobb_douglas_coeffs = [0.4,0.6]
income = 4000

def utilityFunc(x):
    return x[0]**cobb_douglas_coeffs[0]*x[1]**cobb_douglas_coeffs[1]

def optimiseUtility():
    def valueOfChoice(x):
        return -utilityFunc(x)

    # Define the budget constraint
    constraints = (
    {'type': 'ineq', 'fun': lambda x: income-np.dot(relative_prices, x)},
    {'type': 'ineq', 'fun': lambda x: x[0]},
    {'type': 'ineq', 'fun': lambda x: x[1]},
    )
    x0 = [income/2,income/2]
    res = minimize(valueOfChoice, x0, method='SLSQP',
                   options={'disp': True, 'iprint': 1,
                   'eps': 1.0e-09, 'maxiter': 1000,
                   'ftol': 1e-06}, constraints=constraints)

    return np.dot(relative_prices, res.x), res.x

if __name__ == '__main__':
    a = optimiseUtility()
    print(a)
```

Listing 4.3: Code to check correctness of Cobb-Douglas utility maximisation implementation

When this code is run, the following output is produced:

```
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -121.25732524013279
            Iterations: 16
            Function evaluations: 69
            Gradient evaluations: 16
(4000.000000003326, array([ 80.00357688, 159.99523083]))
```

Which confirms the correctness of the implementation. Please note that the objective function value of -121.25732524013279 appears as negative because the library used for optimisation just allows minimisation problems, but as our problem is a maximisation one we use the fact that $\max f(x) \Leftrightarrow -\min f(x)$.

4.6.2 `Firm` Agent

The main aspect of the `Firm` agent that will be the focus of our testing is the correctness of the optimal solution for the profit maximisation problem using Cobb-Douglas' production functions. Recall that each agent in our proposed model has its own production function $q(\vec{y})$ given by:

$$q(\vec{y}) = \beta_0 \prod_{k=1}^{K} y_k^{\beta_k}$$

Without going into much detail, as solving the classical profit optimisation problem has no constraints (because the expenditure is on the objective function), we will show the basic steps into solving one such problem by hand and then compare that solution with one obtained with the help of code.

Let us assume that we have just two production factors K and L, that the prices of these factors are $p_K = 20$ and $p_L = 15$, the price of the product is $p = 500$ and that the coefficients of the utility function are $\beta_0 = 1$, $\beta_1 = 0.25$

and $\beta_2 = 0.25$. The problem that needs to be solved is thus:

$$\max_{K,L\geqslant 0} \pi = p \cdot K^{0.25} L^{0.25} - 20K - 15L \tag{4.15}$$

Imposing first order conditions on the profit objective function gives:

$$\frac{\partial \pi}{\partial K} = 0 \quad \Rightarrow \quad \frac{p}{4} K^{-0.75} L^{0.25} - 20\lambda = 0 \tag{4.16}$$

$$\frac{\partial \pi}{\partial L} = 0 \quad \Rightarrow \quad \frac{p}{4} K^{0.25} L^{-0.75} - 15\lambda = 0 \tag{4.17}$$

Solving this system of equations provides the following solution:

$$\begin{aligned} K &= 45.10 \\ L &= 60.14 \end{aligned}$$

A test was performed by implementing this simple example in the manner used by the proposed agent-based model to check the correctness of the implementation. The code used was:

```
import numpy as np
from scipy.optimize import minimize

relative_prices = [20,15]
cobb_douglas_coeffs = [1, 0.25, 0.25]
price = 500

def utilityFunc(x):
    return price*cobb_douglas_coeffs[0]*x[0]**cobb_douglas_coeffs[1] * x[1]**cobb_douglas_coeffs[2] - np.dot(relative_prices, x)

def optimiseUtility():
    def valueOfChoice(x):
        return -utilityFunc(x)

    # Define the budget constraint
    constraints = (
```

```
    {'type': 'ineq', 'fun': lambda x: x[0]},
    {'type': 'ineq', 'fun': lambda x: x[1]},
    )
    x0 = [1, 1]
    res = minimize(valueOfChoice, x0, method='SLSQP',
                   options={'disp': True, 'iprint': 1,
                   'eps': 1.0e-09, 'maxiter': 1000,
                   'ftol': 1e-06}, constraints=constraints)

    return utilityFunc(res.x), res.x

if __name__ == '__main__':
    a = optimiseUtility()
    print(a)
```

Listing 4.4: Code to check correctness of Cobb-Douglas profit maximisation implementation

When this code is run, the following output is produced:

```
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -1804.2195911186914
            Iterations: 11
            Function evaluations: 47
            Gradient evaluations: 11
(1804.2195911186914, array([45.10622158, 60.14051892]))
```

Which confirms the correctness of the implementation.

Now, if a variant of the previous problem with a budget constraint of $M = 1000$ is enforced, the following code

```
import numpy as np
from scipy.optimize import minimize

relative_prices = [20,15]
```

```python
cobb_douglas_coeffs = [1, 0.25, 0.25]
price = 500
budget = 1000

def utilityFunc(x):
    return price*cobb_douglas_coeffs[0]*x[0]**cobb_douglas_coeffs[1]* x[1]**cobb_douglas_coeffs[2]-np.dot(relative_prices, x)

def optimiseUtility():
    def valueOfChoice(x):
        return -utilityFunc(x)

    # Define the budget constraint
    constraints = (
    {'type': 'ineq', 'fun': lambda x: budget - np.dot(relative_prices, x)},
    {'type': 'ineq', 'fun': lambda x: x[0]},
    {'type': 'ineq', 'fun': lambda x: x[1]},
    )
    x0 = [1, 1]
    res = minimize(valueOfChoice, x0, method='SLSQP',
                   options={'disp': True, 'iprint': 1,
                   'eps': 1.0e-09, 'maxiter': 1000,
                   'ftol': 1e-06}, constraints=constraints)

    return utilityFunc(res.x), res.x

if __name__ == '__main__':
    a = optimiseUtility()
    print(a)
```

Listing 4.5: Code to check correctness of Cobb-Douglas profit maximisation implementation with budget constraint

produces the following output:

```
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -1686.4248295696439
            Iterations: 6
```

```
            Function evaluations: 24
            Gradient evaluations: 6
(1686.4248295696439, array([24.99998406, 33.33335458]))
```

which confirms that adding a budget constraint changes the optimal answer obtained previously.

4.6.3 Economic Sector and Economy Agents

As these agents are really simple, the validation of the logic behind their implementation will simply consist on instantiating some objects and observing them once the code is run. For this purposes, a simple example provided in the following listing has been run:

```
if __name__== '__main__':
    env = simpy.Environment()
    sectorColl = economy(env, print=True)
    for i in range(4):
        sectorColl.__add_val__(sector(env=env, avgprice=rd.uniform(10,20),
        ↪  print=True))

    env.run(until=4)
```

Listing 4.6: Code to check correctness of `Sector` and `Economy` Agents

which produces the following output:

```
Economy Agent, Iteration number 0
Sector ID: S_1 is alive and well at time 0 with average price of
↪  19.37350465031006
Sector ID: S_2 is alive and well at time 0 with average price of
↪  14.753680362575388
Sector ID: S_3 is alive and well at time 0 with average price of
↪  18.693533820150165
```

```
Sector ID: S_4 is alive and well at time 0 with average price of
↪  11.030719284280877
Economy Agent, Iteration number 1
Sector ID: S_1 is alive and well at time 1 with average price of
↪  20.342868010205216
Sector ID: S_2 is alive and well at time 1 with average price of
↪  14.959551484974895
Sector ID: S_3 is alive and well at time 1 with average price of
↪  19.501924440108183
Sector ID: S_4 is alive and well at time 1 with average price of
↪  10.773957796630494
Economy Agent, Iteration number 2
Sector ID: S_1 is alive and well at time 2 with average price of
↪  19.729260941438582
Sector ID: S_2 is alive and well at time 2 with average price of
↪  14.839231133214243
Sector ID: S_3 is alive and well at time 2 with average price of
↪  19.497086057920043
Sector ID: S_4 is alive and well at time 2 with average price of
↪  10.975634340519791
Economy Agent, Iteration number 3
Sector ID: S_1 is alive and well at time 3 with average price of
↪  20.190315207367746
Sector ID: S_2 is alive and well at time 3 with average price of
↪  15.664478235962381
Sector ID: S_3 is alive and well at time 3 with average price of
↪  19.480792426365948
Sector ID: S_4 is alive and well at time 3 with average price of
↪  10.719251003320608
```

Please note that for this example, the `Simpy` cycle that implements the discrete event simulation modifies in a random way the average price on every cycle. The idea is to observe that the objects are created, and that it is possible to modify their state as the iterations progress and observe some change. In the setting of the simulation this price changes are produced by the interactions between buyers and sellers and are taken care by the `Market` object.

4.6.4 `Market` Agent

In order to test the `Market` agent, an implementation was attempted which among other things showed a problem with the initially proposed $\gamma^t_{i,j,k,r}$, which at first was defined as:

$$\gamma^t_{i,j,k,r} = \left(\left(\frac{\bar{p}^t_{k,r}}{p^t_{j,r}}\right)^{\beta^i_k} - 1\right)$$

Starting from the previous definition, it became clear whilst implementing this in the code that $\gamma^t_{i,j,k,r}$ as initially defined by means of using $\bar{p}^t_{k,r}$, would result on a *relative gain* coefficient relative to the average price of the good for a given sector, however rendering the objective of being able to discriminate individual firms not achievable. Consequently, a new proposal considering the ratio $\left(\frac{p^t_{i,k,r}}{p^t_{j,r}}\right)$ is thought as providing a relative gain for each individual consumer, with respect to each firm within a sector. The new proposal for $\gamma^t_{i,j,k,r}$ (denoted by $\Gamma^t_{i,j,k,r}$) is then modified accordingly to become consistent with this observation:

$$\Gamma^t_{i,j,k,r} = \left(\left(\frac{p^t_{i,k,r}}{p^t_{j,r}}\right)^{\beta^i_k} - 1\right)$$

where $p^t_{i,k,r}$ is the willingness to pay for product of sector k by household i and $p^t_{j,r}$ is the price that firm j charges for their product (recall that in our model, every company produces only one product for a sector).

4.6.5 Validation of the Optimisation Model

In order to test the optimisation model, we will create a very simple instance that shows that the optimisation model run once produces the optimal solution. In order to achieve this, a simple implementation is performed in EXCEL and Solver is used to obtain a solution which later we obtain using the method implemented in the class.

The small instance for the problem is comprised of the following agents:

- 2 sectors:

	Sector 1	Sector 2
ID	S_1	S_2
Avg. Price	$13	$16

- 2 companies per sector:

	Firm 1	Firm 2	Firm 3	Firm 4
ID	F_1	F_2	F_3	F_4
Sector	S_1	S_1	S_2	S_2
C0	0.28151	0.14909	0.44311	0.50753
B1	0.1092	0.2977	0.1263	0.9023
B2	0.8908	0.7023	0.8737	0.0977
Budget	$2000	$2000	$2000	$2000

- 6 households:

	House 1	House 2	House 3	House 4	House 5	House 6
ID	H_1	H_2	H_3	H_4	H_5	H_6
Init .Budget	$200	$200	$200	$200	$200	$200
Salary	$150	$150	$150	$150	$150	$150
B1	0.2813	0.5747	0.402	0.4019	0.4138	0.3089
B2	0.7187	0.4253	0.598	0.5981	0.5862	0.6911

4.6.5.1 Solution of the Household Problem

For the instance presented before, an implementation using Microsoft EXCEL and the Solver Add-In allows us to obtain the optimal consumption levels for every household. The structure of such a model is very simple, for example, for household one we need to solve the following problem:

$$\begin{aligned} \max_{x_1,x_2} \quad & x_1^{0.2813} \cdot x_2^{0.7187} \\ s.t. \quad & \\ & \$13x_1 + \$16x_2 \leq \$350 \\ & x_1, x_2 \geq 0 \end{aligned}$$

We will not here elaborate on how such a problem is solved in EXCEL solver as it is a pretty standard procedure. This process was performed for every household, the optimal results are summarised in the following table:

	House 1	House 2	House 3	House 4	House 5	House 6
Objective	12.8017	12.4626	12.1217	12.1220	12.0982	12.5694
x_1^*	7.5735	15.4727	10.8231	10.8204	11.1408	8.3165
x_2^*	15.7216	9.3034	13.0812	13.0834	12.8231	15.1178
Constraint	350	350	350	350	350	350
Budget Cons	350	350	350	350	350	350

4.6.5.2 Solution of the Firm Problem

In a similar manner, the solution of the associated optimisation problem was obtained for every firm to determine the initial offer that every firm will put out. The results of such optimisation are given in the following table:

	Firm 1	Firm 2	Firm 3	Firm 4
Objective	331.4558	140.1869	622.6000	888.9594
x_1^*	16.8000	45.8000	19.4308	138.8154
x_2^*	111.3500	87.7875	109.2125	12.2125
Constraint	2000	2000	2000	2000
Budget Cons	2000	2000	2000	2000
Offer	25.4966	10.7836	38.9125	55.5600

Please note that the quantity being offered is calculated as the value of the objective function divided by the price of the good, for example, the quantity offered by Firm 1 is equal to $331.4558/13 \approx 25.4966$ (as the problem being solved is that of income maximisation with cost constraints). Also note that the optimal solution indicates the quantities needed for each raw material; in this case we are considering the raw materials to be the same as the final products with the same prices, even though they are usually different, but for the purposes of this example we simplify as we want to test the correctness of the implemented model.

4.6.5.3 Solution of the Market Assignment Problem

For solving this problem, and also to illustrate the process involved, we are going to specify the instance data (some of it coming from solving the previous two problems) and other data that needs calculation specifically for this model (such as the γ coefficients). We will explicitly write up the resulting optimisation model and provide its solution, this will later be compared with the solution obtained using the ABM implementation to check its correctness.

It is important to note that at this point in the validation process we are going to solve the optimisation problem just once, to check its correctness, later in the validation process we will check that prices and inventories are updated accordingly which will complete the validation of this very complex `Market` agent.

The assignment model of the `Market` agent requires the following elements:

- $\left\{\vec{d}^t_{i,0}\right\}_{i\in I}$: This was already calculated in a previous subsection and corresponds to the demand for goods. In the current example they are represented by the vector

$$\vec{d}^t = \left(\begin{pmatrix}7.5735\\15.7216\end{pmatrix}, \begin{pmatrix}15.4727\\9.3034\end{pmatrix}, \begin{pmatrix}10.8231\\13.0812\end{pmatrix}, \begin{pmatrix}10.8204\\13.0834\end{pmatrix}, \begin{pmatrix}11.1408\\12.8231\end{pmatrix}, \begin{pmatrix}8.3165\\15.1178\end{pmatrix}\right)$$

- $\left\{B^t_{i,0}\right\}_{i\in I}$: This has already been defined and actually used as the basis for the calculation of $\vec{d}^t$, this can be summarised in the following vector

$$B^t_0 = (\$350, \$350, \$350, \$350, \$350, \$350)$$

- $\left\{\vec{s}^t_{j,0}|_1\right\}_{j\in J}$: This was already calculated in a previous subsection corresponds to the demand for goods. In the current example they are represented by the vector

$$\vec{s}^t_j = (25.4966, 10.7836, 38.9125, 55.5600)$$

- $\left\{p^t_{j,r}\right\}_{j\in J}$: This has been defined previously and is equal to

$$\vec{p}^t_j = (\$13, \$13, \$16, \$16)$$

- $\left\{p^t_{i,k,r}\right\}_{i\in I,k\in K}$: This data has been calculated for the purposes of this example as a random variation around the price, for the purposes of the example the following were used

  $$\vec{p}^t_i = \left(\begin{pmatrix}12.8644\\15.6268\end{pmatrix},\begin{pmatrix}12.8744\\16.0955\end{pmatrix},\begin{pmatrix}12.8359\\16.0810\end{pmatrix},\begin{pmatrix}12.9010\\16.0490\end{pmatrix},\begin{pmatrix}12.6926\\15.7736\end{pmatrix},\begin{pmatrix}13.2202\\16.0071\end{pmatrix}\right)$$

- $\left\{\gamma^t_{i,j,k,r}\right\}_{i\in I,j\in J,k\in K}$: This calculation can be summarised in the following vector:

  $$\vec{\gamma}^t \approx \left(\begin{pmatrix}-0.0029\\-0.0168\end{pmatrix},\begin{pmatrix}-0.0056\\0.0025\end{pmatrix},\begin{pmatrix}-0.0051\\0.0030\end{pmatrix},\begin{pmatrix}-0.0031\\0.0018\end{pmatrix},\begin{pmatrix}-0.0099\\-0.0083\end{pmatrix},\begin{pmatrix}0.0052\\0.0003\end{pmatrix}\right)$$

  It is not difficult to see that all coefficients whose willingness to pay falls below the market prices are negative and the converse is also true

In order to provide a better illustration on how the Γ coefficients are calculated, the explicit calculation for the first household of the previous list is given in detail:

$$\begin{aligned}\Gamma^t_{1,1,1,0} = \Gamma^t_{1,2,1,0} &= \left(\left(\frac{12.8644}{13}\right)^{0.2813} - 1\right)\\ &= -0.002944494\end{aligned}$$

and

$$\begin{aligned}\Gamma^t_{1,3,2,0} = \Gamma^t_{1,4,2,0} &= \left(\left(\frac{15.6268}{16}\right)^{0.7187} - 1\right)\\ &= -0.01681921\end{aligned}$$

The optimisation problem is formulated then as

$$
\begin{aligned}
\max \quad & -0.0029 \cdot x_{1,1,1} - 0.0029 \cdot x_{1,2,1} - 0.0168 \cdot x_{1,3,2} - 0.0168 \cdot x_{1,4,2} \\
& -0.0056 \cdot x_{2,1,1} - 0.0056 \cdot x_{2,2,1} + 0.0025 \cdot x_{2,3,2} + 0.0025 \cdot x_{2,4,2} \\
& -0.0051 \cdot x_{3,1,1} - 0.0051 \cdot x_{3,2,1} + 0.0030 \cdot x_{3,3,2} + 0.0030 \cdot x_{3,4,2} \\
& -0.0031 \cdot x_{4,1,1} - 0.0031 \cdot x_{4,2,1} + 0.0018 \cdot x_{4,3,2} + 0.0018 \cdot x_{4,4,2} \\
& -0.0099 \cdot x_{5,1,1} - 0.0099 \cdot x_{5,2,1} - 0.0083 \cdot x_{5,3,2} - 0.0083 \cdot x_{5,4,2} \\
& 0.0052 \cdot x_{6,1,1} + 0.0052 \cdot x_{6,2,1} + 0.0003 \cdot x_{6,3,2} + 0.0003 \cdot x_{6,4,2}
\end{aligned}
$$

$s.t.$

$$
\begin{aligned}
x_{1,1,1} + x_{1,2,1} &\leq 7.5735 \\
x_{1,3,2} + x_{1,4,2} &\leq 15.7216 \\
x_{2,1,1} + x_{2,2,1} &\leq 15.4727 \\
x_{2,3,2} + x_{2,4,2} &\leq 9.3034 \\
x_{3,1,1} + x_{3,2,1} &\leq 10.8231 \\
x_{3,3,2} + x_{3,4,2} &\leq 13.0812 \\
x_{4,1,1} + x_{4,2,1} &\leq 10.8204 \\
x_{4,3,2} + x_{4,4,2} &\leq 13.0834 \\
x_{5,1,1} + x_{5,2,1} &\leq 11.1408 \\
x_{5,3,2} + x_{5,4,2} &\leq 12.8231 \\
x_{6,1,1} + x_{6,2,1} &\leq 8.3165 \\
x_{6,3,2} + x_{6,4,2} &\leq 15.1178
\end{aligned}
$$

$$13x_{1,1,1} + 13x_{1,2,1} + 16x_{1,3,2} + 16x_{1,4,2} \leq 350$$

$$13x_{2,1,1} + 13x_{2,2,1} + 16x_{2,3,2} + 16x_{2,4,2} \leq 350$$

$$13x_{3,1,1} + 13x_{3,2,1} + 16x_{3,3,2} + 16x_{3,4,2} \leq 350$$

$$13x_{4,1,1} + 13x_{4,2,1} + 16x_{4,3,2} + 16x_{4,4,2} \leq 350$$

$$13x_{5,1,1} + 13x_{5,2,1} + 16x_{5,3,2} + 16x_{5,4,2} \leq 350$$

$$13x_{6,1,1} + 13x_{6,2,1} + 16x_{6,3,2} + 16x_{6,4,2} \leq 350$$

$$x_{1,1,1} + x_{2,1,1} + x_{3,1,1} + x_{4,1,1} + x_{5,1,1} + x_{6,1,1} \leq 25.4966$$

$$x_{1,2,1} + x_{2,2,1} + x_{3,2,1} + x_{4,2,1} + x_{5,2,1} + x_{6,2,1} \leq 10.7836$$

$$x_{1,3,2} + x_{2,3,2} + x_{3,3,2} + x_{4,3,2} + x_{5,3,2} + x_{6,3,2} \leq 38.9125$$

$$x_{1,4,2} + x_{2,4,2} + x_{3,4,2} + x_{4,4,2} + x_{5,4,2} + x_{6,4,2} \leq 55.5600$$

$$x_{1,1,1}, x_{2,1,1}, x_{3,1,1}, x_{4,1,1}, x_{5,1,1}, x_{6,1,1} \geq 0$$

$$x_{1,2,1}, x_{2,2,1}, x_{3,2,1}, x_{4,2,1}, x_{5,2,1}, x_{6,2,1} \geq 0$$

$$x_{1,3,2}, x_{2,3,2}, x_{3,3,2}, x_{4,3,2}, x_{5,3,2}, x_{6,3,2} \geq 0$$

$$x_{1,4,2}, x_{2,4,2}, x_{3,4,2}, x_{4,4,2}, x_{5,4,2}, x_{6,4,2} \geq 0$$

When this problem is solved, the following solution is obtained:

	House 1	House 2	House 3	House 4	House 5	House 6
F1	0	0	0	0	0	4.158270434
F2	0	0	0	0	0	4.158270434
F3	0	4.651719137	6.540624713	6.541718462	0	7.55890527
F4	0	4.651719137	6.540624713	6.541718462	0	7.55890527

The previous with an objective function of 0.134925047. This exercise enabled a review of the model, allowing the validation of the same. To put the usefulness of this type of testing within context, it is noteworthy to mention here that initially the solution of the optimisation model gave different outcomes than the ABM code, which required a careful and lengthy review of the models. Finally, the differences were found to be due to an implementation error using EXCEL and Solver. After the error was found, it was clear that the solutions were equivalent as the next subsection will show. Also important to note is that the previous optimisation model does not have a unique optimal solution, in fact, the following is also a solution (as the interested reader can check):

	House 1	House 2	House 3	House 4	House 5	House 6
F1	0	0	0	0	0	8.316528327
F2	0	0	0	0	0	1.26871E-05
F3	0	0	10.71125097	13.08343692	0	15.11781133
F4	0	9.303438352	2.369998481	0	0	0

The following solution is also optimal:

	House 1	House 2	House 3	House 4	House 5	House 6
F1	0	0	0	0	0	0
F2	0	0	0	0	0	8.316541014
F3	0	9.303438274	13.08124943	13.08343692	0	0
F4	0	0	0	0	0	15.11781054

This situation is not an abnormal one. This usually happens in linear optimisation models, when the level curves of the objective function are parallel to one of the faces of the polyhedron defined by the constraint set. In fact, for those problems there are infinite number of solutions, expressed as convex combinations of the vertices that define the polyhedra.

4.6.5.4 Solution Using the ABM Code

An example was run to test the implementation of the optimisation process. This test actually is checking several things: first is checking that the associated problems for every household are well solved, secondly that the production problem for each firm is appropriately solved and after these elements are provided, then the program checks that the optimisation problem for the market is solved (one iteration only). After this essential test is performed, then it only rest to check that the price updating mechanism works (see subsection (4.6.6)) and that the inventory update reflects the changes submitted to the market (see subsection (4.6.7)).

```
import simpy
import Household as hh
import Economy as ec
import Company as co
import Market as mk
import colorama

colorama.init()

env = simpy.Environment()
sectorColl = ec.economy(env, print=False)
sectorColl.__add_val__(ec.sector(env=env, avgprice=13, print=False))
sectorColl.__add_val__(ec.sector(env=env, avgprice=16, print=False))

firmColl = co.firms(env, print=False)
houseColl = hh.households(env,  print=False)
sectors = sectorColl.returnSectorList()
sectorsexp = list(sectorColl.returnSectorList())
sectorsexp.insert(0, 'CONST')

```

```
firmColl.__add_val__(co.firm(env=env, economy=sectorColl, sector='S_1', print=False, budget=2000, cdCoeffs={'CONST': 0.28151, 'S_1': 0.1092, 'S_2': 0.8908}))
firmColl.__add_val__(co.firm(env=env, economy=sectorColl, sector='S_1', print=False, budget=2000, cdCoeffs={'CONST': 0.14909, 'S_1': 0.2977, 'S_2': 0.7023}))
firmColl.__add_val__(co.firm(env=env, economy=sectorColl, sector='S_2', print=False, budget=2000, cdCoeffs={'CONST': 0.44311, 'S_1': 0.1263, 'S_2': 0.8737}))
firmColl.__add_val__(co.firm(env=env, economy=sectorColl, sector='S_2', print=False, budget=2000, cdCoeffs={'CONST': 0.50753, 'S_1': 0.9023, 'S_2': 0.0977}))

houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=150, print=False, cdCoeffs={'S_1': 0.2813, 'S_2': 0.7187}))
houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=150, print=False, cdCoeffs={'S_1': 0.5747, 'S_2': 0.4253}))
houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=150, print=False, cdCoeffs={'S_1': 0.402, 'S_2': 0.598}))
houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=150, print=False, cdCoeffs={'S_1': 0.4019, 'S_2': 0.5981}))
houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=150, print=False, cdCoeffs={'S_1': 0.4138, 'S_2': 0.5862}))
houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=150, print=False, cdCoeffs={'S_1': 0.3089, 'S_2': 0.6911}))

mercado = mk.market(env=env, economy=sectorColl, households=houseColl, firms=firmColl, print=False)

env.run(until=1)
```

Listing 4.7: Code to check correctness of the optimisation model for the `Market` Agent

The output of the previous program is:

```
Market ID: M_1 is iterating the optimiser: 0
Gamma H_1 F_1 S_1 -0.002944494
Gamma H_1 F_2 S_1 -0.002944494
Gamma H_1 F_3 S_2 -0.016819205
Gamma H_1 F_4 S_2 -0.016819205
Gamma H_2 F_1 S_1 -0.005563634
Gamma H_2 F_2 S_1 -0.005563634
Gamma H_2 F_3 S_2 0.002533059
Gamma H_2 F_4 S_2 0.002533059
Gamma H_3 F_1 S_1 -0.005094033
Gamma H_3 F_2 S_1 -0.005094033
Gamma H_3 F_3 S_2 0.003023462
Gamma H_3 F_4 S_2 0.003023462
Gamma H_4 F_1 S_1 -0.003070217
Gamma H_4 F_2 S_1 -0.003070217
Gamma H_4 F_3 S_2 0.001829828
Gamma H_4 F_4 S_2 0.001829828
Gamma H_5 F_1 S_1 -0.009854719
Gamma H_5 F_2 S_1 -0.009854719
Gamma H_5 F_3 S_2 -0.008319374
Gamma H_5 F_4 S_2 -0.008319374
Gamma H_6 F_1 S_1 0.005201324
Gamma H_6 F_2 S_1 0.005201324
Gamma H_6 F_3 S_2 0.000304989
Gamma H_6 F_4 S_2 0.000304989
H_1 Demand Sector S_1 7.5751 Willlingness to Pay 12.8644342
H_1 Demand Sector S_2 15.7203 Willlingness to Pay 15.62680052
H_2 Demand Sector S_1 15.4727 Willlingness to Pay 12.87440703
H_2 Demand Sector S_2 9.3035 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 10.8206 Willlingness to Pay 12.83589121
H_3 Demand Sector S_2 13.0832 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 10.8181 Willlingness to Pay 12.90091642
H_4 Demand Sector S_2 13.0853 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 12.69256085
H_5 Demand Sector S_2 12.8231 Willlingness to Pay 15.77359466
H_6 Demand Sector S_1 8.3169 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 15.1175 Willlingness to Pay 16.00706144
H_1 Budget: 350
H_2 Budget: 350
H_3 Budget: 350
```

```
H_4 Budget: 350
H_5 Budget: 350
H_6 Budget: 350
F_1 Supply: 25.49700972
F_2 Supply: 10.78352971
F_3 Supply: 38.91225404
F_4 Supply: 55.56022204
2100 130.75301551
Optimal 3.1356629710371
('H_1', 'F_1', 'S_1') 0.0
('H_1', 'F_2', 'S_1') 0.0
('H_1', 'F_3', 'S_2') 0.0
('H_1', 'F_4', 'S_2') 0.0
('H_2', 'F_1', 'S_1') 0.0
('H_2', 'F_2', 'S_1') 0.0
('H_2', 'F_3', 'S_2') 0.0
('H_2', 'F_4', 'S_2') 9.3035
('H_3', 'F_1', 'S_1') 0.0
('H_3', 'F_2', 'S_1') 0.0
('H_3', 'F_3', 'S_2') 0.0
('H_3', 'F_4', 'S_2') 13.0832
('H_4', 'F_1', 'S_1') 0.0
('H_4', 'F_2', 'S_1') 0.0
('H_4', 'F_3', 'S_2') 0.0
('H_4', 'F_4', 'S_2') 13.0853
('H_5', 'F_1', 'S_1') 0.0
('H_5', 'F_2', 'S_1') 0.0
('H_5', 'F_3', 'S_2') 0.0
('H_5', 'F_4', 'S_2') 0.0
('H_6', 'F_1', 'S_1') 0.0
('H_6', 'F_2', 'S_1') 8.3169
('H_6', 'F_3', 'S_2') 0.0
('H_6', 'F_4', 'S_2') 15.1175
```

We can see that despite the fact that the solution is not exactly the same as the ones that can be obtained using Solver with EXCEL, they are equivalent due to the fact that the prices of the goods produced by the firms were chosen equal and hence indistinguishable by the model. The important thing to note is that the ABM optimisation provides the same objective function and a solution that gets product from each sector for each household in the same way as the EXCEL-Solver based solution indicates. This, is believed, validates the implementation of the optimisation model as part of the `Market` agent. The next subsections will deal with the last two key elements of this agent, namely the price change mechanism and the inventory update.

We can also check that all the values have been correctly calculated, in particular the Γ coefficients are correct[4].

4.6.6 Validation of the Price Change Mechanism

In the inner workings of the `Market` agent, it has been outlined previously that in case still exists unsatisfied demand and available supply after any iteration of the underlying optimisation model, then both the selling price and willingness to pay need to be adjusted. The reason for this adjustment relates to the way the γ coefficients are calculated, as for any good or service, if the price that households are willing to pay is less than the actual price asked by the good/service producer, then γ will be negative and hence the underlying optimisation model will not assign any of that good/service to that particular household.

If prices are not changed, then it is not possible to reach an assignment that either allocates all goods/services or satisfy all household demand for

[4]Please note that this assertion is true to some decimal places, with some of the variances due to small differences in the implementation of the numerical algorithms to perform the optimisation

goods/services. Furthermore, it is clear that in the case of an unbalanced assignment problem either one of the two constraints will be saturated first. In this case either there will be either surplus demand or excess supply. One possible assumption of the model could be that surplus supply is not stored, however, due if a price adjustment mechanism is implemented, the relative scarcity of some of the good/services, could be reflected in an bigger willingness to pay, and also very likely reflected in future adjustments of supply levels for those goods whose price has increased.

In order to test that the price changing mechanism is actually working and reflecting the changes outlined before, an experiment has been conducted based on a validation example using the optimiser, printing out informative messages focusing on the prices and willingness to pay, before and after the optimiser is run, thus checking that the price changing mechanism is doing what is supposed to do. For this purpose we will consider again an instance considered before, as the solution for the model considered is already known (as we are recycling a previous example), we will just focus on the change of prices that occur to see if they are consistent with the suggested course of action suggested by the model. The code for the example is exactly the same presented in listing 4.7, the difference will happen in the internal implementation of the `Market` object that will now print the willingness to pay and prices before and after the optimisation run to make the comparison and check the consistency of the change. For the purposes of the example, a multiplicative change was implemented with $\varepsilon = 1.1$ y $\delta = 0.9$ (See Equation 4.9 and successive explanations for a definition). The output of the run of just two iterations is:

```
Market ID: M_1 is iterating the optimiser: 0
H_1 Demand Sector S_1 7.575 Willlingness to Pay 12.8644342
H_1 Demand Sector S_2 15.7203 Willlingness to Pay 15.62680052
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 12.87440703
H_2 Demand Sector S_2 9.3045 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 10.8231 Willlingness to Pay 12.83589121
H_3 Demand Sector S_2 13.0813 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 10.8178 Willlingness to Pay 12.90091642
H_4 Demand Sector S_2 13.0855 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 12.69256085
H_5 Demand Sector S_2 12.8232 Willlingness to Pay 15.77359466
H_6 Demand Sector S_1 8.317 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 15.1175 Willlingness to Pay 16.00706144
2100 130.75301551

Market ID: M_1 is iterating the optimiser: 1
H_1 Demand Sector S_1 7.575 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 15.7203 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 14.161847733000002
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 10.8231 Willlingness to Pay 14.119480331
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 10.8178 Willlingness to Pay 14.191008062000002
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 13.961816935
H_5 Demand Sector S_2 12.8232 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 6.299999999903605e-05 Willlingness to Pay
↪  16.00706144
1182.459208 71.84727851000001
```

We can see that after the first optimisation Household 1 (`H1`) has not bought any goods from any sector, meaning that the price `H1` is willing to pay for those goods is not good enough for `H1`, hence, it needs to offer more in order to acquire goods. After running the optimisation again, we see that the willingness to pay has increased from $(12.8644342, 15.62680052)$ to $(14.15087762, 17.189480572)$ which reflects a 10% increase in the willingness to pay, which shows that the price adjustment mechanism actually works as intended. The same can be seen for other households in the example. Furthermore, it can be seen that the households that do satisfy their demand after one market round, do not change their willingness to pay at all, which constitutes an additional validation of the correctness of the mechanism.

4.6.7 Validation of the Inventory Update Methods

This is probably the easiest operation to validate as it only requires to check that on every iteration the goods/money go in and out of the corresponding inventories/accounts and that no goods/money are lost in the process. In order to check this behaviour, we will use the same model that was used before to test the optimiser, adding some extra operations derived from the execution of the corresponding transactions suggested by the solution of the assignment model. In this way we can check that both the inventory and account balance state is correctly updated for every agent from the situation prior to the assignment to the state that should be enacted after the assignment takes place. In order to simplify the checks, the validation proceeds looking at the balances in aggregated form to check their consistency, an analogous analysis can be performed on an individual basis for each agent.

The output of the example, with additional printed output, has been included in Appendix F. This decision has been taken to make the presentation cleaner and not cluttered with so much detail. The interested reader can observe that by following the detail of every household and firms in terms of their demand and available supply respectively, the history of the steps that the `Market` takes to allocate can be reproduced. For example, it can be observed that after iteration 1 of the `Market` allocation mechanism, the changes contained in Table 4.1 have been produced[5].

Table 4.1: Comparison of changes between iterations, inventory update mechanism

Measure	Starting Value	Final Value
Firm 1 Inventory	25.497	25.497
Firm 2 Inventory	10.784	2.467
Firm 3 Inventory	38.912	38.912
Firm 4 Inventory	55.560	4.972
Household 1 Demand for Sector 1	7.575	7.575
Household 1 Demand for Sector 2	15.720	15.720
Household 2 Demand for Sector 1	15.471	15.471
Household 2 Demand for Sector 2	9.305	0.0
Household 3 Demand for Sector 1	10.823	10.823
Household 3 Demand for Sector 2	13.081	0.0
Household 4 Demand for Sector 1	10.818	10.818
Household 4 Demand for Sector 2	13.086	0.0
Household 5 Demand for Sector 1	11.141	11.141
Household 5 Demand for Sector 2	12.823	12.823
Household 6 Demand for Sector 1	8.317	0.0
Household 6 Demand for Sector 2	15.118	0.0

We can observe that adding the differences in demand at the household level for each sector provides that 50.59 units produced by sector 2 have been allocated, which is reflected on a decrease of the same available amount (rounding considered) by companies that belong to sector 2 (Firms 3 and 4). For the case of product that belongs to sector 1, the situation is that based on allocation to households, 8.317 units have been allocated, which is consistent

[5] Due to numerical solution of optimisation problems, values smaller than 10^{-4} are considered equal to zero for the purposes of the analysis

with the decrease of the inventory of firm 2 by the same amount (rounding considered). The same process can be repeated after each iteration to check that the inventories are updated according to the optimal market allocation mechanism.

4.6.8 Overall Validation of the Computational Model

With the individual elements of the computational having being validated the validation process is almost completed. However, there is still a last joint validation test that will be run to observe all the aspects interoperating together to see how they interact in the simulation runs. In order to achieve this, a first "rough" attempt was performed with a simple example to see the output and determine visually if the corresponding log makes sense.

For this validation attempt we will create an instance comprising households, firms, sectors, etc. They will be created at random an the output of the execution will be analised. Based on all the previous class definitions and previously presented code, we may proceed to present the first validation example in the following code:

```
import simpy
import Household as hh
import numpy as np
import Economy as ec
import Company as co
import Market as mk
import random as rd
from TimeMeasure import measure
import pprint

```

```python

@measure
def main(nSectors, nCompanies, nHouseholds, nIters, printHousehold=False, printHouseholds=False, printSector=False, printEconomy=False, printFirm=False, printMarket=False):
    env = simpy.Environment()
    sectorColl = ec.economy(env, print=printEconomy)
    for i in range(nSectors):
        sectorColl.__add_val__(ec.sector(env=env, avgprice=rd.uniform(10,20), print=printSector))

    firmColl = co.firms(env, print=printFirm)
    houseColl = hh.households(env,  print=printHouseholds)
    sectors = sectorColl.returnSectorList()
    sectorsexp = list(sectorColl.returnSectorList())
    sectorsexp.insert(0, 'CONST')

    for j in range(len(sectorsexp)-1):
        sectorName = sectorsexp[j+1]
        for i in range(nCompanies):
            a = np.random.dirichlet(np.ones(len(sectorsexp)-1), size=1).round(4).tolist()[0]
            a.insert(0, rd.uniform(0, 2))
            dictoF = {key:value for key, value in zip(sectorsexp, a)}
            firmColl.__add_val__(co.firm(env=env, economy=sectorColl, sector=sectorName, print=printFirm, budget=2000, cdCoeffs=dictoF))

    for i in range(nHouseholds):
        a = np.random.dirichlet(np.ones(len(sectors)), size=1).round(4)
        dictoH = {key:value for key, value in zip(sectors, a.tolist()[0])}
        houseColl.__add_val__(hh.household(env=env, economy=sectorColl, budget=200, salary=20, print=printHousehold, cdCoeffs=dictoH))

    mercado = mk.market(env=env, economy=sectorColl, households=houseColl, firms=firmColl, print=printMarket)

    env.run(until=nIters)

    print("**************************************")
    for key, firm in firmColl.collection.items():
```

```
        print(key)
        pprint.pprint(firm.supplyhistory)

if __name__== '__main__':
    main(nSectors=5, nCompanies=3, nHouseholds=100, nIters=5,
    ↪  printEconomy=True, printFirm=False, printHousehold=False,
    ↪  printMarket=True)
```

Listing 4.8: Code to Validate the Overall Operation of the Code

Once this code is run, output like the following is produced:

```
Economy Agent, Iteration number 0
Market ID: M_1 is iterating the optimiser: 0
Market ID: M_1 is alive and well at time 0
Economy Agent, Iteration number 1
Market ID: M_1 is iterating the optimiser: 0
Market ID: M_1 is alive and well at time 1
Economy Agent, Iteration number 2
Market ID: M_1 is iterating the optimiser: 0
Market ID: M_1 is alive and well at time 2
Economy Agent, Iteration number 3
Market ID: M_1 is iterating the optimiser: 0
Market ID: M_1 is alive and well at time 3
Economy Agent, Iteration number 4
Market ID: M_1 is iterating the optimiser: 0
Market ID: M_1 is alive and well at time 4
**************************************
F_1
{0: [50.17723432, 19.84665811999999],
 1: [60.06545943, 79.91211754999999],
 2: [60.07133872, 139.98345626999998],
 3: [60.05102861, 200.03448487999998],
 4: [60.42611158, 260.46059646]}
F_2
{0: [67.95661733, 1.4432899320127035e-15],
 1: [97.55721647, 0],
 2: [140.05126876, 0],
 3: [201.05491524, 118.76611524000003],
```

```
 4: [236.89833817, 299.92745341]}
F_3
{0: [50.98100088, 0.0],
 1: [68.28331705, 14.970270709999996],
 2: [86.37710029, 100.667671],
 3: [86.60778205, 187.27545305],
 4: [86.60778197, 273.88323502000003]}
F_4
{0: [14.15092993, 2.999999793029673e-08],
 1: [15.9670356, 2.999999626496219e-08],
 2: [18.01621716, 18.016217189999995],
 3: [18.01621715, 36.032434339999995],
 4: [18.01621716, 54.04865149999999]}
F_5
{0: [46.41054763, 0],
 1: [65.72878455, 0],
 2: [93.08722149, 6.493521490000002],
 3: [129.12909527, 98.25651675999998],
 4: [144.68121532, 212.36793207999997]}
F_6
{0: [92.21150949, 63.11990949],
 1: [116.92108145, 171.39889094],
 2: [124.26132742, 295.66021836],
 3: [124.26132743, 419.92154579],
 4: [124.26132743, 544.1828732199999]}
F_7
{0: [62.44165926, 40.84965926000001],
 1: [74.9245614, 111.39511739000001],
 2: [77.45623597, 188.85135336000002],
 3: [77.45623599, 266.30758935],
 4: [77.45623598, 343.76382533000003]}
F_8
{0: [72.440285, 7.105427357601002e-15],
 1: [119.72479673, 3.000003401254503e-08],
 2: [197.87762248, 112.36502251000005],
 3: [253.69974003, 325.63576254000003],
 4: [280.09199799, 567.6217605300002]}
F_9
{0: [38.7096632, 38.7096632],
 1: [38.70966318, 77.41932638],
```

```
 2: [38.70966318, 116.12898956000001],
 3: [38.70966319, 154.83865275],
 4: [38.70966318, 193.54831593]}
F_10
{0: [44.38275148, 0.0],
 1: [58.68982543, 8.57596483],
 2: [74.84394082, 83.41990565],
 3: [74.84394081, 158.26384646000002],
 4: [74.84394082, 233.10778728000003]}
F_11
{0: [42.02366333, 2.999999404451614e-08],
 1: [54.71504672, 4.9999992590699094e-08],
 2: [71.23973559, 13.918635639999998],
 3: [88.5519015, 73.33383714000003],
 4: [97.35192627, 146.97656341000015]}
F_12
{0: [11.83821759, 0.0],
 1: [12.85027327, 0.0],
 2: [13.94885919, 13.94885919],
 3: [13.94885916, 27.897718349999998],
 4: [13.94885919, 41.84657754]}
F_13
{0: [50.57379501, 9.999994610154772e-09],
 1: [67.9308016, 0],
 2: [91.24407237, 48.75317236999997],
 3: [105.82627086, 136.86584322999997],
 4: [111.9052345, 234.86057772999996]}
F_14
{0: [31.06040933, 2.999999360042693e-08],
 1: [37.68599313, 18.72929637],
 2: [41.72969977, 60.45899614],
 3: [41.72969977, 102.18869591],
 4: [41.72969977, 143.91839568]}
F_15
{0: [11.50288982, 1.999999987845058e-08],
 1: [12.41674078, 12.4167408],
 2: [12.41674078, 24.833481579999997],
 3: [12.41674075, 37.25022233],
 4: [12.41674077, 49.666963100000004]}
Total execution time for function (main): 9722 ms
```

The numbers reported for each one of the firms are for each period the quantity produced by each firm and then how much product was supplied to the market (i.e., sold). In this particular run the product can pass from one period to another, i.e., it is not discarded. This means that the following relationship must hold:

$$I_t = I_{t-1} + P_t - D_t$$

where I_t is the inventory at time t, P_t is the amount of product produced at time t and D_t is the supply given to the market at time t. Based on this relationship, we can follow what has happened with a particular firm over the 5 iterations, for example let us consider firm `F_15`, the inventory evolves in the following way

$$(11.50, 0.00, 9.59 \times 10^{-14}, -9.95 \times 10^{-14}, 0.00)$$

where $a \times 10^{-14}$ is a number close enough to zero to be considered zero. If we now put our attention into other firm, we will see that the situation is different to that of firm `F_15`, which basically produces inventory on the first step and then sells everything it produces. For example for firm `F_13` we get a declining inventory:

$$(50.57, 67.93, 42.49, 17.71, 13.91)$$

And finally, for firm `F_2` we get a non monotonic behaviour:

$$(67.96, 97.56, 140.05, 82.29, 55.74)$$

so whilst in general we observe that inventory levels tend to decrease with iterations, which make logical sense as prices and budgets are updated, sometimes it could be the case that this behaviour is not observed.

Finally, a test was performed to stress the implemented algorithm to see how it scales with respect to some macro parameters for the same such as number of sectors, number of companies for each sector, number of households and number of iterations[6]. All these results are summarised in the following tables and graphs. The initial base case from which one factor at a time will be changed to perform sensitivity analysis corresponds to the case of 5 sectors, 3 companies per sector, 100 households and 5 iterations (instance is encoded as $(5, 3, 100, 5)$.

Table 4.2: Sensitivity of execution time with respect to number of sectors

Instance	Time (ms)	Difference wrt Base Case (ms)
Base Case $(\mathbf{5}, 3, 100, 5)$	10,996	0
$(\mathbf{3}, 3, 100, 5)$	6,247	-4,749
$(\mathbf{4}, 3, 100, 5)$	9,349	-1,647
$(\mathbf{6}, 3, 100, 5)$	16,322	5,326
$(\mathbf{7}, 3, 100, 5)$	20,086	9,090

We can observe that plot 4.4, at the level that the sensitivity experiment has been conducted, does not exhibit a observable trend. In order to test the intuition behind the scaling according to this factor, a test has been run

[6]Although analytic estimates would be desirable by some readers, the computational model has so many interacting elements involved that discriminating execution threads proves to be a daunting task, due to the diverse structure of the agents involved. To present a basic numerical comparison, at least provide a sense of the complexity escalation of the current implementation.

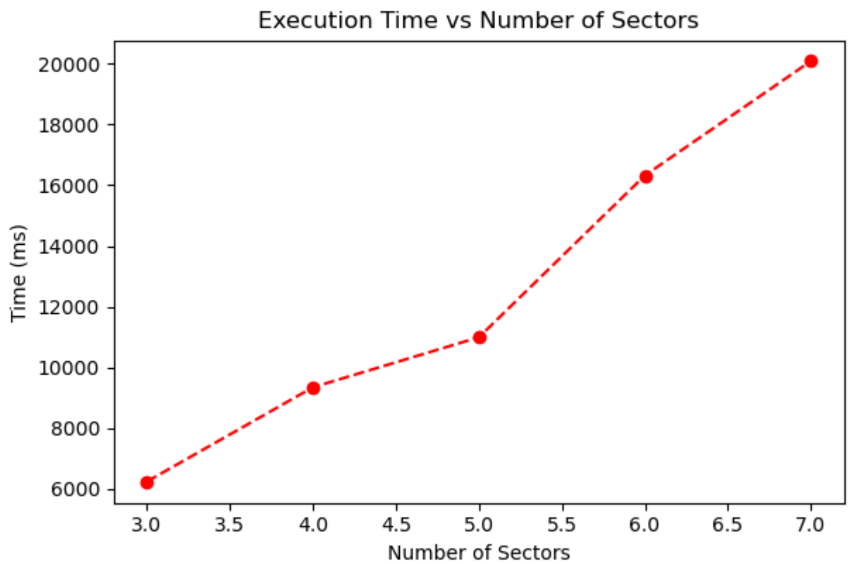


Figure 4.3: Sensitivity with respect to the number of sectors

Table 4.3: Sensitivity of execution time with respect to number of firms per sector

Instance	Time (ms)	Difference wrt Base Case (ms)
Base Case $(5, \mathbf{3}, 100, 5)$	10,996	0
$(5, \mathbf{2}, 100, 5)$	13,413	2,417
$(5, \mathbf{4}, 100, 5)$	14,984	3,988
$(5, \mathbf{5}, 100, 5)$	13,888	2,892
$(5, \mathbf{6}, 100, 5)$	13,738	2,748

separately for a size of 20 firms per sector and the time has been observed, i.e., the experiment has been run for the instance $(5, 20, 100, 5)$ and it took 75,451 ms. It was repeated on a random instance keeping the same number of firms per sector and the time it took was 45,731 ms. It clearly scales as one would expect, but for small differences in size this cannot be appreciated that clearly. Clearly in the newer instances the experiment has been run on an example an order of magnitude larger and from a memory point of view only this would imply larger memory consumption, larger optimisation problems

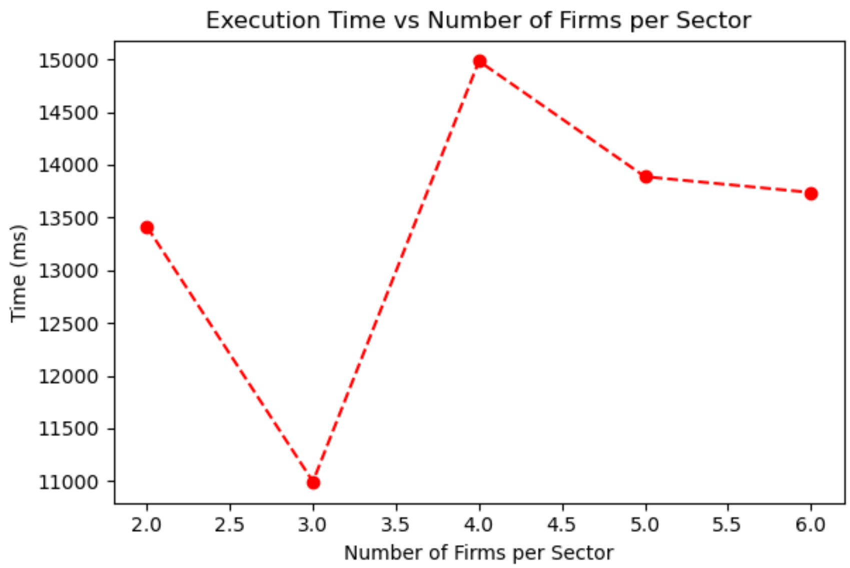


Figure 4.4: Sensitivity with respect to the number of firms per sector

and more degrees of freedom to solve them.

Table 4.4: Sensitivity of execution time with respect to number of households

Instance	Time (ms)	Difference wrt Base Case (ms)
Base Case $(5, 3, \mathbf{100}, 5)$	10,996	0
$(5, 3, \mathbf{80}, 5)$	12,149	1,153
$(5, 3, \mathbf{90}, 5)$	12,685	1,695
$(5, 3, \mathbf{110}, 5)$	14,259	3,269
$(5, 3, \mathbf{120}, 5)$	26,609	15,619

On this example, it can be seen that eventually the number of households make the algorithm to explode in execution time. So this needs to be taken into account in the design of the case study.

At last, an example to see the level of dispersion in between runs was attempted. 50 realisations of a run with the same parameters was attempted.

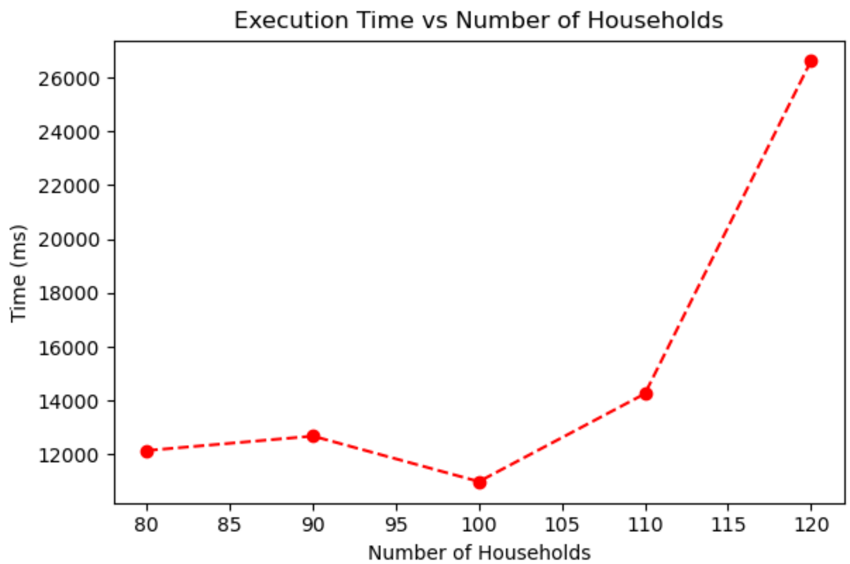


Figure 4.5: Sensitivity with respect to the number of households

Table 4.5: Sensitivity of execution time with respect to number of iterations

Instance	Time (ms)	Difference wrt Base Case (ms)
Base Case $(5, 3, 100, \mathbf{5})$	10,996	0
$(5, 3, 100, \mathbf{6})$	18,953	7,957
$(5, 3, 100, \mathbf{7})$	29,730	18,740
$(5, 3, 110, \mathbf{8})$	42,647	31,651
$(5, 3, 120, \mathbf{9})$	53,235	42,231

Figure 4.7 shows the histogram that summarises the results of this experiment. Here we can observe that variations are observed between individual executions, and it is hypothesised that this is mainly due to the random nature of the instances, which in turn plays a role in the dispersion observed. In any case, previous sensitivity analyses show that growth is pretty much linear with respect to some of the factors, being the number of households a noticeable exception. Also, that what has been used as a base example during this small scaling test takes a reasonable amount of time given the complexity of the

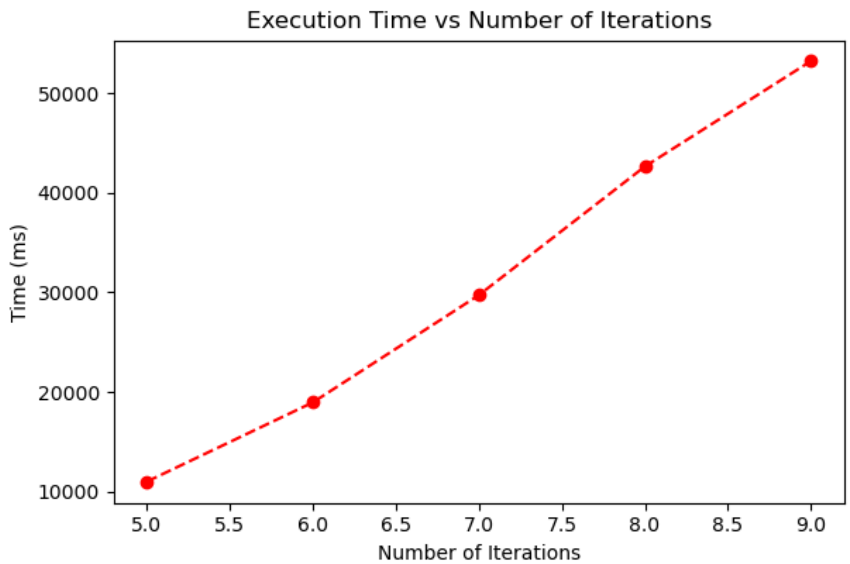


Figure 4.6: Sensitivity with respect to the number of iterations

simulation efforts, hence, times are considered to scale up to a level for bigger instances that will be worked in the next chapters.

4.7 Chapter Concluding Remarks

This chapter has provided the rationale and details of the implementation of the different objects that conform the code that perform the simulation. The code implementation is based on the `SimPy` library, and the implementation reflects the particular requirements this library imposes when used. From an implementation point of view, the code considers some essential properties and methods but complemented with helpers that it is believed make an easier use of the objects in the case studies to follow after validation of the underlying mathematical models (please see Section 4.6).

The software implementation has been partitioned into classes. This way it is possible to modularise the software components, making the code more

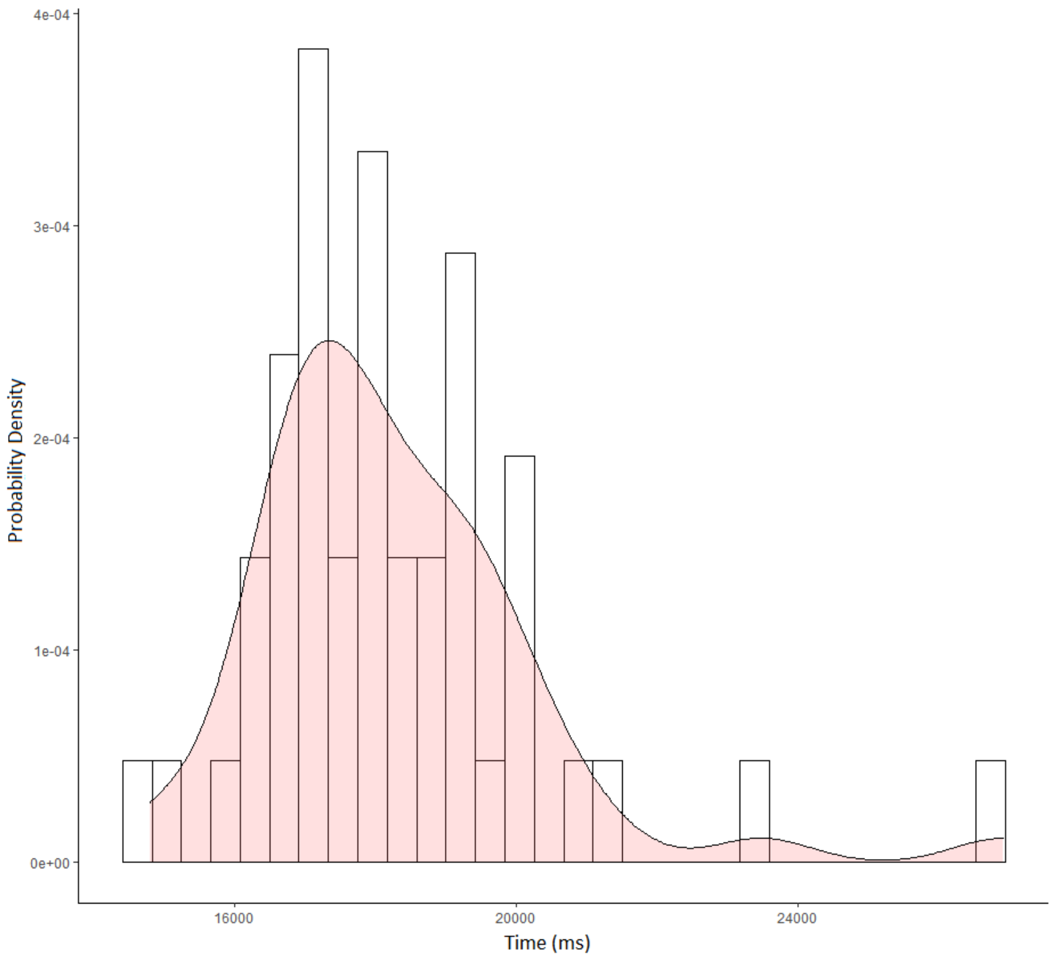


Figure 4.7: Histogram for 50 repetitions for the instance with 7 sectors, 2 firms per sector, 100 households and 5 repetitions

maintainable. Some of the software elements presented herein have been presented in terms of the initial design proposed for them, mainly for the purposes of understanding the structure of the proposed classes. However, some of the classes may suffer from changes and/or additions for which reason the reader is directed to peruse the final implementation available in the appendices.

A brief discussion regarding the use of national accounts information has been conducted, mainly to illustrate the way data will be used. Of particular importance for the thesis is the decision regarding the fitting of producer's/sector's production functions, as it is not possible, due to data not being available to use econometric models to calibrate them. A decision has been taken in order to being able to create a computational representation of firms and/or sectors which does not invalidate the methodology and which can certainly be further improved in the future.

The chapter has also provided some tests of the implementations of the crucial aspects of the objects that have been proposed for the agent based model. The computational model has been decomposed into smaller software elements, denominated classes in usual object oriented parlance, which tested separately can ensure that at least there are no obvious implementation errors.

The main validation effort has been spent on the most complex classes which are `Firm`, `Household` and `Market`. It can be observed that the implementations prove correct for simple examples selected which ensure that the basic constituent elements of these classes are valid.

It can be observed that the implemented algorithm scales to instances of bigger size when compared to the simple example used to validate the model for almost all factors. However, scaling with respect to the number of household seems not to be linear but exponential, which will require some attention in the design of the thesis case study and future examples. Although there is uncertainty with respect to the measured times, it can be seen that in the big picture those differences are not an obstacle for an implementation at the scale of the case study that is performed and analysed in the next chapters.

In the following chapter, the discussion moves into implementing the baseline case study for the thesis, whose implementation will help in the posterior design of experiments and case studies to follow. In case minor changes are introduced in the baseline, they are highlighted if needed.

Chapter 5

Baseline Numerical Model

The implicit research question being attempted in this work can be stated as whether it is possible to achieve enduring community value from mining. The research question itself is a very broad one, and it depends on several conceptual clarifications. To mention just a few of them:

- How can community value be measured? And how is it defined?
- What makes a particular community to be affected by mining? Where are the limits drawn?
- How can enduring benefits be guaranteed? And more importantly, What is the precise meaning of enduring and the temporal extent of the same?

Unfortunately, we do not have clear answers for any of these questions. Answering them would require a deep knowledge of the cultural underpinnings of each community and its culture/value makeup, a daunting task to say the least. As we have already seen in previous chapters, some approaches to measure the impact of economic activities are available from traditional economics, however, the models are usually based on homogeneity assumptions and the pervasive use of a representative economic agent among some other identified problems. We have also reviewed alternative mechanisms to model economic behaviour that allows the use of heterogeneous economic agents, and actually have proposed the skeleton of such a model and illustrated all the complexity required to implement it.

In this chapter, we take a small step back from that complexity and focus our efforts into building a baseline numerical model that will allow us in subsequent chapters to answer specific questions relating to the way the existing "royalties for regions" tax can be used. The idea of the baseline case is to serve as the reference point that can be used in subsequent experiments to compare alternative policies for the use of the royalty. This required a much clearer definition of impact, and how it is measured, as it constitutes the main KPI used to perform the comparison between the baseline case and alternatives, for which different royalty use rules are implemented.

5.1 Introduction

A mentioned already in the text, the idea is to create a case study based on the city of Kalgoorlie-Boulder, in the Goldfields area of inland Western Australia. As we will see shortly, some of the statistical information for the area is available, particularly when it comes down to demographics and income distribution. All the information used in what follows has been obtained from the Australian Bureau of Statistics. The interested reader is encouraged to visit the website `https://quickstats.censusdata.abs.gov.au/census_services/getproduct/census/2016/quickstat/UCL512004`. For completeness some of the demographic data used in this thesis has been reproduced in Appendix G.

According to statistics from the 2016 census (ABS, 2018), there are 12,866 dwellings in the City of Kalgoorlie-Boulder, but not all of them are occupied; just 86% of private dwellings which are 11,543 in total. Based on the same source, the average number of people per dwelling is 2.7, however, care must be used when utilising such numbers, as mistakes can be made by performing wrong extrapolations based on them. For example, the expected number of inhabitants would be 34,738.2 if it were calculated as the total number of dwellings multiplied by the average number per dwelling. The same number would be 31,166.1 if using only the number of private dwellings, and finally, the expected number would be 26,802.9 if we used only the occupied dwellings. The actual number of inhabitants according to the 2016 census was 29,875 people, according to the census community profile for Kalgoorlie-Boulder, as currently shown in the Australian Bureau of Statistics webpage. This data can be downloaded as a `csv` file. Table G.2 contains the information published in the community profile and Table G.3 has been created using the information from the `csv` file. We can observe that there are minimal differences between these two versions. Hence, this information will be used as a reference to generate a realistic instance while understanding that it is possible to find small inconsistencies.

One of the important things that came out from the validation tests performed in Section 4.6 relates to the ability of the model to scale. In general, there was a linear relationship with several of the parameters. However, the complexity of the execution of the model did not scale well with the number of households, and a non linear complexity growth behaviour has been observed in past experiments (see Figure 4.5). For this reason, an instance with 100 households, drawn according to some of the statistical data for the city of Kalgoorlie-Boulder will be utilised. This means that the different levels will need to be scaled down to this base for both producers and consumers. One of the main problems then is the creation of those 100 households in such a way that the age groups, income, sex distribution, and other demographics characteristics are appropriately represented. Each one of the tables in Appendix G represents a marginal distribution, but what is really needed is a joint distribution to link all those different aspects and this last problem is not an easy one to solve.

5.1.1 Interlude 1: Fitting Joint Distributions

Given a set of of random variables $\{X_1, \dots, X_n\}$, each one having its own nature and characteristics (i.e., discrete, continuous, categorical, etc.), its **joint distribution** corresponds to the probability distribution on all possible n-tuples of outputs[1]. The joint distribution encodes information about

[1] The reader is warned here about a possible misunderstanding that could follow which might arise from the notation used. In particular, the common practice of using summations for discrete r.v. and integrals for continuous r.v. The discrete case can easily expressed as a continuous one by using the Delta Dirac function $\delta(x - x_0) = \lim_{\Delta \to 0} U[x_o - \Delta, x_0 + \Delta]$ with $U[a, b]$ denoting a uniform distribution defined in the interval $[a, b]$. In particular, with this notation, we can express a discrete random variable taking the discrete values $\{x_1, x_2, \dots\}$ with point probability masses $\{p_1, p_2, \dots\}$ as a mixture of Delta distributions:

$$p(x) = \sum_i p_i \delta(x - x_i)$$

the **marginal distributions**, i.e., the distributions for each of the individual variables, but also encodes information relating to the **conditional probability distributions** which deals with how the outputs of individual variables are distributed when provided with information of the output of other random variables.

From classical probabilities and statistics, we know that a random variable can be characterised using the probability density function, a similar concept is used for the case of a set of random variables[2]. For a function $f_{X_1,\dots,X_n}(x_1,\dots,x_n)$ to be a proper joint density function for set of random variables $\{X_1,\dots,X_n\}$, the following properties must be satisfied[3]:

1. $f_{X_1,\dots,X_n}(x_1,\dots,x_n) \geq 0$
2. $\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{X_1,\dots,X_n}(x_1,\dots,x_n)\, dx_1,\dots,dx_n = 1$
3. $I\!P[(X_1,\dots,X_n) \in A] = \underset{A}{\int \cdots \int} f_{X_1,\dots,X_n}(x_1,\dots,x_n)\, dx_1 \dots dx_n$

From a joint distribution function we can easily go to marginal and conditional distributions, and this process determine those in a unique way:

1. **Marginal density**:

$$f_{X_i}(x_i) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{X_1,\dots,X_n}(x_1,\dots,x_n)\, dx_1 \dots dx_{i-1} dx_{i+1} \dots dx_n$$

[2]It is assumed that the reader has some familiarity with the concepts discussed, classical references abound, some of them are (Dudley, 2006), (Breiman, 1968), (Nedzela, 1986), (Dodge, 2005), (Walpole et al., 2007) and (Brémaud, 1984) to mention just a few.

[3]Please note that this is just a natural extension to several variables of similar concepts used with uni-dimensional random variables.

2. **Conditional density**:

$$f_{\vec{Y}|\vec{Z}}(\vec{y}|\vec{z}) = \frac{f_{\vec{Y},\vec{Z}}(\vec{y},\vec{z})}{f_{\vec{Z}}(\vec{z})}$$

with $\vec{Y} = (X_{\sigma(1)}, \ldots, X_{\sigma(k)})$, $\vec{Z} = (X_{\sigma(k+1)}, \ldots, X_{\sigma(n)})$, $\sigma(\cdot)$ being any permutation function and k an arbitrary index in the set $\{1, \ldots, n\}$, $f_{\vec{Y},\vec{Z}}(\vec{y},\vec{z})$ the joint density function and $f_{\vec{Z}}(\vec{z})$ the marginal density for the vector $\vec{Z}$. For the particular case that the random variables are independent the following property holds:

$$f_{X_1,\ldots,X_n}(x_1, \ldots, x_n) = \prod_{i=1}^{n} f_{X_i}(x_i)$$

However, it is possible to find that different joint densities provide the same marginals, which makes the reverse process to be complex without additional information being incorporated. For example, it is not difficult to see that if you consider two Bernoulli random variables X and Y each with parameter $\frac{1}{2}$:

- If the two variables are independent, then $I\!P(X = a, Y = b) = \dfrac{1}{4}$ for $(a, b) \in \{0, 1\} \times \{0, 1\}$
- However, if they are dependent, for example $X = 1 - Y$ we have the following relationships: $I\!P(X = 0, Y = 1) = I\!P(X = 1, Y = 0) = \dfrac{1}{2}$ and $I\!P(X = 0, Y = 0) = I\!P(X = 1, Y = 1) = 0$

So, from classical statistics it is clear that obtaining a proper characterisation for the joint density provides all the other useful information. However, this process operates in one direction only, starting from the joint distribution the marginals and conditionals can be obtained but not the other way around. The information provided by the Australian Bureau of Statistics contains the information aggregated at the different variable levels (marginals) but by no means the information is available in order to uniquely determine the joint distribution.

The next subsection looks at one particular technique to try to obtain a joint distribution that respects given marginals. It is not a complete solution to the problem as there is not correlation structure that has been informed that can be imposed, but at least is an option to generate information consistent with the statistical information available.

5.1.2 Interlude 2: Iterative Proportional Fitting

Given the problem of finding a joint distribution with the ability to replicate marginal distributions, one particular technique could come to our aid: Iterative Proportional Fitting (IPF) (Lovelace et al., 2015; Lomax and Norman, 2016).

IPF is a process that allows the adjustment of a table of data cells in such a way that they add up to selected totals for both the columns and rows (for the two-dimensional case) of the table. The same idea can be generalised to tensors (i.e., matrices with more than two dimensions). The IPF algorithm can be described as follows for the two-dimensional case:

For example, let us consider a two dimensional example with row marginals given by $(20, 30, 35, 15)$ and column marginals given by $(35, 40, 25)$ and a seed matrix given by $\begin{pmatrix} 6 & 6 & 3 \\ 8 & 10 & 10 \\ 9 & 10 & 9 \\ 3 & 14 & 8 \end{pmatrix}$.

Algorithm 4: Iterative Proportional Fitting

begin
  Populate table with numbers which will be called seeds
  while *Convergence level is not reached* **do**
    Step 1

      Adjust each row cells to equal the marginal row totals, basically each element of the row is normalised to the row sum and then multiplied by the marginal target.

    Step 2

      Each column cells is proportionally adjusted to equal the marginal column totals

We will illustrate the algorithm on this simple example by a sequence of steps (iterations):

Iteration 1

Row Adjustment → Column Adjustment

	29.62	39.61	30.76
20	8.00	8.00	4.00
30	8.57	10.71	10.71
35	11.25	12.50	11.25
15	1.80	8.40	4.80

	35	40	25
20.78	9.45	8.08	3.25
29.65	10.13	10.82	8.71
35.06	13.29	12.62	9.14
14.51	2.13	8.48	3.90

Iteration 2

Row Adjustment → Column Adjustment

$$\begin{array}{c} \\ 20 \\ 30 \\ 35 \\ 15 \end{array} \begin{array}{ccc} 34.81 & 40.09 & 25.10 \\ \hline 9.10 & 7.77 & 3.13 \\ 10.25 & 10.95 & 8.81 \\ 13.27 & 12.60 & 9.13 \\ 2.20 & 8.77 & 4.03 \end{array} \rightarrow \begin{array}{c} \\ 20.02 \\ 30.00 \\ 35.01 \\ 14.98 \end{array} \begin{array}{ccc} 35 & 40 & 25 \\ \hline 9.15 & 7.76 & 3.12 \\ 10.30 & 10.92 & 8.77 \\ 13.34 & 12.57 & 9.09 \\ 2.21 & 8.75 & 4.02 \end{array}$$

Iteration 3: Finished

Row Adjustment → Column Adjustment

$$\begin{array}{c} \\ 20 \\ 30 \\ 35 \\ 15 \end{array} \begin{array}{ccc} 34.99 & 40.00 & 25.00 \\ \hline 9.14 & 7.75 & 3.11 \\ 10.30 & 10.92 & 8.78 \\ 13.34 & 12.57 & 9.09 \\ 2.21 & 8.76 & 4.02 \end{array} \rightarrow \begin{array}{c} \\ 20.00 \\ 30.00 \\ 35.00 \\ 15.00 \end{array} \begin{array}{ccc} 35 & 40 & 25 \\ \hline 9.14 & 7.75 & 3.11 \\ 10.30 & 10.92 & 8.77 \\ 13.34 & 12.57 & 9.09 \\ 2.21 & 8.76 & 4.02 \end{array}$$

There are implementations of this algorithm for both `Python` and `R`. We will illustrate the use of the `Python` library by looking at a simple example:

```python
import numpy as np
from ipfn import ipfn

m = [[40, 30, 20, 10], [35, 50, 100, 75], [30, 80, 70, 120], [20, 30, 40, 50]]
m = np.array(m)
xip = np.array([150, 300, 400, 150])
xpj = np.array([200, 300, 400, 100])

aggregates = [xip, xpj]
dimensions = [[0], [1]]

IPF = ipfn.ipfn(m, aggregates, dimensions, convergence_rate=1e-6)
m = IPF.iteration()
print(m)
```

Listing 5.1: Example of the IPF algorithm with `Python`

The seed for this example is the following table:

$$\begin{bmatrix} 40 & 30 & 20 & 10 \\ 35 & 50 & 100 & 75 \\ 30 & 80 & 70 & 120 \\ 20 & 30 & 40 & 50 \end{bmatrix}$$

With sum for rows given by $(100, 260, 300, 140)$ and sum for columns given by $(125, 190, 230, 255)$. The output of this code is the matrix:

$$\begin{bmatrix} 64.55852549 & 46.23247384 & 35.38430991 & 3.82473358 \\ 49.96791318 & 68.15934981 & 156.4985403 & 25.37417955 \\ 56.72193658 & 144.42821667 & 145.0824759 & 53.76734831 \\ 28.75162474 & 41.17995969 & 63.03467389 & 17.03373857 \end{bmatrix}$$

With sum for rows given by $(150, 300, 400, 150)$ and sum for columns given by $(200, 300, 400, 100)$, which confirms reproduction of the desired marginals. It is important to note the following:

- The marginal column and row total must sum to the same value for IPF to work
- IPF will work with any non/zero seed values, but a more accurate seed (proportionate) will probably provide a more accurate results
- IPF yield values with decimals. Separate rounding may be necessary if integral values are needed
- The algorithm being numerical will converge to the selected marginals, but in most cases there will be small deviations from the desired marginal totals

- If a marginal value is zero, then IPF does not work, a workaround is to consider a small marginal value such as 0.001
- If a cell is initialised with seed zero, no adjustments will be made to it. A possible workaround is to set the seed to a small value such as 0.01

5.2 Baseline Case Study Definition and Scope

As mentioned before, given the bad scaling that the algorithm exhibits regarding the number of households, and also due to the actual impossibility of having the joint distribution of the data forces making some calls for the implementation of the model in a setting useful enough to draw conclusions. Furthermore, one of the main points of the work is to show that the proposed methodology can be used as a tool to assess the impact of the mining activity in a given region. The exercise of implementing the proposed model in a realistic case study has shown that the data requirements and availabilities are not aligned with the conceptual model developed before. This section outlines the nature of the case study to be utilised in the rest of the work, certain assumptions that were made to end up with an implementation and how this is reflected both at the code and data levels.

As it is not possible to create a representation in silico of the city of Kalgoorlie-Boulder, due to the limitations mentioned before, it has been decided that the available data will be used to create the artificial city of **Fakegoorlie** as an alternative inspired on Kalgoorlie, as much as practical, but scaled down in size to adapt to the limitations of the proposed software, but trying to honour some of the marginal distributions contained in the statistical data obtained from the Australian Bureau of Statistics (ABS).

5.2.1 Assumptions and Scope

The case study consists of **100** households with all other data to be used to be scaled down to be consistent with this definition. Recall that this number is due to the poor scaling of the algorithm with respect to this parameter, hence, this number becomes the bottleneck for the whole simulation and the key driver in the construction of the dataset for Fakegoorlie. Also, it will assumed that only **3** firms per sector currently operate within the region as the main element sought, which is community value is broadly approximated by the concept of local multiplier and contribution to local GDP (more to come on this later in the chapter). One important assumption in the case study regarding all firms is that they cannot store product.

One factor not initially considered in the development/implementation of the computational model, but that could be of relevance, relates to the number of firms that are local to the region and those who are not. If a firm is local to a region it is assumed that it is owned by inhabitants of the region and that the profits it produce are kept as much as possible in the region. This distinction is important as the way impact is characterised depends on measuring the monetary flows in and out of a given region/area. For this purpose, to assist in the delimitation of what constitutes local to the community, the `firm` agents have been modified by adding a property that indicates the percentage of local ownership for firms on every sector. For every company, the local portion of it is accounted in terms of contribution to the local GDP.

Households are assumed to be all local, i.e, the workers are local and they spend their salary in the city of *Fakegoorlie*, but it is acknowledged that further studies could consider FIFO workers as they move money across regions and this has an incidence on the measurable benefits of the mining activity for the local community. Not only the multipliers will change, but also because of "migrant" population the benefits/detriments could affect a different number of people, not directly involved with the extractive industry.

For the purposes of creating data consistent with that provided by ABS, but at a scaled down level (to a 100 households) we will consider the following aspects for which we will attempt to reproduce the marginal distributions:

- Family composition as described by Table G.15
- Dwelling Tenure as described by Table G.20

Please note that family income per family composition given by table G.28, already contains the joint distribution of income for the different family compositions. Hence, what is needed is to add the joint dimension of dwelling tenure altogether with family composition which can be performed using the iterative proportional fitting method. The previous fit is proposed based on the available information available of ABS's data download section. Other pieces of information that are thought be relevant, at the individual disaggregated level, but not readily available are:

- Educational level
- Hobbies
- Travel patterns

- Lifestyle patterns
- Club memberships
- Etc.

The particular fit of the joint distribution was performed (see code in Appendix G.1). We observe that the fitted joint distribution needs to be saved. The idea behind this action is to guarantee that this table will be fixed for future simulations and not changing from run to run, otherwise it will be difficult to isolate this impact in future comparisons. The input to the fitting process is the marginal tables, considering the percentages, rounded to an integer respecting the total sum being 100% (see Tables 5.1 and 5.2):

Table 5.1: Family Composition

Category	%
Couple family without children	35
Couple family with children	50
One parent family	14
Other family	1

Table 5.2: Dwelling Tenure

Category	%
Owned outright	17
Owned with a mortgage	42
Rented	38
Other tenure type	1
Tenure type not stated	2

Table 5.3 shows the seed used by the implementation. And the outcome after the process is performed is given by Table 5.4.

Table 5.3: Initial Seed for Iterative Proportional Fitting, Family Composition - Dwelling Tenure

	Owned	Mortgage	Rented	Other	Not Stated
Couple family without children	6	2	4	5	1
Couple family with children	7	2	3	3	6
One parent family	5	1	7	7	3
Other family	6	5	1	9	7

Table 5.4: Result of Iterative Proportional Fitting, Family Composition - Dwelling Tenure

	Owned	Mortgage	Rented	Other	Not Stated
Couple family without children	5.502699	15.308440	13.606465	0.405385	0.177011
Couple family with children	9.657226	23.028243	15.350991	0.365889	1.597651
One parent family	1.728094	2.884522	8.973383	0.213879	0.200122
Other family	0.111970	0.778751	0.069217	0.014848	0.025213

It is easy to check that the marginal distributions are honoured. For example, adding all the numbers in column "Rented" of Table 5.4 gives:

$$13.606465 + 15.350991 + 8.973383 + 0.069217 = 38.0005 \approx 38$$

Similar test can be performed for every other column, which are not included in the text but prove the implementation correct. Performing the same process for the row "Couple family with children" gives:

$$9.657226 + 23.028243 + 15.350991 + 0.365889 + 1.597651 = 50$$

On the other hand, the industry sectors will be based on the ones defined by the Australian Bureau of Statistics (ABS). They are mentioned in the Table G.29. We shown the final list in Table 5.5, it is worthy to note that this list is consistent with the definition made before during the preliminary model (see subsection 3.1.1), where 35 sectors were identified. In the case of the ABS list, they group economic activity into 19 sectors, which after intersection with the previously considered list provides 17 sectors, which certainly helps to reduce the computational demands on the implemented model.

Table 5.5: Industry Sectors for Case Study

Sector
Agriculture, Forestry and Fishing
Mining
Manufacturing
Electricity, Gas, Water and Waste Services
Construction
Wholesale Trade
Retail Trade
Accommodation and Food Services
Transport, Postal and Warehousing
Information Media and Telecommunications
Financial and Insurance Services
Rental, Hiring and Real Estate Services
Public Administration and Safety
Education and Training
Health Care and Social Assistance
Arts and Recreation Services
Other Services

It is worthy to mention here that in an ideal world, more information from companies would be desirable. Whether the information exists or not is not being questioned, but for sure public availability of anonymised data is so far unbeknown to the author. Some examples of desirable information are:

- Corporate structure
- Debt structure

- Sales
- Buying patterns
- Specific technology
- Employment policies
- Provisioning strategies
- Inventory Management practices
- Etc.

Also, it has been considered important to incorporate distributional characteristics of salaries among the households per industry, firstly, to appropriately reflect the sectorial composition of the town, and secondly, to represent the consistent differences of salaries between sectors[4]. For this reason, two tables in the Appendix, Table G.29 looking at the total column and Table G.28 were considered to fit a joint distribution using the IPF technique. The code is similar to the one already shown, so it will be not reproduced here. The output of the process is shown in section H.2.1 in the Appendix and omitted here for brevity.

For the generation of a "representative" sample of Households, the previous tables were used with the following important decisions that were taken to consider both the salary and household composition:

[4]In the case of a mining town, the salaries of mining employees usually outperform those of the general population. Among other measures, in those mining towns, it is not uncommon to provide subsidised housing in order to attract skilled workers, specially in areas different from mining that cannot compete with those high salaries in those towns. Examples of such subsidies are for teachers, lecturers, doctors, nurses, police, etc.

- If the salary category belongs to any of the following categories: `'NIL'`[5], `'Partial'`[6] and `'Not Stated'`[7], then the salary is drawn from a uniform distribution with minimum 350 and maximum 1500. The minimum has been calculated considering the payments that social security services such as centrelink offer on a weekly basis ($280 base plus $70 in case they are renting). The upper limit has been deemed reasonable but it needs to be acknowledged that is arbitrary.

- Otherwise, the salary assigned to the household does not suffer any change as it is consistent with its category

For the purposes of differentiating the money that stays in the region as opposed to the money that flows outside a given region, a distinction between local and non-local firms has been made. One possible way of doing this is to consider all sectors having the same percentage of local ownership, other way is to assign a specific value for each sector. for the purposes of the baseline case study and all future work it has been decided that a different number will be used for each sector, and within each sector, a normally distributed value with mean centred on the sector percentage of ownership and standard deviation in the order of 2.5% relative to the average value will be used for each firm. For example, for `Agriculture, Forestry and Fishing`, the nominal sector local ownership is 89.9%, with a 2.5% standard deviation relative to this percentage, this is equivalent to 2.2475%, which implies that the value drawn will be in the interval $[83.1575, 96.6425]$ with a confidence of 99.9%. As

[5] In the information provided by ABS there is a category stated as Negative/Nil income corresponding to 0.789% of the families

[6] This category corresponds to "Partial income stated" as per ABS categories

[7] This category corresponds to "All incomes not stated" as per ABS categories

no data indicating the percentage of local ownership is available from public sources, the percentages of local ownership were chosen with the help of a random number generator but with a certain logic where different bounds were used to obtain a final data that seemed consistent. For example, in *Fakegoorlie* services should probably be owned mostly locally whilst transport, postal and warehousing might not. The decision on what should be low, medium or high was driven by intuition. This is not considered to be problematic as the objective of the experiment relating to the introduction of this factor is to assess if local ownership introduces changes in the evolution of the multipliers, but with no intention to precisely measure the influence but just to allow a comparison with other cases where local ownership is not present. Table 5.6 contains the local ownership percentages for every sector that are used in the rest of this work.

Lastly, but by no means least, the process to adjust the production functions for companies has already been explained previously, however, the mechanism for buying the raw materials/services from the different sectors will equally distribute the money spent for every sector at a preferential price of 80% the normal list price, also, it will be assumed that for this market there are no supply constraints, otherwise a model similar to that implemented by the market would be required just for raw materials/services. For the case of consumers the utility functions have been created at random using Dirichlet distributions with the particular care that when the corresponding Household owns the dwelling, the associated coefficient in the utility function is set to zero, thus not participating of the utility calculations but still being considered in the budget constraint of the utility maximisation associated with the

Table 5.6: Ownership of Industry Sectors for Case Study

Sector	Percentage Local
Agriculture, Forestry and Fishing	89.9
Mining	33.0
Manufacturing	68.3
Electricity, Gas, Water and Waste Services	36.7
Construction	33.2
Wholesale Trade	58.3
Retail Trade	15.4
Accommodation and Food Services	78.9
Transport, Postal and Warehousing	9.9
Information Media and Telecommunications	42.3
Financial and Insurance Services	13.4
Rental, Hiring and Real Estate Services	63.2
Public Administration and Safety	11.2
Education and Training	27.2
Health Care and Social Assistance	22.0
Arts and Recreation Services	83.5
Other Services	85.4

Household.

In the same way as has already been explained, a case is created and saved to eliminate the noise that different random realisations of the agents may have when comparing different conditions. The programming technique requires the use of the `pickle` module, to make data persistent. `pickle` is a module that is part of `Python`'s Standard Library.

5.3 Numerical Experiments

A group of simulations was run with some of the parameters fixed in the way it was mentioned before to guarantee consistency across the base case and alternative cases. In fact, the only source of uncertainty relates to a parameter introduced late in the development of the work and that indicates the percentage of local ownership of a given company. In the first subsection the metric will be discussed and in the second subsection the results will be presented and analysed.

5.3.1 Definition of Metric to Measure Impact

For the base case the first thing to note is the way the contribution to the local economy has been measured. Here the work of Sacks (2002) is of great help, and the interested reader is encouraged to peruse this publication. In this work, a methodological proposal for the measurement of the impact in a local economy is proposed. Essentially this proposal implements a measurement based on the distinction of local versus non-local economic agents and the measure of impact is based on the idea of the classical multiplier, but instead of considering an infinite number of rounds of spending for its determination, the LM3 metric (Sacks, 2002) considers only 3 rounds as an approximation of the original infinite series (which usually converges after some steps). This require to introduce small changes in the code that will allow to keep track of expenditure within the local context, furthermore, it is possible to isolate the monetary flows only related to mining, be that originating in salaries or mining expenditures into the local economy. Initially it was thought that adding salaries into the calculation was a good idea, but the reality is that the salary component dominates the multiplier calculation thus reducing substantially its value.

Below, a presentation of the way the LM3 multiplier is calculated is presented. Essentially, when money starts to flow into the local economy we account for it in a manner similar to that of classical multipliers but restricted to the local economy[8]. The calculation of the LM3 multiplier requires three terms:

1. First step:
   - Account for all money flows that occur to and from local entities, i.e., account for all income into the locally owned portion of each firm
   - Let us denote the sum of the local expenditure by firms by

   $$E_1 = \sum_{j=1}^{M} X_j \cdot c_j \cdot \varepsilon_j$$

   with M the number of firms, $X_j = \sum_{k=1}^{M} x_{k,j}$ the quantity of product/services to be bought from firm j by firm k, c_j the cost of product j, ε_j the proportion of local ownership of firm j, and finally, the first round local profits accounted as the sum over all firms proportional to their local participations.

2. Second step:
   - Repeat the previous to obtain E_2

3. Third step:
   - Repeat the previous to obtain E_3

[8] Similarly, the mining equivalent can be defined by further restricting those flows to those related to mining only

The total local multiplier is then defined as:

$$LM3 = \frac{E_1 + E_2 + E_3}{E_1} \tag{5.1}$$

And a local sector multiplier can be defined analogously as:

$$LM3_X = \frac{E_1|_X + E_2|_X + E_3|_X}{E_1|_X} \tag{5.2}$$

with $E_i|_X$ representing the restriction of the local money component to sector X activities only, i.e., just focusing on the flows that belong to sector X, with $i \in \{1, 2, 3\}$. For example, for the mining sector (denoted using M), we have:

$$LM3_M = \frac{E_1|_M + E_2|_M + E_3|_M}{E_1|_M}$$

In order to provide better guidance on the notation used in future examples, it is worthy to note that $LM3$ multiplier refers to the multiplier calculated using Equation (5.1), $LM3_M$ represents the restriction of the $LM3$ multiplier to the mining sector and $LM3_A$ represents the restriction of the $LM3$ multiplier to the accommodation sector, being the sector local multipliers calculated according to the formula presented in Equation (5.2).

The code presented in previous chapters has been adapted then to add this metric and measure it on every iteration to be presented as an indicator that evolves over time. The idea is to use this metric to understand the impact different policies have on the use of the "Royalties for Regions" money.

As the main ingredient in the proposed calculation is the determination of the money flows between firms and between consumers and firms, the modification required for every firm (consumers are assumed local) is essentially to account for all the money a firm receives in one iteration of the algorithm. It is tempting though to try to use techniques similar to those presented in Chapter 3, however, it needs to be noted that the simple ABM presented there is based on the input-output matrix, whose coefficient $a_{i,j}$ is calculated as the expenditure in sector j by sector i per unit of output of sector j in the period, and this requires the accounting of money flows which the classical Leontief multipliers also requires, with the difference that the LM3 implementation uses only two iterations as the first step of the calculation is direct and the starting point. In particular, for every iteration of the main algorithm, the consumption decision of a particular firm into every sector is determined by solving an optimisation problem, this consumption by a given firm remains fixed whilst calculating the LM3 multiplier, hence, it can be normalised and used as template to iterate with it the two times needed to determine the LM3 multiplier of that particular firm.

5.3.2 Results and Analysis

A first run of the main algorithm for the agent based model, consisting of 10 principal iterations was run, firstly to see the type of output and secondly to get an idea of the time required to run each iteration. The output of this run is contained in Appendix H.2.2.

For this run, it can be seen that the times per iteration are on average approximately 43 secs. The results are shown in table 5.7.

However, it needs to be noted that the calculation of impact hasn't been

Table 5.7: Cost per Iteration

Iteration	Time Marker (secs)	Δ (secs)
1	1648484129.333	-
2	1648484166.698	37.364
3	1648484210.629	43.931
4	1648484259.189	48.560
5	1648484297.764	38.576
6	1648484339.774	42.010
7	1648484382.200	42.426
8	1648484421.739	39.539
9	1648484468.710	46.971
10	1648484513.590	44.880
Average		**42.70**

implemented at this point. The output that can be obtained is similar to the one analysed before, be aware though that for this example there is no inventory.

Several KPIs could be monitored when a run is performed. As mentioned previously, Appendix H.2.2 contains an example of such output. For the purposes of understanding the information it conveys an extract of it will be analysed in more detail:

```
F_4
{0: [451.79652791, 0.0],
 1: [1002.80103097, 1002.80103097],
 2: [1390.12440141, 1390.12440141],
 3: [1819.52765462, 1819.52765462],
 4: [2204.72413934, 2204.72413934],
 5: [2543.4477093, 2543.4477093],
 6: [2859.50530874, 2859.50530874],
 7: [3109.27853978, 3109.27853978],
 8: [3330.37067884, 3330.37067884],
 9: [3527.87269266, 3527.87269266]}
F_5
{0: [540.16565814, 540.16565814],
```

```
 1: [888.13852606, 888.13852606],
 2: [1452.03832064, 1452.03832064],
 3: [1916.07926687, 1916.07926687],
 4: [2299.11708841, 2299.11708841],
 5: [2628.92576274, 2628.92576274],
 6: [2937.9268859, 2937.9268859],
 7: [3174.5484729, 3174.5484729],
 8: [3386.3103192, 3386.3103192],
 9: [3578.00572062, 3578.00572062]}
F_6
{0: [645.7845452, 570.4395731000008],
 1: [1052.17877139, 521.2466713900005],
 2: [1744.16683211, 1080.7659321100007],
 3: [2208.15760474, 1373.06690474],
 4: [2563.44257804, 1566.5514780400008],
 5: [2860.68978276, 1678.6489827600014],
 6: [3143.50788854, 1789.3943885400017],
 7: [3344.61713378, 1768.996133780001],
 8: [3532.57472875, 1781.7374287499995],
 9: [3701.11715509, 1748.97235509]}
```

In this partial output, the three firms `'F_4'`, `'F_5'` and `'F_6'` belong to the Mining sector, as the example that built the simulation assigned these identifiers to them. The numbers shown on this output are by column, first the iteration number (here indicated with numbers 0 to 9), followed by a vector of dimension two containing the amount available for sale at the start of the turn on its first component and the total amount sold in that turn on the second component.

It can be observed that `'F_4'` and `'F_5'` tend to sell most of their production on every iteration, on the other hand, `'F_6'` consistently sells to consumers less than it produces but still manages to sell around half of its production. More interestingly, the flows of money of a particular instance can be examined in closer detail, which the next paragraph illustrates.

A test to track the flows of money has been performed, with the objective of checking the calculations, after introducing all the elements that allow to pass contributions between firms and between consumers and firms. One firm was selected (`'F_6'`) and some trackers were put on the code to print some messages to the console to check that the numbers actually do add up. Table 5.8 summarises the evolution between two iterations.

Table 5.8: Money Flows for Firm `'F_6'`

Iteration	Other Producers	Budget	Payments	Balance
0	0.000	50,000.000	40,000.000	10,000.000
1	64,677.209	78102.319	62481.855	15620.464

The list of payments `'F_6'` received from buyers during iteration 0 were:

```
Payment to F_6 369.4437688134416
Payment to F_6 68.98096051308211
Payment to F_6 202.50498338800668
Payment to F_6 5.360584886537961
Payment to F_6 18.192681873540252
Payment to F_6 388.41060736245163
Payment to F_6 629.6909381875066
Payment to F_6 19.69148520453701
Payment to F_6 195.90502317406148
Payment to F_6 620.3605498837339
Payment to F_6 62.81921348565098
Payment to F_6 21.896931547445156
Payment to F_6 522.069657564493
Payment to F_6 221.51683104605564
Payment to F_6 42.29956067479744
Payment to F_6 35.96677903301834
```

The first 4 numbers under `'F_6'` for each iteration are respectively: moneys received from other producers, available budget before payments, expenses (payments to other producers) and finally available budget after payments. It can be seen that initially there are no money received from other producers ($0), initial budget is $50,000.000, payments to other producers amount to $40,000.000 (80% of the available budget has been used) and the budget after payment is $10,000.000. As the code output shows, the lines starting with `Payment` contain records of contributions from consumers to the different sectors (in this case product bought from firm `'F_6'`), their total is approx. $3,425.111. The total of sales made by the firm to other producers from all sectors is approx. $64,677.209, which is recorded in the line corresponding to Iteration 1, which when added with the rolling budget of $10,000.000 and the sales to consumers for a total of $3,425.111 explain the value of $78,102.319 = $64,677.209 + $10,000.000 + $3,425.111. This check is really important as the code has been changed to account for those flows and a correct implementation of the same guarantee a proper calculation for the LM3 multiplier.

The following test analyses what happens with the average duration of iterations when the impact measure (inspired in LM3) is introduced, additionally, some repetitions are presented and a basic test to plan assessment of the uncertainty in the result was conducted.

The evolution of the local multiplier is given by

```
[1.1702497272408505, 1.2801967609151472, 1.1647911428479623,
↪  1.0795650447816776, 1.0306571902208868, 1.0114147843086048,
↪  1.0149270125864405, 1.0174506745568694, 1.0212229666830879,
↪  1.0229476784353762]
[1.225655381409775, 1.3605894682893174, 1.3361702296675935,
↪  1.3410928055808577, 1.3442050677976984, 1.3729967208851293,
↪  1.4427182373669538, 1.468196664184207, 1.509559252421928,
↪  1.4977418764811312]
```

where the first line exhibits the evolution of the multiplier as defined by a first crude idea and that will be called multiplier A[9]. The average time of running each iteration has been calculated for this instance and presented in Table 5.10 it can be seen that it is consistent with the original calculations performed before but apparently the first iteration costs more. In any case, the average is lower probably due to some heavy attempt at optimising the existing codebase.

Table 5.9: Cost per Iteration Inclusive of LM3 Calculations

Iteration	Time Marker (secs)	Δ (secs)
1	1648549003	-
2	1648549078	74.39283991
3	1648549110	32.60148001
4	1648549146	35.82412004
5	1648549173	27.39063001
6	1648549204	30.05397987
7	1648549234	30.46437001
8	1648549262	28.39101005
9	1648549292	29.74662995
10	1648549322	30.21063995
Average		**35.453**

A first run was performed in order to understand the performance of the whole system. The test consists of three instances of the whole Agent Based Model algorithm, each one for 10 periods or iterations and then two KPIs

[9] The very first idea for an LM3 inspired multiplier was to consider

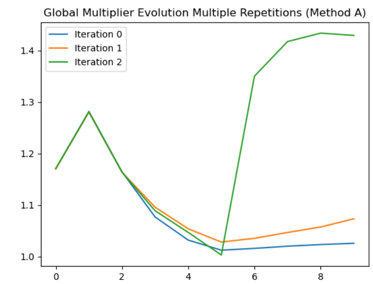
$$\frac{S + (E_1 + E_2 + E_3)}{S + E_1}$$

with S the salaries in the region, but at the test shows, it can be seen that the salary component quickly dominates the calculation making the multiplier to converge to 1 (basically it could end up being proportional to salary divided by salary which is one) were observed, the LM3 inspired measure described before and an alternative which considers salary into the calculation (the very first idea but that proved not good). The results of the test can be seen in Figure 5.1. On the first line of the figure is presented the global multiplier, second line contains the mining multiplier, third line contains an example of accommodation sector. Left column of the figure contains the multiplier calculated with the initial (defective) idea, the right column presents the multiplier as calculated using the final proposal.

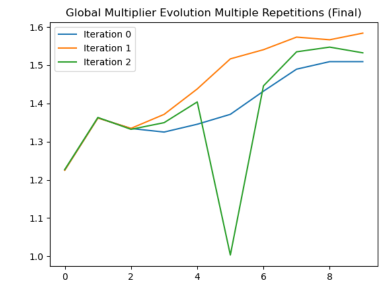
5.3.3 A Larger Example

A larger example was run consisting of 20 repetitions of the whole ABM model with each repetition consisting of 20 epochs/periods. The time taken is the expected due to the tests performed before: in the order of 40 seconds per iteration multiplied by 20 gives a total of 800 seconds (approx. 13 minutes) per ABM run and this is repeated 20 times which gives a total of around 4 hours to run this simple example, which among other things justifies the vast amounts of time used in careful planning and testing of the code before arriving to the point where the iterations can be run with confidence. The results are summarised in the following plots and tables to be found in the next pages.

One of the important things to note at this point is that the results between iterations visually appear to be consistent enough, however the evolution of the multiplier seems to be less chaotic for the global multiplier than for any particular industry. This could be explained for the capabilities of the markets to average out individual variations. For the examples chosen and represented



(a)



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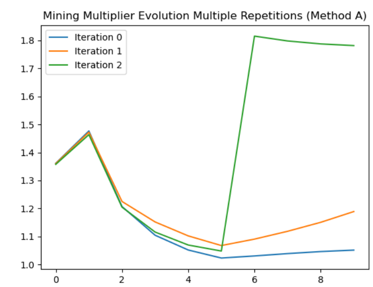

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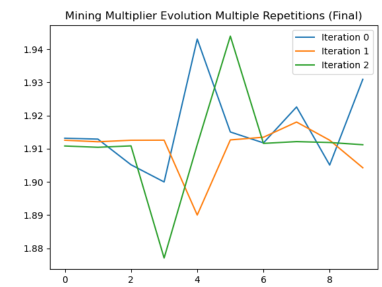

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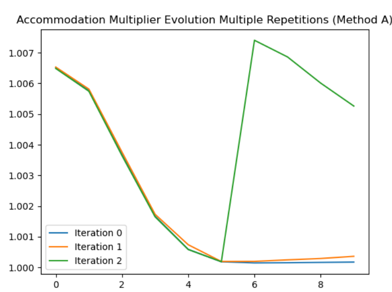

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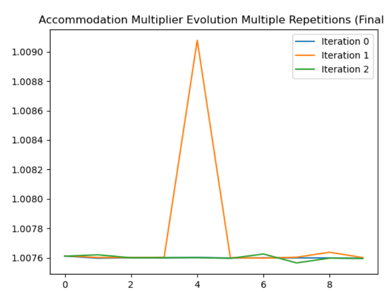
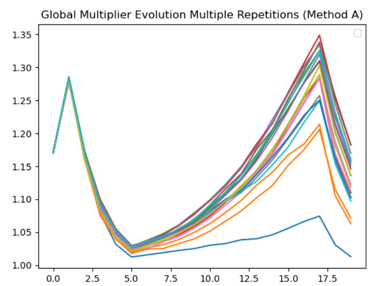


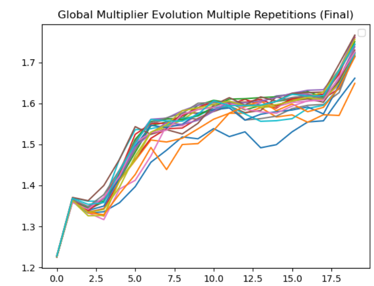
(f)

Figure 5.1: Different Results from Baseline Realistic Test, Three Repetitions, initial testing. (a) and (b) $LM3$ multiplier, (c) and (d) $LM3_M$ multiplier, (e) and (f) $LM3_A$ multiplier. All simulations on the left column ((a), (c) and (e)) used a method for calculating the LM3 multiplier that considers salary, which showed to be inadequate and shown here for illustration purposes. All simulations on the right column ((b), (d) and (f)) use the LM3 multiplier in its final version. Fluctuations observed are due to the stochastic nature of the simulation.

by Figure 5.2 (d) and (f), it can be observed that the clusters of high volatility in the evolution of the multiplier are located in relatively disjoint zones of the timeline.



(a)



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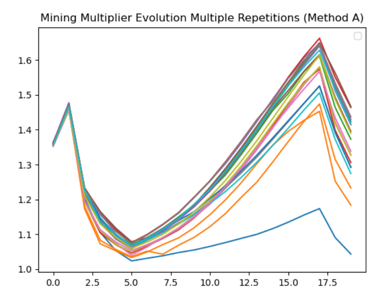

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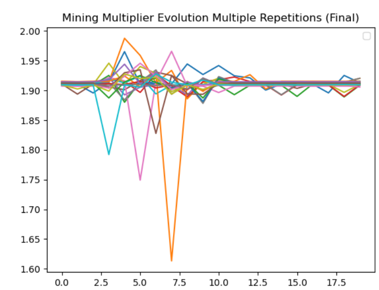

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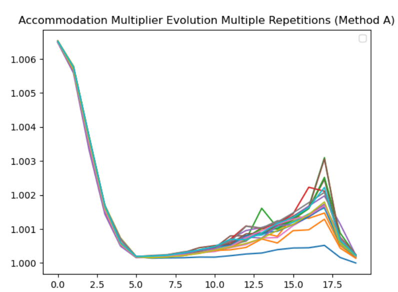

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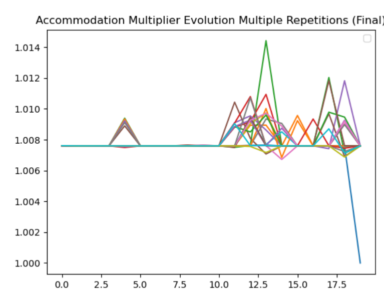


(f)

Figure 5.2: Different Results from Baseline Realistic Test, Twenty Repetitions. (a) and (b) $LM3$ multiplier, (c) and (d) $LM3_M$ multiplier, (d) and (f) $LM3_A$ multiplier. All simulations on the left column ((a), (c) and (e)) used a method for calculating the LM3 multiplier that considers salary, which showed to be inadequate and shown here for illustration purposes. All simulations on the right column ((b), (d) and (f)) use the LM3 multiplier in its final version. Fluctuations observed are due to the stochastic nature of the simulation.

From this moment onwards in the text, no further mention will be made about the initial multiplier idea. This is obviously motivated by the fact that it tends to converge to 1 in the long run with small jumps from time to time, hence, it has been considered that including salaries in this calculation is a mistake. In particular, for the example under study as there was an escalation process to a 100 Households and there were simplifying assumptions such as all companies starting with the same seed capital, then it becomes clear that some of the things observed could be an artefact of these decisions.

One interesting analysis that can be performed is about the variability on each iteration. In order to understand this, the average and standard deviations were calculated among different iterations for the same time period.

This last example is better understood when considering the graphs associated to the realisations with the average in a different colour. They can be found on Figure 5.3.

5.4 Chapter Concluding Remarks

This chapter has presented the reader with the numerical output of the model run in the selected baseline case study. In the chapter, the landing of the proposed model into the realm of practical implementation showed the different challenges that a study of this nature might face. Discussions regarding the use of existing information, strategies to adapt to not readily available sources (particularly the disaggregated information needed from census), and also required ad-hoc assumptions, were conducted. When an answer is sought it is

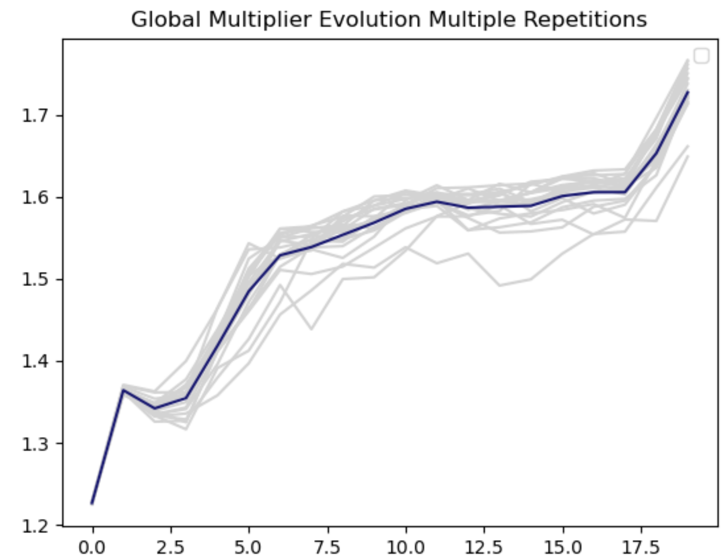
Table 5.10: Average and Standard Deviation Evolution for Multiplier

Step	$LM3$ Avg.	$LM3$ Std.	$LM3_M$ Avg.	$LM3_M$ Std.
1	1.2267	0.0009	1.9112	0.0023
2	1.3641	0.0032	1.9097	0.0047
3	1.342	0.0095	1.9105	0.0041
4	1.3545	0.0208	1.9076	0.0295
5	1.4181	0.0249	1.9184	0.0252
6	1.4846	0.0389	1.9108	0.041
7	1.5285	0.0284	1.9145	0.0231
8	1.5386	0.0304	1.897	0.0687
9	1.5534	0.0235	1.9076	0.0123
10	1.5682	0.026	1.9061	0.0133
11	1.5851	0.02	1.9143	0.0088
12	1.5938	0.02	1.9114	0.0062
13	1.5866	0.0196	1.9123	0.0045
14	1.5877	0.0276	1.9101	0.0036
15	1.5889	0.0274	1.9095	0.0062
16	1.6007	0.024	1.91	0.0055
17	1.6054	0.0223	1.9113	0.0022
18	1.6055	0.0197	1.9105	0.0041
19	1.6528	0.0292	1.9094	0.0085
20	1.7271	0.0301	1.9121	0.0039

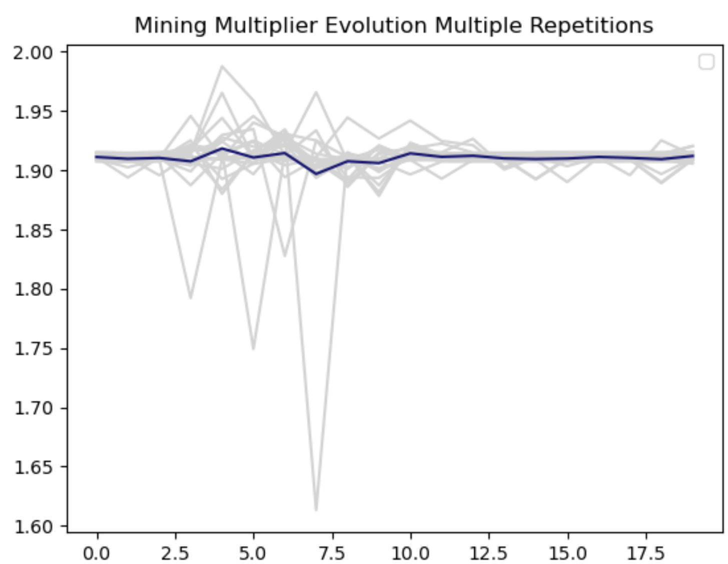
just sufficiently not good enough to say that "I do not have the information hence I cannot continue". Adaptation to challenging informational circumstances seems to be the norm in applied sciences.

Of particular interest is the definition of a metric for measuring impact. The metric adopted is, with some modifications, heavily inspired by the LM3 metric which is intended to measure the multiplier effect within a local context.

The initial results show promise in the sense that they first demonstrate that a multiplier can be calculated, for both the local economy and the mining



(a)



(b)

Figure 5.3: Different Results from Baseline Realistic Test, Twenty Repetitions. (a) global multiplier with average in different colour, (b) mining multiplier with average in different colour

restricted money flows; but also they allow us to assess the capability of the algorithm to run with more realistic data and all the learning derived from attempting to use real sources of information in the context of applying the algorithm.

From the foundation laid in this chapter, the next chapter goes into the analysis of possible options in the allocation of the "Royalties for Regions" funds in a model local economy, to estimate which option presents the best outcome for achieving enduring value from mining activities.

Chapter 6

Application to Benefits Distribution

This chapter is focused on comparing two possible strategies for benefits sharing arising from mining. As mentioned in previous chapters, the idea is to determine if different strategies could have better impact when compared to a base case. This study is inspired on the "Royalties for Regions" program. As has already been mentioned, it is a specific tax with the objective to remain in the region where the mines are located. Funding can be accessed for initiatives that are considered to deliver social and economic benefits for the regional communities. Similar regimes exist in other mining districts; for example in Perú, where the success of the specific ad valorem tax called "Canon Minero" is not clearly established.

One example of the problem when defining what is considered to provide benefits when using public funds is given by Kalgoorlie and what is called the

Clock Tower in the Courthouse. A picture of the same is provided in Figure 6.1.

Figure 6.1: Clock Tower, Kalgoorlie, Western Australia. Picture taken by Marion Halliday, travel blogger.

In the case of Kalgoorlie's Clock Tower, it is not clearly known how much money was spent in the gilding of the tower, but it has appeared in the newspaper that the project suffered a cost blowout in the order of $160,000 that was paid by the Department of the Attorney General. Apparently the project was fundraised and financed by locals, however, it calls to question about the rationality in the use of funding, as 23 ct gold leafs were used in the gilding process. Certainly some people may think that a better use of the money can be planned in order to provide enduring benefits.

Another example, this time in Perú, is given in Figure 6.2.

Figure 6.2: Celendín's Square, Cajamarca, Perú. Picture taken from Cajamarca's Regional Government's webpage.

It is certainly difficult to understand if investing in tourist attractions provide sustaining benefits for the communities. And undoubtedly, it is clear from the zealot support of some and adamant rejection by others for certain projects, that no one single measure of benefit can capture those differing opinions/feelings. This chapter looks into the problem of using funds coming from mining to support regional development in the spirit of “Royalties for Regions”, and uses the framework developed so far to compare different options relating to the distribution of these “benefits”. The main idea is to establish, from a methodological point of view, if the proposed model based on computational agents, is capable of enabling such analyses. There is a clear temptation in attempting the addition of micro-motives into the mix. However, what people wants is usually not captured by crude census’ statistics. We can only observe the “shadows” of such micro-motives by the information contained in any census, and this is seriously constrained as the questions usually never ask about feelings or motivations. Under this point of view, the argument is that a better knowledge of the preferences, necessities and/or motivations of a given population, combined with the modelling presented in this work, has great potential to enhance the qualitative aspects of this type of studies. The work developed in this thesis, as exposed so far, illustrates how to model some heterogeneity of the agents, and in the rest of this work the onus is put on how the RFR tax’s use could affect a given regional population.

6.1 Introduction and Scope

The ABM model so far has been tested on data as close to reality as possible, with considerations of the limitations of the computational tool, that guided the research to adopt a baseline case study where household numbers were scaled to keep the computational cost under some form of control.

This chapter presents two options for the distribution of the royalty for regions (RFR) tax. One possibility is to give every household a share of the royalty so those families receiving the benefit can decide on what to spend the money[1], the other option is to give every company a share of the money collected by the royalty if the company operates in the region, does not necessarily has to be a cash award but it could take the form of a tax deduction or similar.

In the process of scaling down to the level of a 100 households[2], some choices regarding sectors, number of firms per sector and starting capital were made. Several of those parameters were uniformly assumed. For example, every firm starting capital was set at $50,000, as the interest was on the comparison of a baseline against alternative case, also three firms per sector were created. With some of the parameters arbitrarily chosen for *Fakegoorlie*, such as the ones mentioned just in the previous sentence, but with inspiration in Kalgoorlie as much as feasible, it was soon clear that there was a misalignment on some of these choices. For example, it was realised that a starting capital of $50,000 for companies was not really aligned with salaries in the order of $2,000 or more. Hence, to "equalise" the quantities, the scaling has been applied to the RFR money to be distributed to households. Obviously, other equalisation mechanisms could have been chosen, but this option was selected due to the easiness of implementation it presented. A RFR tax of 3% will be applied ad valorem to the sales figures of mining companies in any given period. In order to equalise the distribution of this tax with households, it will be scaled by a factor of 20 (determined heuristically after attempting

[1]Such schemes are not alien to the Australian economic system. In the past (2009), they have been used as part of monetary policy to reactivate the country's economy or avoid crisis. The main idea is that by giving money directly to people, their increased consumption will avoid an otherwise potentially stagnant circulation of cash-flows

[2]Please note that the scaling down process for the number of households is required by the observed substantial increase in time with the number of households (see Section 4.5)

some numbers). In practical terms, this choice implies that if a company earns \$20,000, then this tax will represent \$600 less income for the company, but an equivalent contribution of \$12,000 to be distributed among 100 Households, or \$120 per household, or a contribution in the order of 5% for the average family of *Fakegoorlie*. The other option, consisting in scaling the households importance, from both an income and expenditure point of view, required a more significant intervention in the code, as it required among other measures changing the relatives prices of the products/services that households could consume. Similar reasoning was applied for the experiment consisting on distributing benefits to firms, but as their number is 51, and to keep consistency with the rule used for households – which is double the number of firms – the scaling factor used was set to 10^3.

The `Economy` object was modified to add two properties: the RFR tax rate (a number between 0 and 1) and a boolean marker to see if the RFR is used for Households or used for Firms. Also, it is worth to note that the distribution of the RFR tax can be used with Households as a direct monthly contribution, or with Firms as a subsidy. In both case studies, there is money transferred from the RFR collected Pool to agents, the difference is the destination. However, in the case the rate is set to 0%, the computational model reverts to the baseline case. The households are assumed to be local, hence, the benefits are distributed to all of them. However, for the case of firms, the distribution of benefits is proportional to the local ownership of the firm.

[3] From an implementation point of view, the code got the addition of the Boolean marker `usePeople` which if equal to `True` implies that the distribution rule will be used with households, in the opposite case, i.e., `usePeople = False`, the distribution is then performed on firms.

6.2 Case Study I: Sharing Benefits with Firms

An example was run consisting of 20 repetitions of the whole ABM model with each repetition consisting of 20 epochs/periods. Consistently with the baseline case study, the time taken is the expected: in the order of 40 seconds per iteration multiplied by 20 gives a total of 800 seconds (approx. 13 minutes) per ABM run and this is repeated 20 times which gives a total of around 4 hours to run this instance. The results are summarised in Figure (6.3) and Table (6.1). For this experiment, the `rfr_rate` parameter was set to 0.03 and the `usePeople` parameter was set to `False`.

We present now the graphs associated to the realisations with the average in a color different to that of the individual realisations (blue in this case). They can be found on Figure 6.3.

6.3 Case Study II: Sharing Benefits with People

An example was run consisting of 20 repetitions of the whole ABM model with each repetition consisting of 20 epochs/periods. Consistently with the baseline case study, the time taken is the expected: in the order of 40 seconds per iteration multiplied by 20 gives a total of 800 seconds (approx. 13 minutes) per ABM run and this is repeated 20 times which gives a total of around 4 hours to run this instance. The results are summarised in Figure (6.4) and Table (6.2). For this experiment, the `rfr_rate` parameter was set to 0.03 and

Table 6.1: Average and Standard Deviation Evolution for Multiplier Case Study I

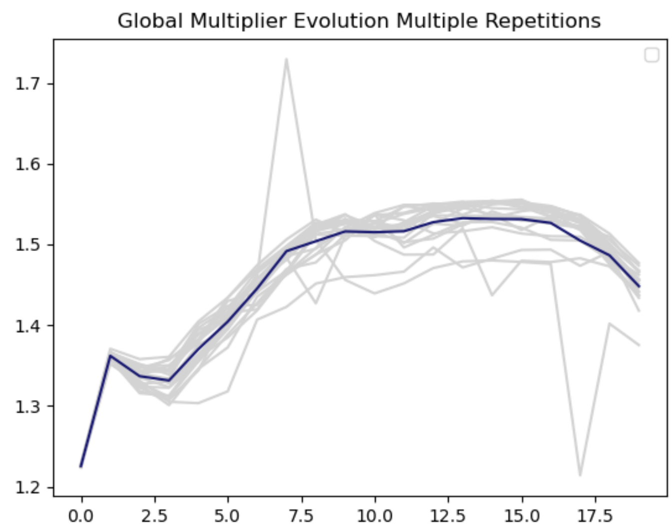
Step	$LM3$ Avg.	$LM3$ Std.	$LM3_M$ Avg.	$LM3_M$ Std.
1	1.2255	0.001	1.8835	0.0023
2	1.3621	0.0045	1.8811	0.0074
3	1.3368	0.012	1.8835	0.0023
4	1.3315	0.0194	1.8833	0.0159
5	1.3704	0.0246	1.8824	0.0126
6	1.4044	0.0263	1.8987	0.0426
7	1.4454	0.0189	1.8858	0.0271
8	1.4917	0.0589	1.8822	0.0172
9	1.5039	0.0271	1.8831	0.0133
10	1.5161	0.0219	1.883	0.0054
11	1.5152	0.0246	1.8844	0.0106
12	1.5163	0.0261	1.8822	0.0047
13	1.5276	0.0227	1.8835	0.0023
14	1.5325	0.023	1.8815	0.0058
15	1.5315	0.0307	1.8681	0.0719
16	1.5313	0.0242	1.8816	0.0057
17	1.5266	0.0221	1.8823	0.0043
18	1.5048	0.0705	1.8824	0.0054
19	1.4865	0.0227	1.8824	0.0047
20	1.4483	0.0226	1.8828	0.0056

the `usePeople` parameter was set to `True`.

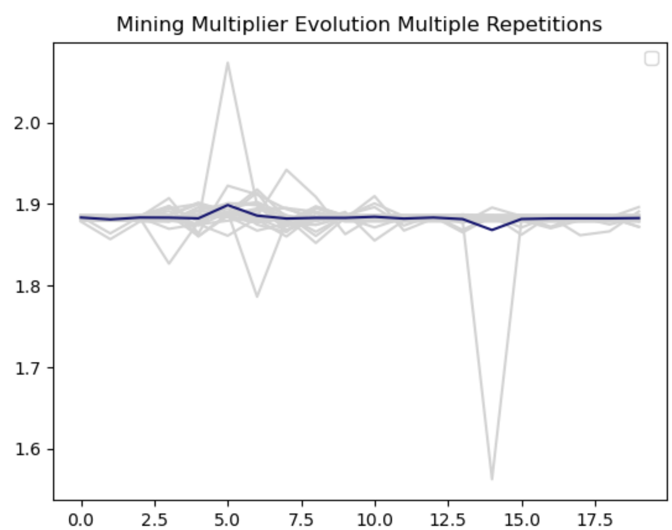
We present now the graphs associated to the realisations with the average in a different colour. They can be found on Figure 6.4.

6.4 Comparison of Results and Analysis

At this point in the experimentation is important to note that the agent based modelling approach to economics is essentially an out of equilibrium approach,



(a)



(b)

Figure 6.3: Different Results from Case Study I, Twenty Repetitions. (a) $LM3$ multiplier with average in blue, (b) $LM3_M$ multiplier with average in blue

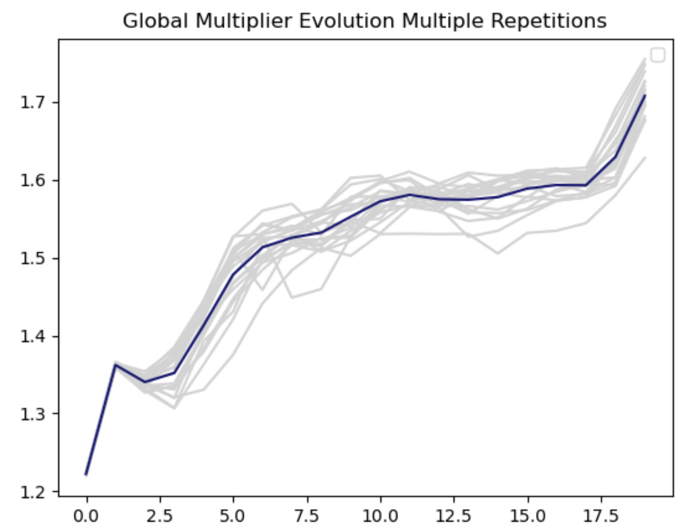
i.e., there is no such concept either theoretical or practical that relates to the classical equilibrium notion. The important consideration when using an ABM

Table 6.2: Average and Standard Deviation Evolution for Multiplier Case Study II

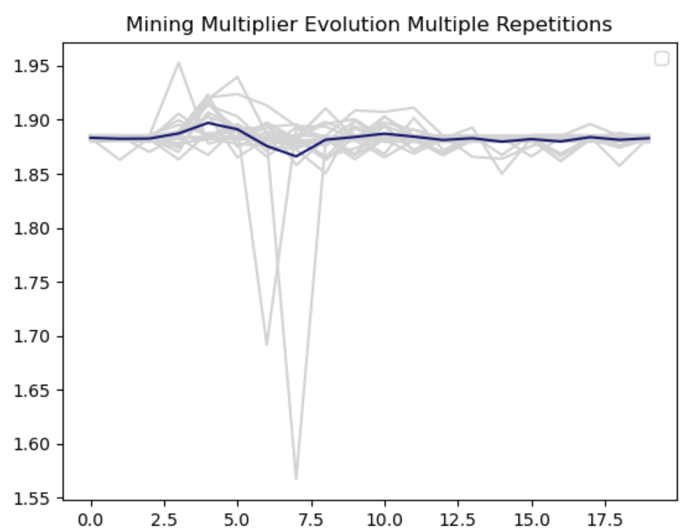
Step	$LM3$ Avg.	$LM3$ Std.	$LM3_M$ Avg.	$LM3_M$ Std.
1	1.2225	0.0011	1.8834	0.0016
2	1.3622	0.0026	1.8825	0.0048
3	1.3403	0.0074	1.8827	0.0033
4	1.3519	0.0256	1.8874	0.0182
5	1.4131	0.0304	1.8972	0.0169
6	1.4782	0.0394	1.8913	0.0164
7	1.513	0.0294	1.8757	0.0445
8	1.5254	0.0255	1.8662	0.0709
9	1.532	0.0255	1.8816	0.0134
10	1.5523	0.0261	1.8841	0.0124
11	1.572	0.0221	1.8872	0.0108
12	1.5804	0.0168	1.8845	0.0095
13	1.5748	0.0148	1.8814	0.0054
14	1.5741	0.0221	1.8829	0.0048
15	1.5774	0.0262	1.8798	0.009
16	1.5884	0.0221	1.8822	0.0044
17	1.593	0.0193	1.88	0.0081
18	1.5927	0.0156	1.884	0.0032
19	1.629	0.0311	1.8815	0.0065
20	1.7073	0.0299	1.883	0.002

is that of emergent behaviour which is not readily inferable from the local characterisation and properties of the agents. Hence, the comparisons are made at a point in time, predefined for the experiment.

A first output containing the comparison between the baseline case, Case Study I and Case Study II is presented in Table 6.3. In this table, Δ_1 is defined as Baseline $-$ Case I, Δ_2 is defined as Baseline $-$ Case II and $\Delta_{2,1}$ is defined as Case I $-$ Case II.



(a)



(b)

Figure 6.4: Different Results from Case Study I, Twenty Repetitions. (a) global multiplier with average in blue, (b) mining multiplier with average in blue

It can be seen that the multiplier for Case I evolves always bounded by the Baseline case with an average difference of 0.0702, the same happens for

Table 6.3: Comparison of Average Evolution for $LM3$ Multiplier

Step	Baseline	Case I	Δ_1	Case II	Δ_2	$\Delta_{2,1}$
1	1.2267	1.2255	0.0012	1.2225	0.0042	-0.003
2	1.3641	1.3621	0.002	1.3622	0.0019	0.0001
3	1.342	1.3368	0.0052	1.3403	0.0017	0.0035
4	1.3545	1.3315	0.023	1.3519	0.0026	0.0204
5	1.4181	1.3704	0.0477	1.4131	0.005	0.0427
6	1.4846	1.4044	0.0802	1.4782	0.0064	0.0738
7	1.5285	1.4454	0.0831	1.513	0.0155	0.0676
8	1.5386	1.4917	0.0469	1.5254	0.0132	0.0337
9	1.5534	1.5039	0.0495	1.532	0.0214	0.0281
10	1.5682	1.5161	0.0521	1.5523	0.0159	0.0362
11	1.5851	1.5152	0.0699	1.572	0.0131	0.0568
12	1.5938	1.5163	0.0775	1.5804	0.0134	0.0641
13	1.5866	1.5276	0.059	1.5748	0.0118	0.0472
14	1.5877	1.5325	0.0552	1.5741	0.0136	0.0416
15	1.5889	1.5315	0.0574	1.5774	0.0115	0.0459
16	1.6007	1.5313	0.0694	1.5884	0.0123	0.0571
17	1.6054	1.5266	0.0788	1.593	0.0124	0.0664
18	1.6055	1.5048	0.1007	1.5927	0.0128	0.0879
19	1.6528	1.4865	0.1663	1.629	0.0238	0.1425
20	1.7271	1.4483	0.2788	1.7073	0.0198	0.259
Avg.			0.0702		0.0116	0.0586

Case II compared to the Baseline case but this time it is much closer to the Baseline multiplier with an average difference of 0.0116.

The first obvious conclusion is that for the purposes of the modelling it is better to give money directly to people rather than giving it to companies. However, the less obvious lecture is that apparently is better not to tax mining companies at all. The potential mechanism in operation here is that as the RFR tax is applied ad valorem, then it reduces the amount of money that comes from mining and multiplies into the economy. For the particular example considered, the increase in consumption power by the households is not enough to compensate for the multiplying loss suffered by the reduction in the mining sector.

A further point of comparison is to acknowledge that the simulation takes some time to reach some form of stationary level. If we perform a comparison of the average differences of Case I and Case II against the Baseline case we observe that on average it increases for Case II and for Case I stays relatively close to the overall value. Those differences come at $\Delta_1|_{20}^{10} = 0.0968$ and $\Delta_2|_{20}^{10} = 0.0146$[4].

Let us consider now the same comparison, but this time performed at the level of the evolution mining multiplier. These results are contained in Table 6.4. These last results are remarkable in the sense that on average the mining multipliers tend to evolve very similarly for both Case I and Case II, with both of them being on average equally far from the Baseline case. All this evidence confirms the initial suspicion that the detriment of the mining sector is not compensated by the benefits to the households, and in particular that these benefits do not ameliorate the situation in the region, but on the contrary make things worse.

[4] The notation $\Delta_\alpha|_q^p$ with $p < q$, represents the average difference measured for comparison α, with α being an index or symbol representing a pair of experiments being compared, starting at epoch p and finishing at epoch q of the ABM runs:

$$\Delta_\alpha|_q^p = \frac{1}{q-p} \sum_{k=p}^{q} [LM3_1(k) - LM3_2(k)]$$

with $LM3_X(t)$ denoting the $LM3$ value for case X at epoch t.

Table 6.4: Comparison of Average Evolution for Mining Multiplier

Step	Baseline	Case I	Δ_1	Case II	Δ_2	$\Delta_{2,1}$
1	1.9112	1.8835	0.0277	1.8834	0.0278	-0.0001
2	1.9097	1.8811	0.0286	1.8825	0.0272	0.0014
3	1.9105	1.8835	0.027	1.8827	0.0278	-0.0008
4	1.9076	1.8833	0.0243	1.8874	0.0202	0.0041
5	1.9184	1.8824	0.036	1.8972	0.0212	0.0148
6	1.9108	1.8987	0.0121	1.8913	0.0195	-0.0074
7	1.9145	1.8858	0.0287	1.8757	0.0388	-0.0101
8	1.897	1.8822	0.0148	1.8662	0.0308	-0.016
9	1.9076	1.8831	0.0245	1.8816	0.026	-0.0015
10	1.9061	1.883	0.0231	1.8841	0.022	0.0011
11	1.9143	1.8844	0.0299	1.8872	0.0271	0.0028
12	1.9114	1.8822	0.0292	1.8845	0.0269	0.0023
13	1.9123	1.8835	0.0288	1.8814	0.0309	-0.0021
14	1.9101	1.8815	0.0286	1.8829	0.0272	0.0014
15	1.9095	1.8681	0.0414	1.8798	0.0297	0.0117
16	1.91	1.8816	0.0284	1.8822	0.0278	0.0006
17	1.9113	1.8823	0.029	1.88	0.0313	-0.0023
18	1.9105	1.8824	0.0281	1.884	0.0265	0.0016
19	1.9094	1.8824	0.027	1.8815	0.0279	-0.0009
20	1.9121	1.8828	0.0293	1.883	0.0291	0.0002
Avg.			0.0273		0.0273	0.0000

6.4.1 Some Comments About the Results

A comparison between the two option can be performed by means of a t-test for the difference of means. Without going into the details of the test, as it is assumed general knowledge, the test conducted for comparing $LM3$ for Case I and Case II provides a p-value for the test with null hypothesis given by $H_0 : \mu_1 - \mu_2 = 0$ of 0.000220254 at significance level $\alpha = 0.05$, which leads to the rejection of the null hypothesis thus confirming that the two cases are statistically significantly different. The result output of the test conducted using EXCEL tools is given in table 6.5. Similar results are obtained if only constrained to the last 11 epochs of the algorithm executions. The detail of the test is presented in table 6.6.

Table 6.5: t-Test: Paired Two Sample for Means, Full Epochs, $LM3$

	Case I	Case II
Mean	1.45542	1.514
Variance	0.00772516	0.013974158
Observations	20	20
Pearson Correlation	0.884510946	
Hypothesized Mean Difference	0	
df	19	
t Stat	-4.547280503	
P(T<=t) one-tail	0.000110127	
t Critical one-tail	1.729132812	
P(T<=t) two-tail	0.000220254	
t Critical two-tail	2.093024054	

Table 6.6: t-Test: Paired Two Sample for Means, Restricted Epochs, $LM3$

	Case I	Case II
Mean	1.512427273	1.594672727
Variance	0.000643282	0.001753902
Observations	11	11
Pearson Correlation	-0.899762246	
Hypothesized Mean Difference	0	
df	10	
t Stat	-4.155648557	
P(T<=t) one-tail	0.000981141	
t Critical one-tail	1.812461123	
P(T<=t) two-tail	0.001962283	
t Critical two-tail	2.228138852	

For the case of the mining multiplier, it can be seen that when performing the same test the difference is statistically equal to zero. In fact the p-value for the test comes out as 0.978561812 which is a great support for the null hypothesis. If just considering a subset of the epochs as before, the p-value for the test is 0.219083779 which still means that the null hypothesis cannot be rejected.

Table 6.7: t-Test: Paired Two Sample for Means, Full Epochs, $LM3_M$

	Case I	Case II
Mean	1.88289	1.88293
Variance	2.57694E-05	3.55043E-05
Observations	20	20
Pearson Correlation	0.299363273	
Hypothesized Mean Difference	0	
df	19	
t Stat	-0.027228033	
P(T<=t) one-tail	0.489280906	
t Critical one-tail	1.729132812	
P(T<=t) two-tail	0.978561812	
t Critical two-tail	2.093024054	

Table 6.8: t-Test: Paired Two Sample for Means, Restricted Epochs, $LM3_M$

	Case I	Case II
Mean	1.881290909	1.882781818
Variance	1.98189E-05	4.62764E-06
Observations	11	11
Pearson Correlation	0.533876573	
Hypothesized Mean Difference	0	
df	10	
t Stat	-1.31124494	
P(T<=t) one-tail	0.10954189	
t Critical one-tail	1.812461123	
P(T<=t) two-tail	0.219083779	
t Critical two-tail	2.228138852	

6.5 Royalty on Profits

It may be hypothesised at this point that the application of an ad valorem tax such as “Royalties for Regions” has far larger detrimental consequences to the local economy than the benefits it may produce. in order to check if applying a tax on profits could create a better situation for the region than the actual ad valorem tax, an experiment was designed where the royalty is applied on profits but the tax is distributed to people, as this proved to be a better mechanism in a previous experiment, and the tax rate is left unchanged. Similarly to past numerical experiences, a series of runs were conducted. The outcomes are shown in the coming graphs and tables, after which some discussion will be conducted.

A first output containing the comparison between the baseline case, Case Study I and Case Study II is presented in Table 6.9. In this table, Δ_2 is defined as Baseline – Case II, Δ_3 is defined as Baseline – Case III and $\Delta_{3,2}$ is defined as Case II – Case III.

Table 6.9: Comparison of Average Evolution for Local Multiplier

Step	Baseline	Case II	Δ_2	Case III	Δ_3	$\Delta_{3,2}$
1	1.2267	1.2225	0.0042	1.2262	0.0005	0.0037
2	1.3641	1.3622	0.0019	1.3493	0.0148	-0.0129
3	1.342	1.3403	0.0017	1.3391	0.0029	-0.0012
4	1.3545	1.3519	0.0026	1.365	-0.0105	0.0131
5	1.4181	1.4131	0.005	1.4219	-0.0038	0.0088
6	1.4846	1.4782	0.0064	1.4835	0.0011	0.0053
7	1.5285	1.513	0.0155	1.5225	0.006	0.0095
8	1.5386	1.5254	0.0132	1.5295	0.0091	0.0041
9	1.5534	1.532	0.0214	1.5417	0.0117	0.0097
10	1.5682	1.5523	0.0159	1.5508	0.0174	-0.0015
11	1.5851	1.572	0.0131	1.5669	0.0182	-0.0051
12	1.5938	1.5804	0.0134	1.5743	0.0195	-0.0061
13	1.5866	1.5748	0.0118	1.5726	0.014	-0.0022
14	1.5877	1.5741	0.0136	1.5652	0.0225	-0.0089
15	1.5889	1.5774	0.0115	1.5699	0.019	-0.0075
16	1.6007	1.5884	0.0123	1.5733	0.0274	-0.0151
17	1.6054	1.593	0.0124	1.5857	0.0197	-0.0073
18	1.6055	1.5927	0.0128	1.5887	0.0168	-0.004
19	1.6528	1.629	0.0238	1.642	0.0108	0.013
20	1.7271	1.7073	0.0198	1.7118	0.0153	0.0045
Avg.			0.0146		0.0182	-0.0037

It can be seen that the average local multipliers for Case II and case III evolve almost always consistently below the Baseline case with an average difference of 0.0146 and 0.0182 respectively. This points to the conclusion that for the local multiplier, there seems to be no difference in the mechanism by which the "Royalties for Regions" tax is collected. Again, a possible simplistic lecture could be not to tax mining companies at all as the average local multiplier is smaller than the baseline case, albeit the difference is relatively minor and in the order of 0.8% of the actual value of the multiplier. Applying the tax on profits apparently does not change the RFR impact on the local economy.

Considering now the same comparison, but this time performed at the level of the evolution mining multiplier provides results such as contained in Table 6.10. The results show that applying the tax on profits does not destroy the mining multiplier as the ad valorem tax does. The rationale behind this result has to do with the way the local multiplier is calculated. In one case, the RFR tax is applied ad valorem, which means that the first term of the local multiplier suffers a "blow" which in turn propagates forward and for the purposes of the calculation, this first term is the most important one. If applied on profits, the RFR tax does not introduces additional costs, meaning that the full amount of money can be used to buy raw materials on the first round of calculation, leaving this quantity unaffected, thus leaving the most important component of the local multiplier untouched.

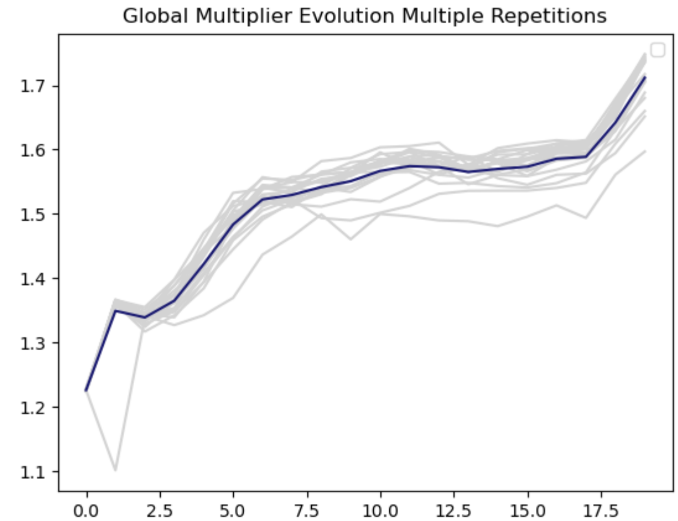
We present now the graphs associated to the realisations with the average in a different colour. They can be found on Figure 6.5.

The current section illustrates the type of additional analyses that can be performed once the foundations of an ABM are laid. The option for a profit

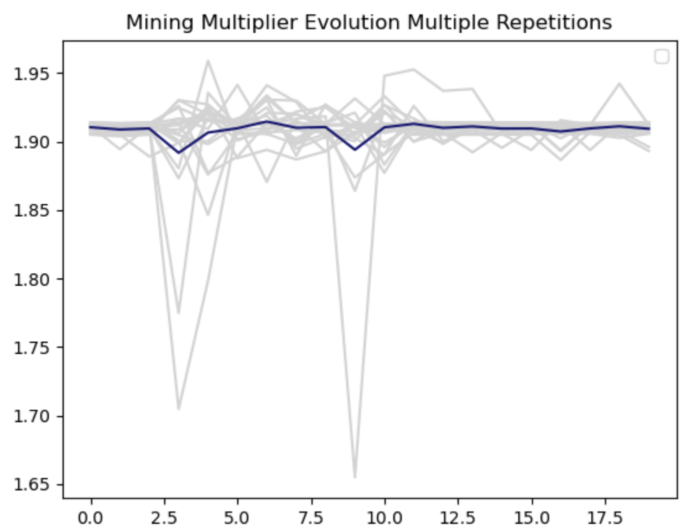
Table 6.10: Comparison of Average Evolution for Mining Multiplier

Step	Baseline	Case II	Δ_2	Case III	Δ_3	$\Delta_{3,2}$
1	1.9112	1.8834	0.0278	1.9103	0.0009	0.0269
2	1.9097	1.8825	0.0272	1.9087	0.001	0.0262
3	1.9105	1.8827	0.0278	1.9095	0.001	0.0268
4	1.9076	1.8874	0.0202	1.8917	0.0159	0.0043
5	1.9184	1.8972	0.0212	1.9065	0.0119	0.0093
6	1.9108	1.8913	0.0195	1.9096	0.0012	0.0183
7	1.9145	1.8757	0.0388	1.9145	0	0.0388
8	1.897	1.8662	0.0308	1.9099	-0.0129	0.0437
9	1.9076	1.8816	0.026	1.9105	-0.0029	0.0289
10	1.9061	1.8841	0.022	1.8939	0.0122	0.0098
11	1.9143	1.8872	0.0271	1.9103	0.004	0.0231
12	1.9114	1.8845	0.0269	1.9128	-0.0014	0.0283
13	1.9123	1.8814	0.0309	1.9099	0.0024	0.0285
14	1.9101	1.8829	0.0272	1.9109	-0.0008	0.028
15	1.9095	1.8798	0.0297	1.9094	0.0001	0.0296
16	1.91	1.8822	0.0278	1.9094	0.0006	0.0272
17	1.9113	1.88	0.0313	1.9072	0.0041	0.0272
18	1.9105	1.884	0.0265	1.9095	0.001	0.0255
19	1.9094	1.8815	0.0279	1.9111	-0.0017	0.0296
20	1.9121	1.883	0.0291	1.9092	0.0029	0.0262
Avg.			0.0273		0.002	0.0253

based RFR does not currently exist, however, by looking at the results of the modified model, it is possible to observe the impact of the new collection mechanism without affecting the existing operation of the system. Certainly the ABM shows that it is flexible enough to consider other options, however this does not mean at all that it is easy to identify the best one. The ABM is just a tool that can be used to assess the relative merit of different RFR collection and distribution mechanisms, however, the exact level at which the RFR should be applied and specific rules pertaining its application will require repeated use of the proposed tool (or improved versions of it) to arrive at a combination of parameters/rules that produce a better otucome when compared to a base case.



(a)



(b)

Figure 6.5: Different Results from Case Study III, Twenty Repetitions. (a) global multiplier with average in blue, (b) mining multiplier with average in blue

6.6 Concluding Remarks

This chapter focused on applying the ABM to a case study which compared two possible strategies for distributing an ad valorem tax such as "Royalties for Regions" in order to see if they were equivalent or not. The results coming out of the comparison can be summarised in the following list:

- There is a difference in impact, as evidenced by the evolution of the local multipliers, obtained with the assistance of the ABM model, for the different distribution mechanism used.

- It is interesting to notice that the mining multiplier remains very stable for the case study considered and that on average the difference between the first two cases is zero.

- This last point suggests that eventually the royalty mechanism may not be appropriate due to its nature and maybe a change could be recommended. And in accordance to this point, a third case study implementing a new collection mechanism was considered, with a change from ad valorem to profit based tax. The results show that whilst the overall local multiplier does not improve, the local mining multiplier becomes comparable with that of the base case.

It is believed that these results are encouraging as they open up discussion avenues and future exploration in the area, eventually with better equipped computational models, that could inform policy development on one side, and on the other provide the opportunity to put the focus on a different information gathering exercise that will change the way impact is assessed.

As a general rounding conclusion coming out of the chapter but that applies to the whole thesis, there is always the possibility of finding evidence of some shortcomings in the modelling that with additional effort could be surmounted, thus creating a better representation of reality aiding to determine what provides benefits to local communities, and ultimately to provide enduring benefits for local mining communities. However, the important lesson of the whole process is that alternative ways of measuring impact can be considered, and implemented in practical terms, thus enhancing the toolbox of the policymaker, government and communities.

Chapter 7

Conclusions

In the conclusions chapter, the problem is re-stated, the main learnings acquired in the process of solving it are described and the main results contained in the thesis are briefly summarised. Additionally, possible extensions and improvements of the work are outlined.

7.1 Discussion and Summary

Let us start recalling the main thesis of this work, which can be stated as: It is possible to approximate the impact of mining related activities in a regional location by focusing on the economic agents operating in that region, by means of using heterogeneous descriptions. The process required to prove or disprove the assertion expressed by this thesis needed a series of stages of development, involving thinking and rethinking, in order to arrive at an implementable definition of how to measure impact, and a review of some of the techniques available for doing so. The development of this work has been a journey with a clear objective, however, there are methodological obstacles that arose.

First, it was thought that the classical tools were appropriate for the task. However, early in the development of the work it was noticed that the assumptions of those models were too simplistic for the task. This motivated the search for alternative tools, and the answer came in the form of agent based models, which remove several of the assumptions that exist when using classical tools. As no readily available software exists for this implementation, a tool for the task had to be developed as there were no libraries that are considered fit for purpose.

This development process started with some initially dispersed ideas, coming from a diverse set of methodological backgrounds, whose integration in a coherent software implementation required time to coalesce. This implementation "step"[1] required a sequence of tasks including, but not limited to, careful planning, consideration, testing and validation.

The implementation task had two main components. First, there are the intricacies of the programming language, which is a user-friendly, entry-level language. Second, there was a series of small obstacles that added to the complexity, making the journey a difficult one. For example, the impossibility of fitting a technology function from the input-output matrices. Furthermore, the fact that the associated optimisation problem heavily depends on the initial point for the numerical solution, as it has multiple local minima and maxima,

[1]Due to the nature of software implementation, this process is never straightforward but a spiral development cycle, where on each "new version" some old problems are fixed, and potentially some new ones are added.

forced a definition to ensure some reproducibility (see Section 4.5.2). The identification of the LM3 multiplier solved, in an operational manner, the need of measuring local impact in a certain regional location, and with adaptation, was expanded to consider the case of measuring the impact of a particular economic sector within that regional location. All things considered, the core of the implementation offered in this work has been extensively tested for consistency, reproducibility and correctness. The case studies presented extend the core implementation, adapting it to answer specific research questions, and as a result of answering these questions the flexibility of the approach has been illustrated. In its current form, this document reflects the care put into the testing activities in order to eliminate implementation bugs where possible.

The first simple ABM was implemented following basic rules, which was able to reproduce the classical multipliers. This demonstrated the applicability of the approach and it was hypothesised that providing more complex rationality and interaction rules could produce better results. In particular, it is believed that "increases in computational power coupled with the flexibility that the ABM paradigm provides, offer this technique as a welcomed addition to the toolbox of every researcher interested in measuring economic impact" (See Section 3.3). Furthermore, the modelling technique utilised is dynamic by construction, i.e., it is not trying to find an equilibrium, producing simulations which attempt to discover how the system evolves, based on initial conditions and behavioural descriptions of economic agents. This characteristic is helpful in a dynamic industry such as mining where constant changes are observed over time.

One important contribution of this thesis is that it paves the road towards new viewpoints to nurture economic discussion. Several economists have stated that the foundations on which economic theory lies are, to say the least, questionable. Starting with the work of Kahneman (1979), where divergences from neo-classical theory on decision making are discussed, “Behavioural economics” has become increasingly more prominent. Aditionally, various articles (Krugman, 2009; Stiglitz, 2011; Napoletano et al., 2012; Korinek, 2018; Stiglitz, 2018; Dosi et al., 2020), criticising the notions of equilibrium, rationality and representative agents have been published. The questioning of economic tradition conducted by those researchers are encouraging more orthogonal thought to explain economic phenomena.

In a sense, this thesis is, an attempt at an incipient step towards other methodological approaches, which in particular enable incorporating heterogeneous agents, micro-motives and deviations from strict rationality, i.e., a more realistic perceived approximation to the problem. As a by-product of the modelling choice, it is possible to obtain emergent behaviour, which in general cannot be predicted from the individual economic agents.

The modelling presented in this thesis has been simplified as an initial attempt, but will improve substantially when provided with more complex knowledge/data. As mentioned, if it were possible to obtain or create anonymised or disaggregated records containing age, sex, educational level, etc., including preferences such as travel patterns, lifestyle patterns and hobbies for people, together with deeper information concerning companies’ corporate structures, provisioning policies, employment policies, etc., the model could be expanded, giving way to a substantially greater number of research possibilities.

The implementation in this thesis has been successfully used on a case study with the main conclusions restated below with a couple of additional points provided:

- There is a difference in impact for the different RFR distribution mechanisms used, as evidenced by the evolution of the $LM3$ multipliers, obtained with the assistance of the ABM model.
- The $LM3_M$ multiplier remains very stable for the case study considered and that on average the difference between the two cases is zero.
- The previous point suggested that eventually the royalty mechanism may not be appropriate due to its *ad valorem* nature, which might suggest considering a change of the taxation basis for the Royalties for Regions. An experiment was conducted to test the idea and it was observed that for a profit based collection mechanism, the overall local impact remained unchanged but the local mining multiplier grew to levels similar to those observed in the baseline case.
- As opposed to other techniques, the technique used in this thesis focuses on building an economy from the bottom up, allowing the modelling efforts to consider factors of heterogeneous nature that are difficult to consider using other models.
- The modelling effort is compatible with a regional focus. Much of the existing research emphasis is on economic measurements and policy at the

national scale. However, the tools to assess sustainable development of regional locations, affected by mining operations, are usually inadequate as there is a lack of appropriate definition of what local means in the context of the regional community. The modelling proposed in this thesis addresses this perceived weakness by enabling heterogeneous behaviour of the agents. Furthermore, as the case study has shown, by modelling local ownership we can observe its impact in a numerical fashion with respect to a base case. Certainly, not all such factors were captured in the modelling: for instance; are FIFO workers in remote operations local to the region where they temporarily live and work? Depending on the point of view adopted and how much the mining operation integrates with the local community, this could be true or false.

The results are encouraging as a first implementation of these tools in the context of identifying the impact of mining in regional communities. They open avenues for discussion and future research in the area. It is believed that the technique used has the potential to produce insight that cannot be obtained using classical tools. For example, using the ABM and a functional definition of impact (LM3), it was possible to simulate LM3 "paths", and from that a dynamic - out of equilibrium - description of impact via the LM3 multiplier, allowing for informative comparisons. A contribution made by this work is that it shows how a different technique brings a different point of view, which can complement and enrich existing views. The intent is not to replace existing techniques, but to complement them, in particular when the proposed methodology has the potential to add a different, more humane, representation of the regional communities and their aspirations.

7.2 Limitations

This section discusses how this work relates to other concepts/ideas. Some were considered through the development of the thesis but were not included as a substantial part of it. This should not reflect on them as inferior or unimportant concepts. In many cases the omission is related to practical difficulties with their inclusion. They are thought to be a substantial component of the phenomenon behind the general research area.

The most important concept/area that needs consideration is the meaning and measurement of happiness. Happiness has elements that are connected to economic reality, as it would be difficult for an average human being to think about being happy when there is no food on the table for himself and/or family. However, reducing all the discussion just to economic elements is an oversimplification. Cultural considerations, which are difficult to measure, play a role in what brings happiness and what does not.

Happiness is a personal experience, and according to some biologists and psychologists (Lykken and Tellegen, 1996; Dalai Lama and Cutler, 1998; Kováč, 2012; Okbay et al., 2016), is an evolutionary mechanism intended to promote behaviour that will improve the chances of your genetic material being passed on to the next generations. We have all experienced situations that under one set of circumstances make us feel “happy” but in others make us feel “unhappy”. Also, happiness as such, seems to be a temporary condition, which produces a chemical high that fades out, driving organisms to seek more of what produces that chemical high. Unfortunately, there is not a good understanding of the full mechanics behind happiness, hence, only assumptions can be made about it and from an economic point of view, a simplification to a collection of simple elements is usually considered.

Human interactions are extremely complex to model. The modelling of human interactions depends on factors such as social connections, common interests, children attending school together, membership to clubs, or a potential long list of connection mechanisms at different levels, which is too long to enumerate exhaustively. This represents a finer layer of complexity that was beyond the scope of this thesis. One of the main arguments behind the use of ABM is to move out from the classical assumption of the representative economic agent, which has so far been used and abused in classical economics. Despite the good intentions behind ABM and the promise of being able to have heterogeneity of agents, having them interact in a regular fashion with all the complexities of human behaviour considered is utopic to say the least. In this sense, ABM provides a mechanism for bounded heterogeneity, as the interactions between agents could more than complicate the model. Things that could be great to model, but so far are out of scope, relate to social linking, connections, influence of agents on others and so on.

Micro-motives are much more powerful than what people tend to believe. At the very basic level, from a biological point of view, some level of selfishness is present for every organism. The underpinning mandate of preserving genes for the next generation make organisms to first and foremost guarantee their existence as much as conditions permit in order to maximise their chances of passing on their genetic information. This implies that the observed societal behaviour is a composition of localised decisions taken by agents, an emergent behaviour of sorts where very likely selfishness is prevalent, even in the case where apparently a social attitude is exhibited. In this regard, the philosophy behind ABM seems to be correct as it is justly trying to observe emergent behaviour from localised decisions/interactions. However, this is easier said than done. One thing that can be asserted is that individual agents/organisms are commonly not able to fully understand their own micro-motives, which means that despite the mechanism for dealing with micro-motives being conceptually available in the ABM approach, the lack of a proper characterisation of those micro-motives makes the modelling elusive in practice.

Finally, a quote popularly attributed to Albert Einstein is: “If idiots could fly, this place will be an airport”. This quote touches on an aspect that is relevant to this work, and that is the level of rationality of the agents. It is usual in ABM to use bounded rationality, i.e., rather than optimising a potential outcome, an attempt is done to satisfice a given objective. This implies that usually decision makers are happy with decisions that will be good enough instead of optimal. This sounds fine, however, daily experience shows examples of bad decisions, and by bad decisions, no implication is made about accidental bad decisions but very likely stupid decisions. Such is the nature of bad decision making that there is actually an award named the Darwin Awards. According to their website, “The Darwin Awards honor those who tip chlorine into our gene pool, by accidentally removing their own DNA from it during the spectacular climax of a ‘great idea’ gone veddy, veddy wrong.” How to model human stupidity? Certainly a daunting task. The current work did not attempt to model deviations from some rationality, in fact household decisions and firm decisions all followed some well established microeconomic principles. However, this aspect is probably one of the most relevant as the lack of rationality, not due to lack of information or even bounded rationality, is in some societies or segments of societies a major component which eventually will require modelling.

Despite the previously mentioned deficiencies, it is believed that the ABM approach is a step forward into the consideration of all human complexity for the purposes of understanding the economic system. It is certainly not perfect, and the application of the technique in the present work is far from achieving its full potential. Nevertheless, in theory the ABM technique is better equipped to incorporate newer behavioural theories that will allow for a better description of reality, and as such, is a modelling framework that can grow in time, which is believed to be its strength.

7.3 Future Work

It is important to note that the modelling in this thesis will always be a work in progress, as it is never possible to perfectly characterise/model reality. It is important to recall that the main objective of the thesis was to try to investigate if the impact of mining related activities in a regional location can be approximated by focusing on the economic agents operating in that region, by means of using heterogeneous descriptions. Starting with the basic idea of a model with minimal elements, it was discovered that there was no publicly available information that allows researchers to model heterogeneous agents based on reality. At best there is information regarding aggregates based on census data. Given the impossibility of understanding potential motivations for day to day decisions, due to inappropriate data availability, this work introduced assumptions that could be lifted in the future as better instruments are developed to improve the characterisation of the agents. This section will attempt to enumerate the areas where it is possible to produce improvements in the future outside of the context of this Ph.D.

The implementation is a first attempt at the creation of a tool to measure impact that can characterise observable emerging behaviour, having a regional focus, the potential to incorporate the heterogeneous characteristics of economic agents, and a high level of possible customisation. The output of the models considered can be incorporated in policy development for mining activities affecting a regional area. In particular, the tool was able to assist in providing a narrative to decide what to do with money obtained through the Royalty for Regions act from 2009. This aspect is important, as the initiative is not exclusive to Australia. Other countries such as Perú have implemented a similar program since at least 1995[2]. Paradoxically, in the Peruvian case the existence of a regional mining royalty has been unable to eliminate numerous social conflicts that can slow the production of some mining operations. During the last few years some of the conflicts have reached previously unobserved dimensions, which brings into question the contribution of mining in that country.

[2]Strictly speaking, the Peruvian constitution from 1978 establishes the right of the communities affected by mining activities to participate from the benefits of the extractive industry. But despite the good intentions, the legal body providing appropriate definitions and more specific ways to access those benefits came into existence at a later time

The following subsections discuss potential enhancements to the modelling in this thesis, as well as avenues for future research. The objective is not to be completely exhaustive, but rather to put in writing some of the ideas that have emerged in the development of the work, as a reference for continuation.

7.3.1 Opportunities in Modelling

While not fully exploited in this work, it is important to note the dynamic and adaptive nature of ABM techniques. It is possible to have agents that adapt to changing environments by means of introducing changes in their production functions in the case of companies, or utility functions in the case of household agents. It is expected that this will produce more realistic models that can be used for both evaluation and design of mining-related public policy. This could be particularly useful for the case of mining and related activities given the super cycles observed in the economic sector.

In the current implementation of the model, inventory can accumulate between periods. However, there are certain goods that cannot be stored. One such example is services. A better model, to be more realistic, would consider the obsolescence characteristics of some goods, to decide in each case if they should be discarded or stored after each epoch of the algorithm.

The model adjusts prices when the market is unable to produce a match. In that case two different factors are used (δ and ε) that try to reach a consensus price for both consumers and producers. Unfortunately, the way this has been implemented produces sustained inflation in the prices. While a great deal of testing was performed to try to identify the cause for such behaviour, unfortunately it was not possible to find the exact cause in the course of this thesis, but is an important future research avenue. The preliminary idea was to consider the production excedents and use a percentage of the excedents as a price lowering factor. However, when attempted in the implementation it did not work as expected. It is believed that the issues lie within the programming of the implementation and the programming language used. And while there were some initial attempts at fixing this situation, a solution was not identified.

7.3.2 Improvements in Implementation

The implementation process followed a well defined path, but was slowed by constant changes and challenges in the journey to arrive at what is presented today. Several re-writings of the code were necessary to arrive at the current version. The code provided in Appendices D and E are just the last link in the chain of events relating to the implementation of the research ideas.

All in all, the journey has been satisfactory as the objective of arriving at an implementation that can be considered usable has been achieved. Some characteristics of the implementation are:

- Use of `Python` as the base language for development using the object oriented paradigm.
- Use of the `simpy` library for the management of the simulation.

- Use of the `pulp` library to solve mixed integer mathematical programs, required for the assignment mechanism implemented in the `Market` class.
- Use of the `scipy` library to solve non-linear optimisation problems in determining production and consumption levels for producers and consumers respectively.
- Use of the library `ipfn` to fit joint distribution functions based on marginals[3].

It was clear that this choice limited the size of the experiment and number of repetitions that could be investigated. As the agent based modelling approach is advanced, it will require larger datasets and more iterations. As such, it would be advantageous to either move the numerically intensive part of the implementation to a language that is more appropriate for numerical simulations, such as `C++`, or adopt an existing ABM library that has efficient implementations. This is a major undertaking that was not part of this thesis which focused on characterising economic impact in a regional location.

The model as implemented does not include the complexity of a labour market. The dynamics of labour markets were deemed too complex to model in this thesis. Part of the complications are due to the complex matching process that happens between firms' needs and worker's skills. The process of negotiating salaries and ultimately the permanence of workers in a regional location, plus the addition of migratory processes makes the problem complex. In this thesis, the intent was to try to understand the impact of mining in a

[3]This modelling technique is part of the lessons and learnings that the development of the work has brought to the researcher's toolbox.

local economy making assumptions regarding local ownership of the firms and the nature of the workforce (considered 100 % local). However, a more realistic model would lift these constraints. This is believed to be a Ph.D. research topic on its own, to understand the determinants of flows of people and skilled workers to and from regional locations with implications far greater than just the impact of a particular economic sector within a regional context.

The model implements a simple market for production inputs, where the pricing has been established as a percentage of the consumer market price. In reality, the materials used in production are of a different nature than the products for final consumers. However, for the purposes of the modelling it was assumed that every sector produces/provides only one product/service. This aspect of the model could be improved if sufficient data and understanding of the interrelationships between producers of different sectors and consumers could be gathered. A first step would be to have differentiation of consumers and producers markets. Similarly, the model should include a financial market with financial products such as savings and debt. While financial markets are extremely complex and a full model may be unrealistic. Saving and debt decisions at a minimum are part of everyday experience and most people make decisions about them on a continuous basis. These aspects, whilst interesting, are of such complexity that have been deemed worthy of being pursued by successive research efforts.

In the experiments that have been run, it was decided that every economic sector would have the same number of firms. This is an unrealistic assumption. However, not having appropriate information regarding the number and local ownership composition for companies, which is required for modelling the regional context, it was decided for the purpose of this research that the number of companies would be fixed. Ideally, the number of companies operating within a certain regional location, the number of local and non-local people they hire and ultimately their spending decisions within the local context could be measured and included within the modelling.Also, for simplicity the same budget for all companies has been used, however, a more realistic setting should exhibit differences.

7.3.3 Regarding Information

One constant in the development of the thesis has been the fact that micromotives should ideally be modelled, as ABM offer the possibility of heterogeneity. This suggests that there is a good opportunity for changing the data collection exercise and focus less on quantitative aspects and more on qualitative ones. The meaning of what enduring entails is somewhat subjective, can become a contentious issue, and it is believed the perceptions of the local community should inform policy development. Along the same lines, access to more disaggregated information would be ideal for developing more realistic models that can better inform these policy decisions.

Things that cannot be measured cannot be considered. Wellbeing is one of those elusive measures, which is relative to the subject. Obtaining better measures of wellbeing could be ideal, however, this is difficult as the focus on wellbeing is just starting. Some work has already been undertaken by countries such as Bhutan, who has a special preoccupation with improving happiness of all the subjects of their realm, and for all practical purposes has embarked on the difficult task of measuring wellbeing and happiness within the realm in a way that is raising international awareness.

7.4 Closing Remarks

The thesis previously stated for the work was:

> "It is possible to approximate the impact of mining related activities in a regional location by focusing on the economic agents operating in that region, by means of using heterogeneous descriptions"

In this thesis a model has been proposed, implemented and tested in an attempt to assess the truthfulness of the assertion. This chapter focused on the review of what has been achieved and possible avenues for improvement that are derived from this work. In particular, we can summarise the main contributions of this work as:

- The adoption of a novel viewpoint to study the problem: the use of ABM as a tool to study the impact of mining activities within a regional location.
- The modelling technique used being able to be customised to incorporate heterogeneous characteristics of the economic agents: shown in the numerical experiments.

  - As an example of this point, a market clearance mechanism was implemented in an `Agent` actor. Later, once implemented, the model was expanded to consider local ownership.

- The presentation of a case study inspired by reality which allowed the exploration of questions regarding the redistribution of royalties for regions:

  - It was observed that the distribution mechanism has an impact on the evolution of the regional system, with direct distribution to inhabitants being more effective from a cash circulation/multiplier point of view.

  - The question of the nature of the royalty for regions (ad valorem or profit based) being investigated, showing an increased multiplier for the mining sector without detriment to the global sector multiplier.

- The adoption of the LM3 multiplier and its adaptation to measure sector local multiplier which proved essential to performing the experiments. In this case, the use of a local ownership factor in combination with the LM3 multiplier permitted comparisons between a base case and alternative situations.

It is acknowledged that future work can be conducted to address some of the limitations encountered throughout the development of this work. However, as a starting point, it is believed that the model has achieved, in a methodological way, capabilities that allow for the verification of the validity of the assertion forming the body of the work in this thesis.

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Appendix A

Utility Functions

The purpose of this chapter is to give the reader a brushstroke about Utility functions. Although it is a "basic" economic concept, it is one of great importance for the present work. Every model that will be presented in this thesis is constructed around a specific utility function. Next, a Utility Function definition followed by some of its more common functionality forms will be given.

A.1 Definition and Properties

A Utility Function is *"a way of assigning a number to every possible consumption bundle such that the more-preferred bundles get assigned larger numbers than less-preferred bundles. That is, a bundle* (x_1, x_2) *is preferred to a bundle* (y_1, y_2) *if and only if the utility of* (x_1, x_2) *is larger than the utility of* (y_1, y_2)*: in symbols,* $(x_1, x_2) \succ (y_1, y_2) \iff u(x_1, x_2) > u(y_1, y_2)$*"*(Varian, 2010).

The previous definition is derived from the Theory of Consumer Behaviour and as such is just a way to describe the agents preference, leaving behind the ancient idea that utility was a way to quantify happiness.

It is also worth remembering the notion of **indifference curves**, concept first introduced by Francis Ysidro Edgeworth in 1881 (Lenfant, 2012; Chaigneau, 2014) which are all the different combinations of goods that report the same level of utility (Mas-Colell et al., 1995; Varian, 2010). That is,

$$\text{Indifference curve} = \{x \in X / u(x) = k\} \qquad \text{with k = constant}$$

Every agent has a map of indifference curves, each of them representing a different level of utility. And every indifference curve complies with the following five properties: (i) indifference curves are thin, (ii) they never cross, (iii) they have negative slope, (iv) they are continuous and (v) they are convex respect to the origin (Mas-Colell et al., 1995).

Because utility functions are based on preference it is relevant to mention a couple of definitions and two important properties of the said preferences (Mas-Colell et al., 1995; Varian, 2010):

- If an agent weakly prefers x to y ($x \succcurlyeq y$) and weakly prefers y to x ($y \succcurlyeq x$) then the agent will be indifferent between both goods ($x \backsim y$); and the utilities of both of them will be equal, i.e $u(x) = u(y)$
- If an agent weakly prefers x to y ($x \succcurlyeq y$) but is not indifferent between x and y, then the agent strictly prefers x to y ($x \succ y$); and in consequence $u(x) > u(y)$

- Preferences are **monotone** if more quantity of any good improves the agent's position.

  Let $X = (x_1, ..., x_n)$ and $Y = (y_1, ..., y_n)$ represent two bundles of goods,

  $$\left.\begin{array}{ll} x_i \geqslant y_i & \text{for each } i \\ x_i > y_i & \text{for some } i \end{array}\right\} \quad \text{implies} \qquad x \succ y$$

  which implies that the indifference curves are on one hand thin and downwards sloping; and on the other rule out inflexion points on the utility function.

- Preferences are **convex**, that is if $x \succcurlyeq y$ then:

  $$tx + (1-t)y \succcurlyeq y \qquad \forall t \in [0, 1]$$

  and interms of utilities this is if $u(x) \geqslant u(y)$ then:

  $$u(tx + (1-t)y) \geqslant u(y) \qquad \forall t \in [0, 1]$$

  which is known as quasi-concave utility function (Mas-Colell et al., 1995; Varian, 2010)

A.1.1 Marginal Rate of Substitution

The classical results presented in this subsection can be found in almost every microeconomics textbook, the interested reader is encourage to consult those (Mas-Colell et al., 1995; Pindyck and Rubinfeld, 2001; Gravelle and Rees, 2004; Krugman and Wells, 2005; Board, 2009; Varian, 2010; Frank et al., 2012). The slope of the indifference curve is the marginal rate of substitution (MRS), which suggest how much of one good, the agent is willing to sacrifice to obtain one more unit of the other. This is,

$$MRS = -\frac{dx_2}{dx_1}\bigg|_{u(x_1,x_2)}$$

To consider the effect of changes on the agent's vector of goods, it is necessary to calculate the total differential of the utility function, so

$$du(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1}dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2}dx_2$$

but, because it is an indifference curve, it is possible to replace $du(x_1, x_2)$ for zero, then

$$\begin{aligned} 0 &= \frac{\partial u(x_1, x_2)}{\partial x_1}dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2}dx_2 \\ \Longleftrightarrow -\frac{dx_2}{dx_1} &= \frac{\partial u(x_1, x_2/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} \end{aligned}$$

where, $\frac{\partial u(x_1,x_2)}{\partial x_i}$ is the marginal utility of good i ($MU_i(x_1, x_2)$). Consequently, it can be seen that,

$$MRS = -\frac{dx_2}{dx_1} = \frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = \frac{MU_1}{MU_2}$$

A.1.2 Examples of Functional Utility Function Forms

For example purposes, a non-exhaustive list of utility functions (in two dimensions) is presented (Nicholson, 2005; Board, 2009; Varian, 2010; Wills, 2011)

(i) Perfect Substitutes: $u(x_1, x_2) = \beta_1 x_1 + \beta_2 x_2$ with β_i constants $\forall i$

(ii) Perfect Complements: $u(x_1, x_2) = min\{\beta_1 x_1, \beta_2 x_2\}$ with β_i constants $\forall i$

(iii) Cobb-Douglas: $u(x_1, x_2) = {x_1}^{\beta_1} \cdot {x_2}^{\beta_2}$ with β_i constants $\forall i$

(iv) Quasi-Linear: $u(x_1, x_2) = f(x_1) + x_2$

(v) Constant Elasticity of Substitution (CES): $u(x_1, x_2) = (\alpha \cdot x_1^\rho + \beta \cdot x_2^\rho)^{\frac{1}{\rho}}$

(vi) Additive Preferences: $u(x_1, x_2) = f_1(x_1) + f_2(x_2)$

(vii) Bliss Point: $u(x_1, x_2) = -\frac{1}{r}[(x_1 - c_1)^2 - (x_2 - c_2)^2]$ with (c_1, c_2) the bliss point[1]

(viii) Stone-Geary: $u(x_1, x_2) = (q_1 - \gamma_1)^{\beta_1} \cdot (q_2 - \gamma_2)^{\beta_2}$ with q_i consumption of good i, and γ_i,β_i parameters

[1]Note that this utility function contravene the monotonicity property, and it can some times have a upward sloping

Appendix B

Auctions

In this appendix we provide some information initially considered when building the model presented in 3.1.1. It is believed that it is important mentioning this topic, as it is in the opinion of the author a potential future research avenue to expand on the current work.

B.1 Basics of Auctions

The origin of auctions as a selling method dates back to the V century BC (Macey, 1990; Wong et al., 2014). At that time, it was common to utilize this method on the "Human Being Market" either for acquiring a wife (Babylon)(Rowbotham, 1895) or to acquire slaves (Roman Empire, Antique Greece, Persia, and other civilizations) (Jones, 1956). Nevertheless, it was not until the XVII century AD (Carretero Peñalva et al., 2015) that auctions gained real importance. It is also, by this time that new methods to present offers emerge.

Furthermore, the *Theory of Auctions* as such, had its beginnings in 1948 with Chamberlin's empiric work about the competitive equilibrium of Walras (Chamberlin, 1948) and it was perfected by Lawrence Friedman (1956), with his academic work presenting an optimal strategy to submit bids. It was in 1961, thanks to William Vickrey, that Game Theory started to be utilised to analyse auctions and to determine strategies to submit bids (EMAR, 2014).

Now, a brief explanation of the basic auction families will be provided (Shoham and Leyton-Brown, 2008):

- **English Auctions:** The auctioneer sets a starting price for the good, and the agents then have an option to announce successive bids, each of which must be higher than the previous. The final bidder, who is, by definition, the agent with the highest bid, must purchase the good for the amount of the final bid
- **Japanese Auctions:** The auctioneer sets a starting price for the good, and the agents must choose whether they are willing to purchase the good at that price or not. Then, the auctioneer calls out successively increasing price, and each time the agents must announce if they are in or not (when a bidder drops out it is irrevocable). The auction ends when there is just one agent left in, and at that point the bidder must buy the good for the current price
- **Dutch Auctions:** This type of auctions is also called descending auction. Here the auctioneer first announces a high price and then proceeds to announce successively lower prices in a regular fashion. The auction ends when the first agent signals the auctioneer by pressing a buzzer; the agent must purchase the good for the last displayed price

- **Sealed-bid Auctions:** In this case, each agent submits to the auctioneer a secret, sealed bid that is not available to any other agents. The agent with the highest bid must purchase the good, but the price at which the good is bought depends on the sub-type of auction. In a first-price sealed bid auction the winning agent pays the amount of his own bid. In a second-price sealed bid or Vickrey auction, the agent pays an amount equal to the next highest bid (i.e. the highest rejected bid). In general, in a k-th price auction the winning agent buys the good for a price equal to the k-th highest bid.

Now,the more common approaches to Auctions Theory will be presented in a brief manner. The next portion of the literature review is heavily based on the work of Durá (2002).

1. **Models with non-strategist competitors:** As mentioned before, in 1956 Friedman introduced a model in which the auction participants do not act strategically, thus it is not possible to apply Game Theory on the equilibrium analysis. Friedman showed a methodology to obtain optimal bids in first price auctions; the key in this model is the capacity that any agent has to analyse the behaviour of the other agents in presenting his own bids. This is, each agent has an expected value (v) and presents a bid (b) for the good. The agent's expected utility is

$$\mathbb{E}\{U\} = (v - b) \cdot \mathbb{P}(b)$$

where $\mathbb{P}(b)$ represent the probability that his bid wins (i.e. that b is the highest bid). The difficulty of this model lies on how to calculate this probability.

The first approach to calculate the said probability was, in a way, to observe the past behaviour of each participant and extrapolate the present behaviour. Once the bidder had estimated the distribution of every potential rival, $\mathbb{P}(b)$ would be the product of probabilities of defeating each one.

Subsequently, Friedman extended his model to include an unidentified number of competitors, simultaneous auctions, and other variations. The criticism is that the solution given by Friedman assumes that the probabilities of defeating each competitor are independent and more or less easy to calculate.

2. **Auctions with perfect information:** These models are characterized by the fact that the agents do not just have knowledge of their own bid, but also know the bids of the different participants. For this reason it is possible to solve the problem with Game Theory, i.e. the auction is seen as a game with perfect information, and the result will be a Nash Equilibrium.

3. **Benchmark Model for Auctions:** This model was presented by McAfee and McMillan (1987) and is based on four assumptions:

    a) **Bidders have symmetrical information:** The valuations of each agent have the same probability distribution, thus all agents will be equal “ex ante” and the seller would perceive all the participants in a similar fashion.

b) **Bidders are risk-neutral:** With this assumption one can say that the utility for the monetary function of each agent is linear, thus the expected utility value maximization problem is equivalent to maximize the expected profit.

c) **Bidders have independent private valuations:** This means, on one hand, that each agent knows exactly how much his valuation for the good is, and on the other, the random variables from which the valuations are derived are independent and therefore their correlations are zero.

d) **Payments are a function of only the bids:** The seller’s expectation of payment is a function of the bids and it is not possible to make it dependent on posterior variables showing the “real” value that will be reported to the buyer.

Other assumptions are that the bid preparation costs of buyers and the analysis of cost offers by the seller are zero; there is no collusion between buyers, thus they have a non-cooperative behaviour; the number of buyers, their risk propensity and their distribution of probability functions are all of public knowledge; and buyers are confident that the seller will comply with the rules.

The four basic auction families, then, will have dominants equilibrium strategies that will constitute a Bayes-Nash Equilibrium. These strategies will be discussed in the following subsection.

B.2 Bayes-Nash Equilibriums

B.2.1 English Auctions

The dominant strategy for agent i is to remain on the auction until the price equals his valuation v_i. If all agents are rational and follow their dominant strategy, then the winner agent will pay a price that is the second highest bid, v_j, which is the price at which agent j will retire[1]. Having said that, the seller will not obtain the maximum possible price and the winner agent i will have a net profit just shy of $(v_i - v_j)$.

B.2.2 Second Price Sealed-bid

The dominant strategy in this case is to "say the true" (i.e. to make a bid for the same amount of the valuation of each agent). In this case, the price at which the winner (agent i) will buy the good should be completely independent of his own bid if and only if the bids of the other agents are independently placed and not showing any correlation structure. If the agent present a bid for a value less than v_i, it only contributes to lowering his chances to win. Similar to an English Auction, the winner will have a net profit of $(v_i - v_j)$ with v_j the second highest bid.

B.2.3 Dutch Auctions

The result in this case is that each agent will be willing to pay his own valuation v_i, and if he wins his net profit will be zero. Given this result, it is possible to

[1]Reasoning by contradiction, let us suppose that there are two players, i and j, and that player i is willing to pay a maximum of \$100 and player j's willingness to pay is \$90, when player j retires at \$90+δ with $\delta > 0$ a very small number, then it does not make any economic sense to pay anything more than \$90+δ, it follows that a payment of \$100 being substantially higher than \$90+δ is not rational.

see that it is the same as that derived from a First Price Sealed-bid. As such, the focus will be on explaining the latter.

B.2.4 First Price Sealed-bid Auctions

The competitors' strategy, in this case, is not as simple and direct as in the previous cases. The bigger the bid, the greater the probability of winning but, at the same time, the expected profit is lower. In a First Price Sealed-bid Auction, there are no dominant strategies. The agents will need, necessarily, to make conjectures about other agents' behaviour. As showed in the work of McAfee and McMillan (1987) the competitors have incentives to present bids that are at the same time higher than those of the competitor, but no greater than their own valuation. Thus, the maximization problem will be solved by presenting a bid as follows:

$$b_i = B(v_i) = v_i - \frac{1}{F(v_i)^{n-1}} \int_{v_{min}}^{v_i} [F(v_i)]^{n-1} dv_i \tag{B.1}$$

where b_i and v_i represent the bid and valuation of agent i respectively, v_{min} is the valuation that gives zero profit to agent i, and $F(\cdot)$ is the distribution function of the valuations with n being the number of bidders.

Important characteristics to note on the bids' functions are the following:

(a) The second term of equation (B.1) is positive, thus in the case of a Dutch or a First Price Sealed-bid Auction, $b_i \leqslant v_i$

(b) The function $B(\cdot)$ increases when the number of competitors grows

(c) When $n \to \infty$ then $b_i \to v_i$, which is consistent with zero profits on perfect competition

B.3 Revenue Equivalence

There is a classical result explained in the following quote:

> *Of the large (in fact, infinite) space of auctions, which one should an auctioneer choose? To a certain degree, the choice does not matter* (Shoham and Leyton-Brown, 2008)

The result formally known as the *Revenue Equivalence Theorem* is formalised in the following theorem.

Theorem B.3.1 (Revenue Equivalence Theorem) *Assume that each of n risk neutral agents has an independent private valuation (IPV) for a single good at auction, drawn from a common cumulative distribution $F(v)$ that is strictly increasing and atomless on $[\underline{v}, \bar{v}]$. Then any efficient auction mechanism in which any agent with valuation $\underline{v}$ has an expected utility of zero yields the same expected revenue, and hence results in any bidder with valuation v_i making the same expected payment.*

Proof: For the proof the reader is encouraged to read (Shoham and Leyton-Brown, 2008). ■

It needs to be noted that the result established above is for the private-value, single-good case. Similar theorems hold for other cases (however, not all).

B.4 Benchmark Model's Variations

In this subsection some of the common variations of the Benchmark Models will be discussed concisely. These variations are produced by relaxing different assumptions.

- **Risk attitudes:** (Maskin and Riley, 1985; Janssen and Karamychev, 2005) As discussed, one of the classical assumptions is that agents are risk neutral, but what happens when this changes. In short, the Revenue Equivalence Theorem does not hold in every auction's type.

  Facing agents that are risk averse, expected revenue on English and second-price auctions do not vary; but in the case of Dutch or first-price auctions, the expected revenue would be higher. On the other hand, when the agents are risk lovers, they will bid less aggressively in a first-price auction, and the seller will have incentives to hold a second-price auction.

  In a similar fashion, if the auctioneer is risk averse he will be inclined to hold a first-price auction, and if he is a risk lover a second-price auction would be preferred.

- **Asymmetries among buyers:** (Güth et al., 2005; Cantillon, 2008) As mentioned before the assumption of symmetry implies that all the competitors are equal "ex ante". A way of modeling the asymmetries is to assume that the valuation of each agent comes from different probability's distributions, and also assume that there is a "strong" buyer to whom the other agents attribute higher odds of winning.

Under those premises, the result of an English auction will be affected, because it is already known that in an ascendant auction the winner will be the agent with the highest valuation, since as the price increases other participants will drop out. Hence, with the perception of a "strong" participant, the incentives to enter the bid are lower and consequently the final price will be lower.

In the case of a first-price auction, the effects of the said asymmetry are, somehow, gentler. In this type of auction there is a probability that the agent with the highest valuation does not succeed. Because of the asymmetry, the participants can have different strategies and that could lead to a lower bidder winning.

- **Collusion:** (Robinson, 1985; Marshall and Marx, 2007) This case happens when the assumption of no cooperation or independence is lifted. The Cartel's members can be organized in order to lower the final price and to distribute the difference between them. Another curse of action could be to make "phantom" bids in order to alter the auctions' results.

Appendix C

Basic Implementation Details

C.1 SimPy

SimPy is a Python library for process-oriented discrete-event simulation. On the process-oriented discrete-event simulation paradigm each simulation activity is modelled by a process. The other two paradigms are:

- Activity-Oriented: The time is broken down into tiny increments. At each time point, the code will look at all activities and check for the possible occurrences of events
- Event-Oriented: This paradigm implements a future event list (FEL) where future activities are stored in an ordered manner. Then the clock is moved to the next event time in the FEL, thus reducing substantially the time required by the activity-oriented paradigm

Processes in SimPy are defined by Python generator functions and can be used in a multitude of applications, and in particular in the modelling of active components like customers, vehicles or agents. `SimPy` also provides shared resources code facilities which can be used to model limited capacity congestion points. Typical examples are servers, checkout counters, front-end loaders, shovels, etc., which are not being used in the context of the current thesis but are worth of awareness.

`SimPy` is based in of simulation environment concept. A simulation environment is in charge of running the simulation time and managing the scheduling and processing of events. A very basic example of how an environment is initialised and used is presented in code Listing (C.1) (taken from `SimPy`'s documentation).

```python
import simpy

def my_proc(env):
    yield env.timeout(1)
    return "Monty Python's Flying Circus"

env = simpy.Environment()
proc = env.process(my_proc(env))
env.run(until=proc)
```

Listing C.1: Minimal `SimPy` Example

It can be seen that `SimPy` is declared using the directive `import simpy`. After that the environment is declared using `env = simpy.Environment()` which later is run for a specified amount of time as exemplified by the line `env.run(until=proc)`. The rest of the lines shown in the example are used to create a process that is then executed according to the logic of the process.

A better way to use the `SimPy` library is by integrating it into the classes (i.e., the process logic becomes part of the class design). For example, Listing (C.2), which has been adapted from the documentation, illustrates the main structure of a code that runs the logic of processing inside a class in `SimPy`.

```
import simpy
import random as rd


class Car(object):
    def __init__(self, env, id):
        self.env = env
        self.action = env.process(self.run())
        self.id = id

    def run(self):
        while True:
           print('Car', self.id, 'Start parking and charging at %f' %
           ↪  self.env.now)
           charge_duration = rd.uniform(1, 3)
           yield self.env.process(self.charge(charge_duration))

           print('Car', self.id, 'Start driving at %f' % self.env.now)
           trip_duration = rd.uniform(2, 4)
           yield self.env.timeout(trip_duration)

    def charge(self, duration):
        yield self.env.timeout(duration)


if __name__== '__main__':
    env = simpy.Environment()
```

```
    carList = []
    for i in range(3):
        carList.append(Car(env, i))
    env.run(until=12)
```

Listing C.2: `SimPy` Class Integration Example

The output of this Listing is given by:

```
Car 0 Start parking and charging at 0.000000
Car 1 Start parking and charging at 0.000000
Car 2 Start parking and charging at 0.000000
Car 1 Start driving at 1.194972
Car 2 Start driving at 1.996193
Car 0 Start driving at 2.279162
Car 0 Start parking and charging at 4.427830
Car 1 Start parking and charging at 4.825655
Car 2 Start parking and charging at 5.875815
Car 0 Start driving at 6.847299
Car 1 Start driving at 7.590702
Car 2 Start driving at 7.884991
Car 0 Start parking and charging at 9.352278
Car 2 Start parking and charging at 10.237889
Car 1 Start parking and charging at 10.471736
Car 1 Start driving at 11.950355
```

In general, we can see that the minimal structure that a Python class must have in order to support `SimPy` is given by:

```
import simpy

class Class_Name(object):
    def __init__(self, env):
        self.env = env
        self.action = env.process(self.run())

    def run(self):
```

```
        while True:
            do_some_actions
            yield self.env.timeout(duration)
```

In the last code snippet, the line "`def __init__(self, env):`" initialises the class definition and it can be observed that requires `env` to be passed as argument. This environment variable (`env`) is a `SimPy` object that requires instantiation in the main program, usually done in python by using the assignment instruction `env = simpy.Environment()):` which instructs the execution of the program to create an object of the type `Environment` and assigning it to the variable `env`. By following that sequence of declarations and use in the class definition, causes the objects that are instantiated from the class template to be linked to the environment's execution thread. The next line in terms of importance for the code is `self.action = env.process(self.run())` which declares the action that is attached to the environment which is defined on its logic by using `self.run()`.

C.2 Containers and Stores

In this sub-subsection we briefly mention a couple of special objects defined in `SimPy` which can be of use in the development of the implemented model: Containers and Stores. Containers are `SimPy` constructs that assist in the modelling of production and consumption of a homogeneous, undifferentiated bulk material. It may either be continuous (like water) or discrete (like apples). On the other hand Stores can be used to model the production and consumption of concrete objects, in contrast to containers which store abstract amounts.

Containers and Stores both expose `capacity` as a property of the object, and in the particular case of Containers, it is possible to query the level of the Container at any point of time by using the `level` attribute.

C.3 Implementation Considerations

C.3.1 Implementation of the `Household` Agent

For the `Household` agent the main implementation decision relates to the implementation of properties and the main action, which is performed in every cycle, where the agent needs to decide how much to consume for each different item in order to maximise its utility function.

The code for the object is given in section E.1 in the appendix. The `Household` object is characterised by:

- Identifier for the instance
- Cobb-Douglas coefficients for the utility function
- Budget
- Simulation environment to which the instantiated object belongs
- Action logic required by `SimPy`
- Auxiliary properties used for counting/logical purposes

Also, the object implements the following methods:

- Payment of bills

- Addition of salary for the household
- `string` representation for printing purposes
- Different methods to define the optimisation problem and schedule its execution within the simulation thread

In a previous section (see Section 4.6), a particular validation example instance was analysed to check the correctness of the optimisation methods associated with the economic decisions of the agents belonging to this class.

C.3.2 `Firm` Agent

For the `Firm` agent the main implementation decision also relates to the implementation of properties and the main action that will be performed in every cycle of the simulation, where the agent needs to decide how much to spend on every item of the list of possible items in order to maximise its utility function.

The code for the object is given in section E.2 in the appendix. The properties that characterise the `Firm` object are:

- Identifier for the instance
- Cobb-Douglas coefficients for the utility function
- Budget
- Simulation environment to which the instantiated object belongs
- Sector to which the firm belongs
- Economy that contains the sector for which the firm belongs

- Action logic required by `SimPy`
- Auxiliary properties used for counting/logical purposes

Also, the object implements the following methods:

- Payment of bills
- Addition of payments coming from consumers of the goods
- `string` representation for printing purposes
- Different methods to define the optimisation problem and schedule its execution within the simulation thread

In a previous section (see Section 4.6), a particular validation example instance was analysed to check the correctness of the optimisation methods associated with the economic decisions of the agents belonging to this class. Please note that this class is similar to the `Household` class, however, the difference lies in the form of the utility function, which in the current case has a constant that multiplies the familiar utility function form and also requires calculation of the expenses for raw materials used for production purposes (this is a utility maximisation problem).

C.3.3 `Economic Sector` and `Economy` Agents

These two agents are very simple ones and basically they are constituted of code that keeps track of the different sectors in the economy, average prices of the different goods, etc. Their implementation is provided in in section E.3 in the appendix.

C.3.4 `Market` Agent

This is by far the most complex of all the classes. This class implements the market assignation mechanism that has been described previously which requires the iterated solution of an assignation problem to define who buys from whom, processes payments, updates inventories, etc.

The code is given in section E.4 in the appendix. The `Market` class exposes the following properties:

- Identifier for the instance
- A class variable to count the number of markets (if there is more than one)
- Simulated environment to which the instantiated object belongs
- Action logic required by `SimPy`
- `Economy` object containing all sectors for the simulation
- The list of households
- the list of firms
- The average prices
- The average willingness to pay for items on every sector
- Auxiliary properties used for counting/logical purposes

and also implements the following methods:

- Optimisation model

- Main greedy loop of the hybrid optimisation process
- `string` representation for printing purposes

In a previous section (see Section 4.6), a particular validation example instance was analysed to check the correctness of the optimisation process and model associated with the assignment decisions of the market. More importantly, we noted and discussed the problems faced when validating the initial design and the changes that were introduced to fix those.

Appendix D

Code for First Example

In this appendix we provide the code for the first example found in chapter 3.

D.1 EconAgent.py

```
class EcAgent(object):
    n_ec_agents = 0
    __slots__ = ['id', 'tech_mat', 'ac_money', 'income', 'part_income']

    def __init__(self, tech_mat, ext_demand):
        self.id = EcAgent.n_ec_agents
        self.tech_mat = tech_mat
        self.ac_money = ext_demand
        self.income = ext_demand
        self.part_income = 0.0
        EcAgent.n_ec_agents += 1

class Simula(object):
    __slots__ = ['agent_list']

    def __init__(self, agent_list):
        self.agent_list = agent_list

    def iterate(self):
        for a in self.agent_list:
            i = 0
            for b in a.tech_mat:
```

```
                self.agent_list[i].part_income += a.income*float(b)
                i += 1
            a.ac_money += a.part_income
            a.income = a.part_income
            a.part_income = 0
```

Listing D.1: EconAgent.py

D.2 EconAgents.py

```
class EcAgent(object):

    n_ec_agents = 0
    __slots__ = ['id', 'tech_mat', 'ac_money', 'income', 'weight']

    def __init__(self, tech_mat, ext_demand, weight):
        self.id = EcAgent.n_ec_agents
        self.tech_mat = tech_mat
        self.ac_money = ext_demand * weight
        self.income = ext_demand * weight
        self.weight = weight
        EcAgent.n_ec_agents += 1


class Simula(object):
    __slots__ = ['sector_list']

    def __init__(self, sector_list):
        self.sector_list = sector_list

    def iterate(self):
        for a in self.sector_list:
            for b in a.agent_list:
                for k in range(len(b.tech_mat)):
                    self.sector_list[k].part_income += b.income * float(b.tech_mat[k])

        for a in self.sector_list:
            a.ac_money += a.part_income
            a.income = a.part_income
            for b in a.agent_list:
                b.ac_money += a.part_income * b.weight
                b.income = a.part_income * b.weight
            a.part_income = 0
```

```python


class EcSector(object):
    n_ec_sector = 0
    __slots__ = ['id', 'tech_mat', 'ac_money', 'income', 'part_income',
    ↪  'agent_list']

    def __init__(self, tech_mat, ext_demand, agent_list):
        self.id = EcSector.n_ec_sector
        self.tech_mat = tech_mat
        self.ac_money = ext_demand
        self.income = ext_demand
        self.part_income = 0.0
        self.agent_list = agent_list
        EcSector.n_ec_sector += 1
```

Listing D.2: `EconAgents.py`

D.3 Main.py

```python
from EconAgent import *
import copy
from timing import timing
import pprint
import matplotlib.pyplot as plt
import numpy as np

nsector = 3
io_matrix = [
[0.410241249, 0.06244087, 0.123580889],
[0.030104632, 0.374961757, 0.158783577],
[0.025658586, 0.10496279, 0.191891522]
]


def run(demand):
    agent_list = []
    for sector in range(nsector):
        agent_list.append(EcAgent(io_matrix[sector], demand[sector]))

    a = Simula(agent_list)

    history = []
    old_scores = []
```

```
    new_scores = []

    for agent in a.agent_list:
        old_scores.append(agent.ac_money)
    history.append(old_scores)

    tolerance = 1.0

    while tolerance > 0.000000001:
        a.iterate()
        new_scores = [a.agent_list[i].ac_money for i in range(len(a.agent_list)) ]
        v = [new_scores[i] - old_scores[i] for i in range(len(old_scores))]
        tolerance = max(v)
        history.append(new_scores)
        old_scores = copy.deepcopy(new_scores)
        # print(old_scores)
    return new_scores, history


@timing
def main(printa):
    production, hist1 = run([39.24, 60.02, 130.65])
    if printa:
        print(f'Final Production due to Initial Demand: {production}\n')
        pprint.pprint(hist1)

    income_mult, hist2 = run([1, 1, 1])
    if printa:
        print(f'Production Income Multiplier: {income_mult}\n')
        pprint.pprint(hist2)

    return np.array(hist1), np.array(hist2), income_mult, production  #, percentage


def plot(series, title, ylabel, xlabel):
    plt.plot(series)
    plt.title(title)
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    plt.savefig(f'G:/Paulina_PhD_Arreglos/Chapter3_Arreglos/Primera_Corrida/{title}')
    plt.close()

if __name__ == '__main__':
    hist1, hist2, income_mult, production = main(True)

    plot(hist1[:, 0], 'Agriculture Production', 'Evolution', 'Iterations')
    plot(hist1[:, 1], 'Manufacture Production', 'Evolution', 'Iterations')
    plot(hist1[:, 2], 'Service Production', 'Evolution', 'Iterations')
```

```

    plot(hist2[:, 0], 'Agriculture Income Multiplier', 'Evolution', 'Iterations')
    plot(hist2[:, 1], 'Manufacture Income Multiplier', 'Evolution', 'Iterations')
    plot(hist2[:, 2], 'Service Income Multiplier', 'Evolution', 'Iterations')
```

Listing D.3: `Main.py`

D.4 Input-Output Matrix for Larger Example

```
[0.0153146518199961, 0.0176196768440526, 0.000175890490410429, 0.0024324144000079,
↪  0.0169486688378651, 0.000137112207097847, 0.00038638102250946,
↪  0.000234489251935134, 0.0023855056292599, 0.000102751884724635,
↪  0.0325813800218376, 0.00136335363995128, 0.00217453343611647,
↪  0.00122875935390685, 0.00018057400604148, 0.00953288067554069,
↪  0.00909146554817231, 0.0095330529659737, 0.000632981206607034,
↪  0.00241648435443526, 0.00394802878579249, 0.024055443240899, 0.0136771209258618,
↪  0.00317451965616088, 0.00936464618737065, 0.00503945290053959,
↪  0.0103432578012391, 8.37228234864658E-05, 0.0179196366828436,
↪  0.00179625680315356, 0.000764803106499304, 0.000422382008037116,
↪  0.000933778944774406, 0.0201942077066504, 0.000399052634506235],
[2.58383971320765E-12, 0.0671581567284489, 0.000788002161285402,
↪  0.000251830294074144, 0.000290272599131723, 1.39630203511498E-05,
↪  0.000150392391823567, 1.24851958392399E-05, 0.000430893401901278,
↪  0.000121680147576262, 0.0074969909301441, 0.0247630909526946,
↪  0.000382676227658367, 0.0021717501248718, 0.000120482495474942,
↪  0.00543006870837942, 0.000205497961880429, 0.0100410136180718,
↪  6.81529407018751E-05, 0.00223997611032303, 0.0207584843585361,
↪  0.0188018606950646, 0.00997854087378635, 0.00135069589433389,
↪  0.0122218913919028, 0.00446786213770329, 0.0367340378248146,
↪  0.00142226297720577, 0.00728488597929276, 0.00113347609665441,
↪  0.000336036335732209, 6.62032553110239E-05, 0.000359367036770517,
↪  0.0286098016734021, 0],
[1.23703887981921E-13, 0.000303519671381709, 0.0844508723444515,
↪  8.09616469958218E-06, 8.17789396536846E-06, 3.63097934944794E-07,
↪  2.10779870498975E-06, 4.0836872332361E-07, 9.31812662686124E-06,
↪  8.78508813186489E-06, 0.00148658108267857, 0.00361833561015902,
↪  4.52736850600532E-05, 0.000160709339316673, 9.62807129851168E-05,
↪  0.00156464854226206, 4.23055982051812E-06, 0.000869581581186203,
↪  2.60961648159973E-06, 0.00125272159952065, 0.00884024254399223,
↪  0.000754451112821021, 0.000522561094387156, 0.000461727851603634,
↪  0.000902320354964223, 0.000822585852073402, 0.0016696911916348,
↪  0.000851241022386102, 0.0030151129212795, 0.000472796115660397,
↪  0.000124702014840487, 1.34951954143289E-06, 3.56918695186777E-05,
↪  0.00273877057234649, 3.58450480667399E-05],
```

```
[3.31339933246901E-09, 0.0399257181539992, 0.00238376780142846, 0.0306250084681214,
↪  0.0014089398130071, 0.000263300678232551, 0.000232514642235045,
↪  2.38595814099053E-05, 7.28045985738929E-05, 0.000586964867703573,
↪  0.000606128927411356, 0.000969775792226543, 0.00141188314934354,
↪  0.000168364250061706, 3.77780451908291E-05, 0.00185356881280247,
↪  2.08628865907893E-05, 0.0012369267499973‌8, 8.68181860246955E-05,
↪  0.0106885506758763, 0.0021365046601621, 0.0210133052607574, 0.00785827765707848,
↪  0.0013209724710474, 0.0596047938814779, 0.00936641417682709,
↪  0.00473807476526613, 0.00627038475056778, 0.0142763251925268,
↪  0.00105479778807169, 0.000729224833836012, 5.27880766898661E-05,
↪  2.32621199071066E-05, 0.00299848544470573, 0.000124629172606973],
[7.06440553368058E-10, 0.00148815650326726, 0.00560937766159772, 0.0369904085153668,
↪  0.022377617105659, 0.00201513862778852, 0.000611794117268397,
↪  0.000140366875530838, 0.000374204019402294, 0.00209613811173305,
↪  0.0018885374864488, 0.00431799348238908, 0.00720966206240633,
↪  0.00101007283244528, 0.00031141993272885‌2, 0.00304659791629405,
↪  0.000111567769328964, 0.00280126209124863, 0.00152468941822443,
↪  0.0091308605243122, 0.00473053013668484, 0.0571736598700885, 0.0205387406212921,
↪  0.00577837411219683, 0.0356010278758524, 0.0193353699261692, 0.0149782662951528,
↪  0.00205021258709738, 0.0289372784374401, 0.00270508003930117,
↪  0.00112167138492067, 3.19611676574111E-05, 0.000211737896120664,
↪  0.00812341863420678, 5.28361534217938E-05],
[1.23749616359791E-13, 0.000189712832387515, 0.00429508342187397,
↪  0.000653643580730678, 0.00190282126719189, 0.0553846930833539,
↪  0.000578943145045589, 0.000106398036862254, 0.000971960631395193,
↪  0.00456006390216614, 0.00250240225738683, 0.00343623154174053,
↪  0.0458366611602875, 0.0134979243666926, 0.000389100169514113,
↪  0.0228979773837046, 0.000175115223193277, 0.00967003582902392,
↪  0.000795718519626079, 0.0148021200813204, 0.00681671506316456,
↪  0.0345629151653544, 0.0110482068280104, 0.0487780035289619, 0.026988266909114,
↪  0.0316014349012862, 0.0225808472275731, 0.00260329240063207, 0.0472249716353282,
↪  0.00494391037445234, 0.00195271539452815, 0.00038554141858528,
↪  0.00152893878303547, 0.0144923601293823, 4.46404273356402E-05],
[2.54699728248297E-14, 0.0328975528637865, 0.00147010664227085, 0.0329207440655801,
↪  0.00048742123705554, 3.79280280752995E-05, 0.0136443719018457,
↪  0.000341632371409266, 0.00123643570622209, 0.000694727953111651,
↪  0.000949822871167546, 0.00905342349274798, 0.00437022071459713,
↪  0.000609913857993445, 0.00199937304781753, 0.00407498933357667,
↪  7.66371024394251E-05, 0.00294367449368563, 0.000492633452647072,
↪  0.00661405564758503, 0.00317271202460385, 0.0468690637286115,
↪  0.0127222011006219, 0.00267406240360155, 0.0291139670786891,
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Appendix E

One Potential Implementation of the Proposed Model

In this appendix we provide the code for the implementation of the proposed model, most of it is referenced from chapter 4

E.1 Household Agent

This section of the appendix contains the relevant code for the implementation of the household agent as discussed in section 4.1.

```
import numpy as np
import simpy
import Economy as ec
import Company as co
import random as rd
from colorama import init, Back, Fore, Style
```

```python
from scipy.optimize import minimize
from TimeMeasure import measure

init(convert = True)

class household(object):
    def __init__(self, **kwargs):
        if kwargs['name'] != '':
            self.id = kwargs['name']
        else:
            self.id = 'H_' + str(households.counter)
        self.cdCoefficients = kwargs['cdCoeffs']
        self.budget = kwargs['budget']
        self.salary = kwargs['salary']
        self.industry = kwargs['industry']
        self.composition = kwargs['composition']
        self.dwelling = kwargs['dwelling']
        households.aggregatedIncome += self.budget
        self.print = kwargs['print']
        households.counter += 1
        self.env = kwargs['env']
        self.action = self.env.process(self.run())
        self.niter = 0
        self.economy = kwargs['economy']
        self.willingnessToPay = {}
        self.sectorDemand = {}
        for key, value in self.economy.avgprices.items():
            self.willingnessToPay[key] =  value + rd.normalvariate(0,0.02)
            self.sectorDemand[key] = 0

    def billPayment(self, amount):
        if (self.budget-amount)>=0:
            self.budget -= amount
            households.aggregatedIncome -= amount
        else:
            self.budget = 0
            households.aggregatedIncome = 0

    def salaryAdd(self, amount):
        self.budget += amount
        households.aggregatedIncome += amount

    def __str__(self):
        return 'House ID: ' + self.id

    def utilityFunc(self, x):
        counter = 1
        prod = 1
        for key in self.economy.returnSectorList():
```

```
            prod *= x[counter]**self.cdCoefficients[key]
            counter += 1
        return prod

    def optimiseUtility(self):
        def valueOfChoice(x):
            return -self.utilityFunc(x)

        # Define the budget constraint
        constraints = (
            {'type': 'ineq', 'fun': lambda x: self.budget - np.dot(list(self.economy.avgprices.values()), x[1:len(list(self.economy.avgprices.values()))+1])},
            {'type': 'ineq', 'fun': lambda x: x},
            {'type': 'eq', 'fun': lambda x: x[0] - 1}
        )
        x0 = [1 for i in range(len(self.economy.avgprices)+1)]
        res = minimize(valueOfChoice, x0, method='SLSQP',
                       options={'disp': False, 'iprint': 1,
                                'eps': 1.0e-09, 'maxiter': 1000,
                                'ftol': 1e-06}, constraints=constraints)

        return self.utilityFunc(res.x), res.x, np.dot(list(self.economy.avgprices.values()), res.x[1:len(list(self.economy.avgprices.values()))+1])

    def run(self):
        while True:
            self.salaryAdd(self.salary)
            utility, solution, budget = self.optimiseUtility()

            counter = 1
            for key, values in self.sectorDemand.items():
                self.sectorDemand[key] = round(solution[counter],4)
                counter += 1
            if self.print:
                print(f'{Fore.LIGHTYELLOW_EX}{self.__str__()}{Style.RESET_ALL} has budget {self.budget} at time {self.env.now}')
            duration = 1
            yield self.env.timeout(duration)
            self.niter += 1

class households(object):
    counter = 1
    aggregatedIncome = 0
    epsilonfactor = 1.1

    def __init__(self, env, print=False):
        self.collection = {}
```

```
        self.print = print
        self.env = env
        self.action = env.process(self.run())

    def __add_val__(self, house):
        self.collection[house.id] = house
        self.counter += 1

    def __return_coll__(self):
        return self.collection

    def run(self):
        while True:
            if self.print:
                print(f'Households going on {self.env.now}')
            duration = 1
            yield self.env.timeout(duration)
```

Listing E.1: Code for the Household Agent

E.2 Firm Agent

This section of the appendix contains the relevant code for the implementation of the firm agent as discussed in section 4.2.

```
import numpy as np
import simpy
import Economy as ec
import random as rd
from colorama import init, Fore, Style
from TimeMeasure import measure
from scipy.optimize import minimize
import sys

if not sys.warnoptions:
    import warnings
    warnings.simplefilter("error")
```

```
    warnings.filterwarnings("ignore")

init(convert = True)

class firm(object):
    def __init__(self, **kwargs):
        if kwargs['name'] != '':
            self.id = kwargs['name']
        else:
            self.id = 'F_' + str(firms.counter)
        self.cdCoefficients = kwargs['cdCoeffs']
        self.budget = kwargs['budget']
        self.print = kwargs['print']
        firms.counter += 1
        self.env = kwargs['env']
        self.action = self.env.process(self.run())
        self.sector = kwargs['sector']
        self.economy = kwargs['economy']
        self.niter = 0
        self.productPrice = self.economy.avgprices[self.sector] + \
                            rd.normalvariate(0,0.02)
        self.supply = 0
        self.supplyhistory = {}
        self.localperc = kwargs['localperc']
        self.currentIterSales = 0 # We will account for all sales in a
        # given iteration
        self.currentIterRawDemand = {} # We need to save the percentages of
        ↪  each
        # raw material bought to apply template
        self.intermediateFlow = 0
        self.initialBudgets = []
        self.finalBudgets = []
        self.accumMoneysReceived = 0
        self.moneysReceived = []
        self.accumExpenditures = 0
        self.moneysSpent = []

    def billPayment(self, amount):
        self.budget -= amount

    # TODO: Change this one for payments coming from other agents
```

```python
    def paymentAdd(self, amount):
        self.budget += amount
        if (self.supply-amount/self.productPrice) >= 0:
            self.supply -= amount/self.productPrice
            firms.aggregatedOffer -= amount/self.productPrice
        else:
            self.supply = 0

    def moneyAdd(self, amount):
        self.budget += amount

    def moneyTake(self, amount):
        budget_now = self.budget
        self.budget -= max(amount, amount - budget_now)

    def __str__(self):
        return 'Firm ID: ' + self.id

    def utilityFunc(self, x):
        counter = 1
        prod =  self.cdCoefficients['CONST'] * self.economy.collection[self.sector].avgprice
        for key in self.economy.returnSectorList():
            prod *= x[counter]**self.cdCoefficients[key]
            counter += 1
        return prod

    def optimiseUtility(self):
        def valueOfChoice(x):
            return -self.utilityFunc(x)
        # Define the budget constraint
        constraints = (
            {'type': 'ineq', 'fun': lambda x: self.budget - np.dot(list(self.economy.avgprices.values()), x[1:len(list(self.economy.avgprices.values()))+1])},
            {'type': 'ineq', 'fun': lambda x: x},
            {'type': 'eq', 'fun': lambda x: x[0] - 1}
        )
        x0 = [1 for i in range(len(self.economy.avgprices)+1)]
        res = minimize(valueOfChoice, x0, method='SLSQP',
                       options={'disp': False, 'iprint': 1,
```

```
                                'eps': 1.0e-09, 'maxiter': 1000,
                                'ftol': 1e-06}, constraints=constraints)

        return self.utilityFunc(res.x), res.x,
        ↪  np.dot(list(self.economy.avgprices.values()),
        ↪  res.x[1:len(list(self.economy.avgprices.values()))+1])

    def run(self):
        while True:
            # We start discarding all inventory from previous periods,
            # equivalent to be able to provide raw materials to other
            # producers at a preferential price with 20% discount
            self.supply = 0
            self.currentIterSales = 0
            self.currentIterRawDemand = {}
            self.accumMoneysReceived = 0
            self.accumExpenditures = 0
            self.initialBudgets.append(self.budget)
            utility, solution, budget = self.optimiseUtility()
            # Raw materials are 20% cheaper than average price,
            # total quantity is guaranteed because the production problem
            ↪  has
            # a budget constraint on it
            counter = 1
            for key, value in self.economy.collection.items():
                expenditure = value.avgprice*solution[counter]*0.8
                value.toDistribute += expenditure
                self.currentIterRawDemand[key] = expenditure
                self.accumExpenditures += expenditure
                self.budget -= expenditure
                counter +=1
            normFactor = 1.0 / sum(self.currentIterRawDemand.values())
            for k in self.currentIterRawDemand.keys():
                self.currentIterRawDemand[k] = self.currentIterRawDemand[k]
                ↪  *\
                                                    normFactor
            self.finalBudgets.append(self.budget)
            self.moneysSpent.append(self.accumExpenditures)

            Quantity = round(utility /
            ↪  self.economy.collection[self.sector].avgprice, 8)
```

```
            self.supply += Quantity
            self.supplyhistory[self.niter] = []
            self.supplyhistory[self.niter].append(Quantity)
            firms.aggregatedOffer += Quantity
            if self.print:

                ↪ print(f'{Fore.LIGHTCYAN_EX}{self.__str__()}{Style.RESET_ALL}
                ↪ has budget {self.budget} at time {self.env.now}')
            duration = 1
            yield self.env.timeout(duration)
            self.niter += 1
```

Listing E.2: Code for the Firm Agent

E.3 Economy and Sector Agents

This section of the appendix contains the relevant code for the implementation of the economy and sector agents as discussed in section 4.3.

```
import simpy
import random as rd
from colorama import init, Back, Fore, Style
import time
import copy

init(convert=True)

class sector(object):
    def __init__(self, **kwargs):
        if kwargs.get('name') == '':
            self.id = 'S_' + str(economy.counter)
        else:
            self.id = kwargs['name']
        self.print = kwargs['print']
        economy.counter += 1
```

```
        self.env = kwargs['env']
        self.action = self.env.process(self.run())
        self.avgprice = kwargs['avgprice']
        self.technology = {}
        self.firms = {}
        self.toDistribute = 0

    def __str__(self):
        return 'Sector ID: ' + self.id


    def run(self):
        while True:
            self.toDistribute = 0
            yield self.env.timeout(0.1)
            for key, value in self.firms.items():
                value.accumMoneysReceived += self.toDistribute / 3
                value.budget += self.toDistribute / 3
            if self.print:
                print(f'{Fore.LIGHTBLUE_EX}{self.__str__()}{Style.RESET_ALL} is alive and well at time {self.env.now} with average price of {self.avgprice}')
            duration = 0.6
            yield self.env.timeout(duration)
            for key, value in self.firms.items():
                value.moneysReceived.append(value.accumMoneysReceived)
            duration = 0.3
            yield self.env.timeout(duration)


class economy(object):
    counter = 1

    def __init__(self, env, print=False, rfr_rate = 0.03, usePeople = True):
        self.collection = {}
        self.avgprices = {}
        self.print = print
        self.env = env
        self.action = env.process(self.run())
```

```
        self.sectmultiplier = []
        self.globalmultiplier = []
        self.sectmultiplieralt = []
        self.globalmultiplieralt = []
        self.globalsalary = 0
        self.localsalary = {}
        self.iteramarkers = []
        self.rfr_rate = rfr_rate
        self.rfr = 0
        self.usepeople = usePeople

    def __add_val__(self, sector):
        self.collection[sector.id] = sector
        self.avgprices[sector.id] = sector.avgprice
        self.counter += 1

    def returnSectorList(self):
        return self.collection.keys()

    def calculateLocalMultiplier(self):
        # Initialisation of the multiplier
        localTmpMultIni = {}
        localTmpMultInter = {}
        localTmpMultFin = {}
        # We calculate the initial moneys
        for key, value in self.collection.items():
            localTmpMultIni[key] = 0
            for key2, value2 in value.firms.items():
                localTmpMultIni[key] += value2.accumExpenditures * \
                                        value2.localperc
                if key == 'Mining':
                    contrib = value2.accumExpenditures*self.rfr_rate
                    self.rfr += contrib
                    value2.accumExpenditures -= contrib

        # We go with the second round of calculations
        for key, value in self.collection.items():
            localTmpMultInter[key] = 0
            for key2, value2 in value.firms.items():
                localTmpMultInter[key] += value2.currentIterRawDemand[key]
                ↪  * \
```

```
                                          value2.localperc * \
                                          value2.accumExpenditures

        # We go with the third round of calculations
        # We divide the previous reminder by three (three companies= and
        # apply the rule
        for key, value in self.collection.items():
            localTmpMultFin[key] = 0
            for key2, value2 in value.firms.items():
                localTmpMultFin[key] += value2.currentIterRawDemand[key] * \
                                          value2.localperc * \
                                          localTmpMultInter[key]/3


        return localTmpMultIni, localTmpMultInter, localTmpMultFin

    def run(self):
        while True:
            self.iteramarkers.append(time.time())
            if self.print:
                print(f'{Back.BLUE}{Fore.BLACK}Economy Agent, Iteration number {self.env.now}{Style.RESET_ALL}')
            duration = 0.9
            yield self.env.timeout(duration)

            a = self.calculateLocalMultiplier()
            tmpGlobalMult = (self.globalsalary + sum(a[0].values()) + \
                             sum(a[1].values()) + sum(a[2].values())) / (self.globalsalary + sum(a[0].values()) )
            tmpGlobalMultAlt = (sum(a[0].values()) + \
                             sum(a[1].values()) + sum(a[2].values())) / (sum(a[0].values()) )
            self.globalmultiplier.append(tmpGlobalMult)
            self.globalmultiplieralt.append(tmpGlobalMultAlt)

            tmpSectMult = {}
            tmpSectMultAlt = {}

            for key, value in self.collection.items():
                tmpSectMult[key] =  (self.localsalary[key] + a[0][key] +
```

```
                             a[1][key] + a[2][key]) / (
                        self.localsalary[key] + a[0][key] )
                tmpSectMultAlt[key] = (a[0][key] + a[1][key] + a[2][key]) /
                ↪  (
                    a[0][key])

            self.sectmultiplier.append(tmpSectMult)
            self.sectmultiplieralt.append(tmpSectMultAlt)

            duration = 0.1
            yield self.env.timeout(duration)
```

Listing E.3: Code for the Economy and Sector Agents

E.4 Market Agent

This section of the appendix contains the relevant code for the implementation of the market agent as discussed in section 4.4.

```
import numpy as np
import simpy
import random as rd
from colorama import Fore, Style
from TimeMeasure import measure
import Economy as ec
import Company as co
import Household as hh
from pulp import *
import itertools as it
import copy
import warnings
warnings.filterwarnings("ignore")

class market(object):
    counter = 1
```

```python

    def __init__(self, env, economy, households, firms, print=False):
        self.id = 'M_' + str(market.counter)
        self.print = print
        market.counter += 1
        self.env = env
        self.action = env.process(self.run())
        self.economy = economy
        self.households = households
        self.firms = firms
        self.niter = 0
        self.noimprov = 0
        self.nahouseholds = households # Will keep a list of not assigned households
        self.nafirms = firms # Will keep a list of not assigned households
        self.average_prices = copy.deepcopy(self.economy.avgprices) # We keep a list of the average price based on producers
        self.average_willingess_to_pay = copy.deepcopy(self.average_prices)

    def __str__(self):
        return 'Market ID: ' + self.id

    def run(self):
        while True:
            # Distribute the RFR tax, first iteration will be zero so nothing
            # to distribute
            if not self.economy.usepeople:
                to_distribute = self.economy.rfr*10
                for key, value in self.firms.collection.items():
                    value.moneyAdd(to_distribute/len(list(
                        self.firms.collection.keys())))
            else:
                to_distribute = self.economy.rfr * 20
                for key, value in self.households.collection.items():
                    value.salaryAdd(to_distribute / len(list(
                        self.households.collection.keys())))
            # Once assigned it is set to zero
            self.economy.rfr = 0
            # On every iteration we produce the sales and market interactions using the model
```

```python
            self.process_optimisation()
            if self.print:
                print(f'{Fore.LIGHTGREEN_EX}{self.__str__()} {Style.RESET_ALL} is alive and well at time {self.env.now}')
            duration = 1
            yield self.env.timeout(duration)
            self.niter += 1

    def Optimise(self, iteration):
        prob = LpProblem("ABM_Market_Assignation_Model", LpMaximize)
        Households_Nodes = list(self.nahouseholds.__return_coll__().keys()) # Household nodes initialised to remaining households
        Firms_Nodes = [(x[0], x[1].sector) for x in self.nafirms.__return_coll__().items()] # firms nodes initialised to remaining firms
        Firms_Per_Sector = {} # Dictionary with key = sector name and value = list of firms belonging to sector
        Sector_Nodes = [] # List of sector names
        # This loop fill the two lists: Sector_Nodes and Firms_Per_Sector
        for (j, k) in Firms_Nodes:
            if k not in Firms_Per_Sector.keys():
                Firms_Per_Sector[k] = []
                Sector_Nodes.append(k)
            Firms_Per_Sector[k].append(j)
        # These are all the possible combinations of buyers and firms
        Indexes = list(it.product(Households_Nodes, Firms_Nodes))

        # X{i,j,k} = a if Household i buys a units from firm j of sector k
        Buy_Vars = LpVariable.dicts("X", ((i, j, k) for i, (j, k) in Indexes), lowBound = 0, cat='Continuous') # print(Buy_Vars)

        # Need to calculate the coefficients $\gamma_i,j,k$
        Gamma = {}

        for i, (j,k) in Indexes:
            Gamma[i, j, k] = round(((self.nahouseholds.collection[i].willingnessToPay[k] / self.nafirms.collection[j].productPrice)
```

```
                                ** (self.nahouseholds.collection[i].cdCoefficients[k]) - 1), 9)

        # We add the objective function to the problem first
        prob += lpSum([Gamma[i, j, k]*Buy_Vars[i, j, k] for i, (j, k) in Indexes])

        # The constraints are added to 'prob'
        # Households_Nodes and Sector_Nodes are based on nahouseholds and nafirms
        for i in Households_Nodes:
            for k in Sector_Nodes:
                prob += lpSum([Buy_Vars[i, j, k] for j in Firms_Per_Sector[k]]) <= \
                        self.nahouseholds.collection[i].sectorDemand[k], \
                        "Cannot exceed demand " + str(i) + ',' + str(k)

        # Household_Nodes is based on nahouseholds
        for i in Households_Nodes:
            prob += lpSum([self.nafirms.collection[j].productPrice*Buy_Vars[i, j, k] for (j, k) in Firms_Nodes]) <= \
                    self.nahouseholds.collection[i].budget, "Cannot exceed household budget " + str(i)

        # Firm_Nodes is based on nafirms
        for (j, k) in Firms_Nodes:
            if self.nafirms.collection[j].supply > 0:
                prob += lpSum([Buy_Vars[i, j, k] for i in Households_Nodes]) <= self.nafirms.collection[j].supply, "Cannot exceed firm supply " + str(j)
            else:
                prob += lpSum([Buy_Vars[i, j, k] for i in Households_Nodes]) <= 0, "Cannot exceed firm supply " + str(j)

        # prob.writeLP("Tmp_Prob.lp", writeSOS=1, mip=1)
        prob.solve(PULP_CBC_CMD(msg=0))
        numVarOnes = False
```

```python

        addition = 0
        for i, (j,k) in Indexes:
            if (Buy_Vars[i, j, k].value() >= 0.01):
                addition += Buy_Vars[i, j, k].value()
        if addition > 0.1:
            self.noimprov = 0
        else:
            self.noimprov += 1

        for k in Sector_Nodes:
            for j in Firms_Per_Sector[k]:
                demandFirm = 0
                for i in Households_Nodes:
                    # Here we determine how much is sold by the firm as per
                    # the solution of the LP model
                    demandFirm += Buy_Vars[i, j, k].value()
                    # billPayment() of object Household reduces available
                    ↪  budget and aggregatedIncome
                    self.nahouseholds.collection[i].billPayment(Buy_Vars[i,
                    ↪  j, k].value() *
                    ↪  self.nafirms.collection[j].productPrice)
                    # paymentAdd() of object Firm increases available
                    ↪  budget, reduces supply and reduces aggregatedOffer
                    self.nafirms.collection[j].paymentAdd(Buy_Vars[i, j,
                    ↪  k].value() *
                    ↪  self.nafirms.collection[j].productPrice)
                    # We now take demand for sector k for household i
                    if (self.nahouseholds.collection[i].sectorDemand[k] -
                    ↪  Buy_Vars[i, j, k].value()) >= 0:
                        self.nahouseholds.collection[i].sectorDemand[k] -=
                        ↪  Buy_Vars[i, j, k].value()
                    else:
                        self.nahouseholds.collection[i].sectorDemand[k] = 0

        if (self.nafirms.aggregatedOffer >= 0.1) and
        ↪  (self.nahouseholds.aggregatedIncome >= 0.1):
            if self.noimprov <= 1:
                numVarOnes = True
            else:
                numVarOnes = False
```

```
            for j in self.nafirms.collection:
                if self.nafirms.collection[j].supply >= 0.1:
                    self.nafirms.collection[j].productPrice *= self.nafirms.deltafactor
            for i in self.nahouseholds.collection:
                for k in Sector_Nodes:
                    if self.nahouseholds.collection[i].sectorDemand[k] >= 0.1:
                        self.nahouseholds.collection[i].willingnessToPay[k] *= self.nahouseholds.epsilonfactor

        return numVarOnes

    def process_optimisation(self):
        # The Greedy Loop!
        iterating = True
        counter = 0
        # We reset de list of firms and households to start the optimisation
        self.nahouseholds = self.households
        self.nafirms = self.firms

        while iterating:
            iterating = bool(self.Optimise(counter))
            counter += 1
        # Postprocessing
        for key, firm in self.nafirms.collection.items():
            firm.supplyhistory[self.niter].append(firm.supply)
        # Actualisation of average price
        for key in self.average_prices.keys():
            self.average_prices[key] = 0
        for key, firm in self.nafirms.collection.items():
            self.average_prices[firm.sector] += \
                self.average_willingess_to_pay[firm.sector]/3
            delta_prod = (firm.supplyhistory[self.niter][0] - \
                          firm.supplyhistory[self.niter][1]) \
                         / firm.supplyhistory[self.niter][0]
            self.average_prices[firm.sector] -= delta_prod/3
        # Actualisation of willingness to pay
        for key in self.average_willingess_to_pay.keys():
            self.average_willingess_to_pay[key] = 0
```

```
        for key, house in self.nahouseholds.collection.items():
            for key2, value in self.economy.avgprices.items():
                self.average_willingess_to_pay[key2] += \
                    house.willingnessToPay[key2]/100
```

Listing E.4: Code for the Market Agent

Appendix F

Validation Details for Inventory Update Methods

In this appendix we provide the detailed output for a validation test performed for checking the consistency of the inventory update methods. It is provided in this appendix as it has been considered that the level of detail of this test could detract from the main history being told within the body of the thesis.

```
Market ID: M_1 is iterating the optimiser: 0
H_1 Demand Sector S_1 7.575 Willlingness to Pay 12.8644342
H_1 Demand Sector S_2 15.7203 Willlingness to Pay 15.62680052
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 12.87440703
H_2 Demand Sector S_2 9.3045 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 10.8231 Willlingness to Pay 12.83589121
H_3 Demand Sector S_2 13.0813 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 10.8178 Willlingness to Pay 12.90091642
H_4 Demand Sector S_2 13.0855 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 12.69256085
```

```
H_5 Demand Sector S_2 12.8232 Willlingness to Pay 15.77359466
H_6 Demand Sector S_1 8.317 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 15.1175 Willlingness to Pay 16.00706144
F_1 Supply: 25.4970097
F_2 Supply: 10.78352972
F_3 Supply: 38.91225404
F_4 Supply: 55.56022204
Aggregated Households Income: 2100 , Aggregated Offer Firms: 130.7530155
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 13
S_1 F_2 Firm Optimal Supply: 8.317 , Transaction Price: 13
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 16
S_2 F_4 Firm Optimal Supply: 50.588737 , Transaction Price: 16
Aggregated Households Income: 1182.459208 , Aggregated Offer Firms:
↪ 71.8472785
---------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 1
H_1 Demand Sector S_1 7.575 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 15.7203 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 14.161847733000002
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 10.8231 Willlingness to Pay 14.119480331
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 10.8178 Willlingness to Pay 14.191008062000002
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 13.961816935
H_5 Demand Sector S_2 12.8232 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 6.299999999903605e-05 Willlingness to Pay
↪ 16.00706144
F_1 Supply: 25.4970097
F_2 Supply: 2.4665297200000005
F_3 Supply: 38.91225404
F_4 Supply: 4.971485039999996
Aggregated Households Income: 1182.459208 , Aggregated Offer Firms:
↪ 71.8472785
S_1 F_1 Firm Optimal Supply: 25.4970097 , Transaction Price:
↪ 11.700000000000001
S_1 F_2 Firm Optimal Supply: 2.4665297 , Transaction Price:
↪ 11.700000000000001
S_2 F_3 Firm Optimal Supply: 28.5435 , Transaction Price: 14.4
```

```
S_2 F_4 Firm Optimal Supply: 5.5555555e-07 , Transaction Price: 14.4
Aggregated Households Income: 444.2593890199999 , Aggregated Offer Firms:
↪  15.340238544444448
---------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 2
H_1 Demand Sector S_1 0.0 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 0.0 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 15.578032506300003
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 1.2523605999999998 Willlingness to Pay
↪  15.531428364100002
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 0.0 Willlingness to Pay 14.191008062000002
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 15.3579986285
H_5 Demand Sector S_2 0.0 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 6.244444444903605e-05 Willlingness to Pay
↪  16.00706144
F_1 Supply: 0.0
F_2 Supply: 2.000000032253979e-08
F_3 Supply: 10.368754039999999
F_4 Supply: 4.971484484444446
Aggregated Households Income: 444.2593890199999 , Aggregated Offer Firms:
↪  15.340238544444448
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_1 F_2 Firm Optimal Supply: 2e-08 , Transaction Price: 11.700000000000001
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 12.96
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 12.96
Aggregated Households Income: 444.2593887859999 , Aggregated Offer Firms:
↪  15.340238524444448
---------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 3
H_1 Demand Sector S_1 0.0 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 0.0 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 17.135835756930003
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 1.2523605799999997 Willlingness to Pay
↪  17.084571200510002
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
```

```
H_4 Demand Sector S_1 0.0 Willlingness to Pay 14.191008062000002
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 16.89379849135
H_5 Demand Sector S_2 0.0 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 6.244444444903605e-05 Willlingness to Pay
↪  16.00706144
F_1 Supply: 0.0
F_2 Supply: 3.225397900131684e-16
F_3 Supply: 10.368754039999999
F_4 Supply: 4.971484484444446
Aggregated Households Income: 444.2593887859999 , Aggregated Offer Firms:
↪  15.340238524444448
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_1 F_2 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 11.664000000000001
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 11.664000000000001
Aggregated Households Income: 444.2593887859999 , Aggregated Offer Firms:
↪  15.340238524444448
---------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 4
H_1 Demand Sector S_1 0.0 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 0.0 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 15.4714 Willlingness to Pay 18.849419332623004
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 1.2523605799999997 Willlingness to Pay
↪  18.793028320561003
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 0.0 Willlingness to Pay 14.191008062000002
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 11.1407 Willlingness to Pay 18.583178340485002
H_5 Demand Sector S_2 0.0 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 6.244444444903605e-05 Willlingness to Pay
↪  16.00706144
F_1 Supply: 0.0
F_2 Supply: 3.225397900131684e-16
F_3 Supply: 10.368754039999999
F_4 Supply: 4.971484484444446
Aggregated Households Income: 444.2593887859999 , Aggregated Offer Firms:
↪  15.340238524444448
```

```
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_1 F_2 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 10.497600000000002
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 10.497600000000002
Aggregated Households Income: 444.2593887859999 , Aggregated Offer Firms:
↪  15.340238524444448
----------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 0
H_1 Demand Sector S_1 4.0036 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 8.3096 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 15.5225 Willlingness to Pay 20.734361265885305
H_2 Demand Sector S_2 9.3334 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 5.5261 Willlingness to Pay 20.672331152617105
H_3 Demand Sector S_2 6.6801 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 5.0718 Willlingness to Pay 14.191008062000002
H_4 Demand Sector S_2 6.1332 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 10.0377 Willlingness to Pay 20.441496174533505
H_5 Demand Sector S_2 11.5535 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 3.5643 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 6.479 Willlingness to Pay 16.00706144
F_1 Supply: 29.30008012
F_2 Supply: 11.52209043
F_3 Supply: 57.27798991
F_4 Supply: 83.01747842444445
Aggregated Households Income: 1344.2593887859998 , Aggregated Offer Firms:
↪  181.11763888444443
S_1 F_1 Firm Optimal Supply: 29.3000802 , Transaction Price:
↪  11.700000000000001
S_1 F_2 Firm Optimal Supply: 11.522090399999998 , Transaction Price:
↪  11.700000000000001
S_2 F_3 Firm Optimal Supply: 48.4888 , Transaction Price:
↪  9.447840000000003
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 9.447840000000003
Aggregated Households Income: 408.5255685739995 , Aggregated Offer Firms:
↪  93.37857788444445
----------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 1
H_1 Demand Sector S_1 0.0 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 0.0 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 0.0 Willlingness to Pay 20.734361265885305
```

```
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 0.0 Willlingness to Pay 20.672331152617105
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 2.9038294 Willlingness to Pay 15.610108868200003
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 0.0 Willlingness to Pay 20.441496174533505
H_5 Demand Sector S_2 0.0 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 0.0 Willlingness to Pay 16.00706144
F_1 Supply: 0
F_2 Supply: 3.0000001371988105e-08
F_3 Supply: 8.789189909999997
F_4 Supply: 83.01747842444445
Aggregated Households Income: 408.5255685739995 , Aggregated Offer Firms:
↪  93.37857788444445
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_1 F_2 Firm Optimal Supply: 3.0000001e-08 , Transaction Price:
↪  11.700000000000001
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 8.503056000000003
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 8.503056000000003
Aggregated Households Income: 408.5255682229995 , Aggregated Offer Firms:
↪  93.37857785444446
---------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 2
H_1 Demand Sector S_1 0.0 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 0.0 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 0.0 Willlingness to Pay 20.734361265885305
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 0.0 Willlingness to Pay 20.672331152617105
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 2.9038293699999986 Willlingness to Pay
↪  17.171119755020005
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 0.0 Willlingness to Pay 20.441496174533505
H_5 Demand Sector S_2 0.0 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 0.0 Willlingness to Pay 16.00706144
F_1 Supply: 0.0
F_2 Supply: 3.7198810440110656e-16
F_3 Supply: 8.789189909999997
```

```
F_4 Supply: 83.01747842444445
Aggregated Households Income: 408.5255682229995 , Aggregated Offer Firms:
↪  93.37857785444446
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_1 F_2 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 7.652750400000002
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 7.652750400000002
Aggregated Households Income: 408.5255682229995 , Aggregated Offer Firms:
↪  93.37857785444446
---------------------------------------------------------------
Market ID: M_1 is iterating the optimiser: 3
H_1 Demand Sector S_1 0.0 Willlingness to Pay 14.150877620000001
H_1 Demand Sector S_2 0.0 Willlingness to Pay 17.189480572
H_2 Demand Sector S_1 0.0 Willlingness to Pay 20.734361265885305
H_2 Demand Sector S_2 0.0 Willlingness to Pay 16.0954581
H_3 Demand Sector S_1 0.0 Willlingness to Pay 20.672331152617105
H_3 Demand Sector S_2 0.0 Willlingness to Pay 16.08097748
H_4 Demand Sector S_1 2.9038293699999986 Willlingness to Pay
↪  18.88823173052201
H_4 Demand Sector S_2 0.0 Willlingness to Pay 16.04898052
H_5 Demand Sector S_1 0.0 Willlingness to Pay 20.441496174533505
H_5 Demand Sector S_2 0.0 Willlingness to Pay 17.350954126
H_6 Demand Sector S_1 0.0 Willlingness to Pay 13.22017312
H_6 Demand Sector S_2 0.0 Willlingness to Pay 16.00706144
F_1 Supply: 0.0
F_2 Supply: 3.7198810440110656e-16
F_3 Supply: 8.789189909999997
F_4 Supply: 83.01747842444445
Aggregated Households Income: 408.5255682229995 , Aggregated Offer Firms:
↪  93.37857785444446
S_1 F_1 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_1 F_2 Firm Optimal Supply: 0.0 , Transaction Price: 11.700000000000001
S_2 F_3 Firm Optimal Supply: 0.0 , Transaction Price: 6.8874753600000025
S_2 F_4 Firm Optimal Supply: 0.0 , Transaction Price: 6.8874753600000025
Aggregated Households Income: 408.5255682229995 , Aggregated Offer Firms:
↪  93.37857785444446
---------------------------------------------------------------
```

Appendix G

Some Statistical Tables for Kalgoorlie-Boulder

In this appendix we reproduce the data available for Kalgoorlie-Boulder available on the Australian Bureau of Statistics website. In what follows KB means Kalgoorlie-Boulder, WA represents Western Australia, AU respresents Australia and ABTSI represents Aboriginal and/or Torres Strait Islander People.

Table G.1: Sex Distribution

People	KB	%	WA	%	AU	%
Male	15,538	52.0	1,238,419	50.0	11,546,638	49.3
Female	14,340	48.0	1,235,994	50.0	11,855,248	50.7
ABTSI	2,180	7.3	75,978	3.1	649,171	2.8

Table G.2: Age Distribution

Age	KB	%	WA	%	AU	%
Median age	33	–	36	–	38	–
0-4 years	2,568	8.6	161,727	6.5	1,464,779	6.3
5-9 years	2,388	8.0	164,153	6.6	1,502,646	6.4
10-14 years	1,937	6.5	150,806	6.1	1,397,183	6.0
15-19 years	1,796	6.0	149,997	6.1	1,421,595	6.1
20-24 years	2,037	6.8	160,332	6.5	1,566,793	6.7
25-29 years	2,568	8.6	184,908	7.5	1,664,602	7.1
30-34 years	2,703	9.0	194,267	7.9	1,703,847	7.3
35-39 years	2,252	7.5	173,041	7.0	1,561,679	6.7
40-44 years	2,247	7.5	171,996	7.0	1,583,257	6.8
45-49 years	2,149	7.2	172,520	7.0	1,581,455	6.8
50-54 years	2,020	6.8	162,438	6.6	1,523,551	6.5
55-59 years	1,862	6.2	149,899	6.1	1,454,332	6.2
60-64 years	1,271	4.3	132,145	5.3	1,299,397	5.6
65-69 years	886	3.0	116,755	4.7	1,188,999	5.1
70-74 years	494	1.7	82,911	3.4	887,716	3.8
75-79 years	337	1.1	61,509	2.5	652,657	2.8
80-84 years	217	0.7	42,590	1.7	460,549	2.0
85 years and over	153	0.5	42,420	1.7	486,842	2.1

G.1 Iterative Proportional Fitting for Family Composition and Dwelling Tenure

```python
from ipfn import ipfn
import numpy as np
import pandas as pd

fam_comp = ['couple_nc', 'couple_wc', 'one_pf', 'other_f',
    'couple_nc', 'couple_wc', 'one_pf', 'other_f',
    'couple_nc', 'couple_wc', 'one_pf', 'other_f',
    'couple_nc', 'couple_wc', 'one_pf', 'other_f',
    'couple_nc', 'couple_wc', 'one_pf', 'other_f']
dwell_ten = ['owned', 'owned', 'owned', 'owned',
             'mortgage', 'mortgage', 'mortgage', 'mortgage',
             'rented', 'rented', 'rented', 'rented',
             'other', 'other', 'other', 'other',
             'not_stated', 'not_stated', 'not_stated','not_stated']

```

Table G.3: Age Distribution by Sex

Age	Males	Females	Total	% Male	% Female
0-4 years	1,317	1,238	2,555	51.55	48.45
5-9 years	1,203	1,188	2,391	50.31	49.69
10-14 years	973	977	1,950	49.9	50.1
15-19 years	903	893	1,796	50.28	49.72
20-24 years	1,093	940	2,033	53.76	46.24
25-29 years	1,307	1,261	2,568	50.9	49.1
30-34 years	1,356	1,347	2,703	50.17	49.83
35-39 years	1,157	1,108	2,265	51.08	48.92
40-44 years	1,180	1,070	2,250	52.44	47.56
45-49 years	1,146	1,002	2,148	53.35	46.65
50-54 years	1,084	933	2,017	53.74	46.26
55-59 years	1,014	844	1,858	54.57	45.43
60-64 years	705	572	1,277	55.21	44.79
65-69 years	508	370	878	57.86	42.14
70-74 years	279	211	490	56.94	43.06
75-79 years	159	191	350	45.43	54.57
80-84 years	97	123	217	44.7	56.68
85 years and over	54	99	153	35.29	64.71

Table G.4: Registered Marital Status Distribution

Marital Status	KB	%	WA	%	AU	%
Married	10,063	43.8	975,062	48.8	9,148,218	48.1
Separated	840	3.7	63,205	3.2	608,059	3.2
Divorced	1,719	7.5	167,361	8.4	1,626,890	8.5
Widowed	719	3.1	88,619	4.4	985,204	5.2
Never married	9,657	42.0	703,482	35.2	6,668,910	35.0

Table G.5: Social Marital Status Distribution

Social Marital Status	KB	%	WA	%	AU	%
Registered marriage	8,362	43.6	835,938	48.4	8,001,141	47.7
De facto marriage	3,341	17.4	201,709	11.7	1,751,731	10.4
Not married	7,461	38.9	688,868	39.9	7,024,973	41.9

```
m = np.random.randint(1,10,20)
```

```

df = pd.DataFrame()
df['comp'] = fam_comp
df['dwell'] = dwell_ten
df['total'] = m

print(df)

xip = df.groupby('dwell')['total'].sum()
xpj = df.groupby('comp')['total'].sum()

xip.loc['owned'] = 17
xip.loc['mortgage'] = 42
xip.loc['rented'] = 38
xip.loc['other'] = 1
xip.loc['not_stated'] = 2

xpj.loc['couple_nc'] = 35
xpj.loc['couple_wc'] = 50
xpj.loc['one_pf'] = 14
xpj.loc['other_f'] = 1

aggregates = [xip, xpj]
dimensions = [['dwell'], ['comp']]

IPF = ipfn.ipfn(df, aggregates, dimensions)
df = IPF.iteration()

print('--------------------------------')
print(df)
f=open("comp_dwell.txt","wb")
pickle.dump(df,f)
f.close()
```

Listing G.1: IPF fitting with `Python`

Table G.6: Education Distribution

Education	KB	%	WA	%	AU	%
Preschool	420	4.3	30,247	3.9	347,621	4.8
Primary - Government	2,187	22.4	144,988	18.9	1,314,787	18.2
Primary - Catholic	548	5.6	36,865	4.8	380,604	5.3
Primary - other non Government	159	1.6	28,046	3.7	231,490	3.2
Secondary - Government	1,053	10.8	88,176	11.5	827,505	11.5
Secondary - Catholic	631	6.5	34,065	4.4	338,384	4.7
Secondary - other non Government	127	1.3	34,773	4.5	280,618	3.9
Technical or further education institution	648	6.6	46,835	6.1	424,869	5.9
University or tertiary institution	607	6.2	106,811	13.9	1,160,626	16.1
Other	163	1.7	18,547	2.4	198,383	2.8
Not stated	3,233	33.1	197,644	25.8	1,707,023	23.7

Table G.7: Level of Highest Educational Attainment

Level	KB	%	WA	%	AU	%
Bachelor Degree level and above	2,775	12.1	410,272	20.5	4,181,406	22.0
Advanced Diploma and Diploma level	1,455	6.3	177,631	8.9	1,687,893	8.9
Certificate level IV	793	3.4	62,656	3.1	551,767	2.9
Certificate level III	4,159	18.1	279,448	14.0	2,442,203	12.8
Year 12	3,396	14.8	318,674	16.0	2,994,097	15.7
Year 11	1,620	7.0	107,858	5.4	941,531	4.9
Year 10	3,481	15.1	235,001	11.8	2,054,331	10.8
Certificate level II	18	0.1	940	0.0	13,454	0.1
Certificate level I	3	0.0	209	0.0	2,176	0.0
Year 9 or below	1,450	6.3	117,996	5.9	1,529,897	8.0
No educational attainment	61	0.3	10,572	0.5	145,844	0.8
Not stated	3,246	14.1	220,701	11.0	1,974,794	10.4

Table G.8: Employment

Category	KB	%	WA	%	AU	%
Worked full-time	10,519	67.0	715,287	57.0	6,623,065	57.7
Worked part-time	3,354	21.4	376,590	30.0	3,491,503	30.4
Away from work	901	5.7	65,859	5.2	569,276	5.0
Unemployed	919	5.9	97,966	7.8	787,452	6.9

Table G.9: Hours Worked

Category	KB	%	WA	%	AU	%
1-15 hours per week	1,238	8.4	138,263	11.9	1,218,823	11.4
16-24 hours per week	896	6.1	114,814	9.9	1,079,236	10.1
25-34 hours per week	1,219	8.3	123,517	10.7	1,193,445	11.2
35-39 hours per week	1,938	13.1	198,785	17.2	2,031,263	19.0
40 hours or more per week	8,580	58.1	516,501	44.6	4,591,801	43.0

Table G.10: Occupation

Category	KB	%	WA	%	AU	%
Technicians and Trades Workers	3,138	21.2	187,396	16.2	1,447,414	13.5
Machinery Operators and Drivers	2,551	17.3	86,392	7.5	670,106	6.3
Professionals	2,115	14.3	237,230	20.5	2,370,966	22.2
Clerical and Administrative Workers	1,600	10.8	150,408	13.0	1,449,681	13.6
Community and Personal Service Workers	1,368	9.3	122,889	10.6	1,157,003	10.8
Managers	1,295	8.8	139,350	12.0	1,390,047	13.0
Labourers	1,281	8.7	112,599	9.7	1,011,520	9.5
Sales Workers	1,154	7.8	102,337	8.8	1,000,955	9.4

Table G.11: Industry of Employment, Top Responses

Category	KB	%	WA	%	AU	%
Gold Ore Mining	2,810	19.1	12,768	1.1	20,141	0.2
Primary Education	478	3.2	29,683	2.6	231,198	2.2
Road Freight Transport	406	2.8	15,673	1.4	129,528	1.2
Other Mining Support Services	399	2.7	5,122	0.4	9,359	0.1
Hospitals (except Psychiatric Hospitals)	374	2.5	41,706	3.6	411,808	3.9

Table G.12: Median Weekly Income

Category	KB	%	WA	%	AU	%
Personal	970	–	724	–	662	–
Family	2,377	–	1,910	–	1,734	–
Household	2,082	–	1,595	–	1,438	–

Table G.13: Unpaid Work

Category	KB	%	WA	%	AU	%
Did unpaid domestic work (last week)	15,144	65.9	1,387,280	69.4	13,143,914	69.0
Cared for child/children (last two weeks)	6,885	30.0	568,406	28.5	5,259,400	27.6
Provided unpaid assistance to a person with a disability (last two weeks)	1,602	7.0	196,328	9.8	2,145,203	11.3
Did voluntary work through an organisation or group (last 12 months)	3,976	17.3	379,578	19.0	3,620,726	19.0

Table G.14: Unpaid Domestic Work, Number of Hours

Category	KB	%	WA	%	AU	%
Less than 5 hours per week	4,950	21.5	447,726	22.4	4,298,593	22.6
5 to 14 hours per week	5,831	25.4	521,733	26.1	4,944,578	26.0
15 to 29 hours per week	2,300	10.0	228,248	11.4	2,189,776	11.5
30 hours or more per week	2,066	9.0	189,571	9.5	1,710,970	9.0

Table G.15: Family Composition

Category	KB	%	WA	%	AU	%
Couple family without children	2,515	34.8	247,841	38.5	2,291,987	37.8
Couple family with children	3,605	49.9	292,133	45.3	2,716,224	44.7
One parent family	995	13.8	93,344	14.5	959,543	15.8
Other family	109	1.5	10,869	1.7	102,559	1.7

Table G.16: Single Parents

Category	KB	%	WA	%	AU	%
Male	–	19.5	–	18.3	–	18.2
Female	–	80.5	–	81.7	–	81.8

Table G.17: Employmenmt Status of Couple Families

Category	KB	%	WA	%	AU	%
Both employed, worked full-time	1,722	28.1	106,951	19.8	1,084,006	21.6
Both employed, worked part-time	106	1.7	22,177	4.1	203,596	4.1
One employed full-time, one part-time	1,449	23.7	119,913	22.2	1,086,460	21.7
One employed full-time, other not working	1,207	19.7	88,558	16.4	749,886	15.0
One employed part-time, other not working	210	3.4	31,917	5.9	302,037	6.0
Both not working	421	6.9	93,686	17.4	1,006,697	20.1
Other (includes away from work)	416	6.8	29,973	5.6	264,145	5.3
Labour force status not stated (by one or both parents in a couple family)	591	9.7	46,793	8.7	311,381	6.2

Table G.18: Dwelling Count

Category	KB	%	WA	%	AU	%
Occupied private dwellings	9,922	86.0	866,767	86.7	8,286,073	88.8
Unoccupied private dwellings	1,621	14.0	132,874	13.3	1,039,874	11.2

Table G.19: Dwelling Structure

Category	KB	%	WA	%	AU	%
Separate house	7,960	80.2	685,824	79.1	6,041,788	72.9
Semi-detached, row or terrace house, townhouse etc	1,553	15.7	122,562	14.1	1,055,016	12.7
Flat or apartment	200	2.0	49,086	5.7	1,087,434	13.1
Other dwelling	161	1.6	6,314	0.7	64,425	0.8

Table G.20: Dwelling Tenure

Category	KB	%	WA	%	AU	%
Owned outright	1,681	16.9	247,050	28.5	2,565,695	31.0
Owned with a mortgage	4,151	41.8	344,014	39.7	2,855,222	34.5
Rented	3,816	38.5	245,705	28.3	2,561,302	30.9
Other tenure type	26	0.3	9,181	1.1	78,994	1.0
Tenure type not stated	249	2.5	20,823	2.4	224,869	2.7

Table G.21: Dwelling Household Composition

Category	KB	%	WA	%	AU	%
Family households	7,088	71.4	629,882	72.7	5,907,625	71.3
Single (or lone) person households	2,428	24.5	204,202	23.6	2,023,542	24.4
Group households	406	4.1	32,692	3.8	354,917	4.3

Table G.22: Rent Weekly Payments

Category	KB	%	WA	%	AU	%
Median rent	290	–	347	–	335	–
Households where rent payments are less than 30% of household income	–	93.5	–	90.3	–	88.5
Households with rent payments greater than or equal to 30% of household income	–	6.5	–	9.7	–	11.5

Table G.23: Mortgage Monthly Repayments

Category	KB	%	WA	%	AU	%
Median mortgage repayments	1,800	–	1,993	–	1,755	–
Households where mortgage repayments are less than 30% of household income	–	94.9	–	91.4	–	92.8
Households with mortgage repayments greater than or equal to 30% of household income	–	5.1	–	8.6	–	7.2

Table G.24: Number of Motor Vehicles

Category	KB	%	WA	%	AU	%
None	457	4.6	42,620	4.9	623,829	7.5
1 motor vehicle	3,024	30.5	274,198	31.6	2,881,485	34.8
2 motor vehicles	3,794	38.3	337,355	38.9	2,999,184	36.2
3 or more vehicles	2,283	23.0	186,678	21.5	1,496,382	18.1
Number of motor vehicles not stated	358	3.6	25,934	3.0	285,197	3.4

Table G.25: Internet Connection

Category	KB	%	WA	%	AU	%
Internet not accessed from dwelling	1,483	14.9	108,489	12.5	1,172,415	14.1
Internet accessed from dwelling	8,190	82.5	737,659	85.1	6,892,165	83.2
Not stated	253	2.5	20,625	2.4	221,494	2.7

Table G.26: Weekly Personal Income per Age Group - Males

	15-19 years	20-24 years	25-34 years	35-44 years	45-54 years	55-64 years	65-74 years	75-84 years	85 and over	Total
Negative/Nil income	361	87	67	36	50	63	46	6	0	724
$1-$149	155	25	19	17	3	4	14	3	0	250
$150-$299	63	56	58	39	42	53	71	24	3	411
$300-$399	24	37	40	30	58	58	103	57	10	426
$400-$499	34	42	33	34	41	52	107	66	14	427
$500-$649	45	52	37	33	29	43	56	22	3	323
$650-$799	43	78	68	53	54	40	52	13	3	415
$800-$999	31	102	139	80	101	96	31	3	0	584
$1,000-$1,249	20	114	175	145	164	148	45	4	0	816
$1,250-$1,499	6	92	229	157	151	140	30	0	0	807
$1,500-$1,749	5	100	316	243	215	157	34	0	0	1,064
$1,750-$1,999	0	67	322	225	220	148	24	3	0	1,005
$2,000-$2,999	3	96	654	664	555	327	42	5	3	2,343
$3,000 or more	0	13	164	252	249	133	25	5	0	846
Total Personal income not stated	106	130	353	318	296	251	107	38	13	1,614
Total	904	1,096	2,668	2,323	2,234	1,721	789	251	49	12,039

Table G.27: Weekly Personal Income per Age Group - Females

	15-19 years	20-24 years	25-34 years	35-44 years	45-54 years	55-64 years	65-74 years	75-84 years	85 and over	Total
Negative/Nil income	306	105	399	248	202	190	48	8	3	1,498
$1-$149	204	32	98	71	50	33	18	4	3	519
$150-$299	92	73	126	126	79	84	44	21	7	659
$300-$399	42	46	150	114	86	75	117	56	15	705
$400-$499	46	77	138	125	99	111	111	100	31	832
$500-$649	40	88	172	149	155	117	54	18	7	804
$650-$799	30	95	207	175	163	110	30	11	3	829
$800-$999	12	101	182	192	212	116	22	4	4	842
$1,000-$1,249	10	89	223	203	220	122	18	6	0	887
$1,250-$1,499	3	64	200	170	135	77	10	3	0	659
$1,500-$1,749	0	24	173	134	115	59	9	3	0	524
$1,750-$1,999	0	5	111	85	95	54	9	0	0	354
$2,000-$2,999	0	15	101	125	101	50	6	0	0	395
$3,000 or more	0	3	29	31	30	18	3	3	0	118
Total Personal income not stated	104	121	300	227	186	190	81	60	38	1,313
Total	890	943	2,608	2,175	1,929	1,410	586	302	100	10,945

Table G.28: Weekly Family Income per Composition

	Couple family with no children	Couple family with children	One parent family	Other family	Total
Negative/Nil income	26	12	23	0	57
$1-$149	11	4	12	0	31
$150-$299	13	8	29	0	50
$300-$399	3	0	52	0	55
$400-$499	38	16	51	3	107
$500-$649	22	15	82	3	122
$650-$799	139	39	88	9	269
$800-$999	69	39	95	11	220
$1,000-$1,249	112	85	87	11	293
$1,250-$1,499	135	135	73	11	347
$1,500-$1,749	112	168	68	6	352
$1,750-$1,999	149	218	43	6	418
$2,000-$2,499	361	589	92	15	1,054
$2,500-$2,999	308	487	37	15	844
$3,000-$3,499	239	375	20	9	645
$3,500-$3,999	187	310	13	11	520
$4,000 or more	274	571	14	5	866
Partial income stated	248	497	87	3	843
All incomes not stated	57	40	31	5	127
Total	2,515	3,605	995	109	7,224

Table G.29: Industry of Employment by Age

	15-19 years	20-24 years	25-34 years	35-44 years	45-54 years	55-64 years	65-74 years	75-84 years	85 and over	Total
Agriculture, Forestry and Fishing	0	3	4	0	8	7	0	0	0	23
Mining	44	248	1,190	980	796	438	54	0	0	3,754
Manufacturing	28	59	159	139	133	107	15	0	3	644
Electricity, Gas, Water and Waste Services	3	8	26	32	46	24	6	0	0	147
Construction	48	116	243	201	200	132	26	0	0	970
Wholesale Trade	8	35	83	81	90	41	10	0	0	342
Retail Trade	272	152	230	194	208	147	22	0	0	1,227
Accommodation and Food Services	178	114	189	140	145	83	24	0	0	872
Transport, Postal and Warehousing	11	52	123	174	237	196	42	3	0	836
Information Media and Telecommunications	18	16	22	14	15	6	3	0	0	99
Financial and Insurance Services	9	10	56	33	29	8	4	0	0	154
Rental, Hiring and Real Estate Services	13	34	65	62	45	40	6	0	0	270
Professional, Scientific and Technical Services	15	45	116	75	86	55	14	5	0	417
Administrative and Support Services	17	37	82	98	99	67	9	0	0	409
Public Administration and Safety	12	43	216	203	188	122	29	3	0	825
Education and Training	4	66	257	270	285	166	28	4	0	1,072
Health Care and Social Assistance	26	91	289	296	275	215	30	7	0	1,237
Arts and Recreation Services	16	22	30	32	33	14	3	0	0	148
Other Services	66	108	171	173	136	75	15	0	0	740
Inadequately described/Not stated	35	67	115	139	127	86	23	3	0	591
Total	836	1,325	3,673	3,339	3,159	2,035	367	30	7	14,768

Appendix H

Numerical Experiment 1 Details

In this appendix we provide the detailed output for the first numerical test performed for the agent based model with a view to create the baseline case study.

H.1 Firms

A list of 51 firms has been created and saved for future experiments. Each firm that belong to a sector share the same technology which is presented first via its Cobb-Douglas coefficients. After the coefficients, a list of the companies belonging to the sector (recorded on the second column) are presented with their initial budget (which for simplicity has been assumed equal to everyone and equal to $200,000 in this particular example). The output below provides a glimpse of the data.

```
{'Accommodation and Food Services': 0.0050842496765277,
 'Agriculture, Forestry and Fishing': 0.252794749003854,
 'Arts and Recreation Services': 0.00135271881194765,
 'CONST': 1.0,
 'Construction': 0.0781384731366425,
 'Education and Training': 0.00126489806995477,
 'Electricity, Gas, Water and Waste Services': 0.00843165193691891,
 'Financial and Insurance Services': 0.138273179347274,
 'Health Care and Social Assistance': 0.00024920035358389,
 'Information Media and Telecommunications': 0.0168177947497665,
 'Manufacturing': 0.195553328873432,
 'Mining': 0.0029661744407876,
 'Other Services': 0.107692170296671,
 'Public Administration and Safety': 0.00426659727995333,
 'Rental, Hiring and Real Estate Services': 0.0327751933743324,
 'Retail Trade': 0.0375609290522499,
 'Transport, Postal and Warehousing': 0.046005282554122,
 'Wholesale Trade': 0.0707734090419807}
------------------------------
F_1 Agriculture, Forestry and Fishing 200000
F_2 Agriculture, Forestry and Fishing 200000
F_3 Agriculture, Forestry and Fishing 200000
------------------------------
{'Accommodation and Food Services': 0.00401008332885296,
 'Agriculture, Forestry and Fishing': 0.00263605318686217,
 'Arts and Recreation Services': 0.00030998210403671,
 'CONST': 1.0,
 'Construction': 0.0767770649432509,
 'Education and Training': 0.00108303077028909,
 'Electricity, Gas, Water and Waste Services': 0.0108798245210561,
 'Financial and Insurance Services': 0.0145011846018228,
 'Health Care and Social Assistance': 1.17205098117125e-05,
 'Information Media and Telecommunications': 0.00714411703884291,
 'Manufacturing': 0.0684852367646544,
 'Mining': 0.733451608170021,
 'Other Services': 0.0240973980771372,
 'Public Administration and Safety': 0.00410621064934968,
 'Rental, Hiring and Real Estate Services': 0.0335790908845736,
 'Retail Trade': 0.0045384170039367,
 'Transport, Postal and Warehousing': 0.00783660721388079,
```

```
 'Wholesale Trade': 0.00655237023162136}
------------------------------
F_4 Mining 200000
F_5 Mining 200000
F_6 Mining 200000
------------------------------
{'Accommodation and Food Services': 0.00541088772369398,
 'Agriculture, Forestry and Fishing': 0.00565944360605616,
 'Arts and Recreation Services': 0.000132461030468383,
 'CONST': 1.0,
 'Construction': 0.00749230728992199,
 'Education and Training': 0.00134076966317622,
 'Electricity, Gas, Water and Waste Services': 0.023558571890328,
 'Financial and Insurance Services': 0.00699666416664401,
 'Health Care and Social Assistance': 0.0002734640475679,
 'Information Media and Telecommunications': 0.0179329241961311,
 'Manufacturing': 0.146123148080103,
 'Mining': 0.679249473142975,
 'Other Services': 0.00691531543177183,
 'Public Administration and Safety': 0.00443474200170633,
 'Rental, Hiring and Real Estate Services': 0.0324565778624055,
 'Retail Trade': 0.00776598041116893,
 'Transport, Postal and Warehousing': 0.0314237030584393,
 'Wholesale Trade': 0.0228335663974424}
------------------------------
F_7 Manufacturing 200000
F_8 Manufacturing 200000
F_9 Manufacturing 200000
------------------------------
{'Accommodation and Food Services': 0.00949810953011484,
 'Agriculture, Forestry and Fishing': 0.000116473473684534,
 'Arts and Recreation Services': 0.00155404248558866,
 'CONST': 1.0,
 'Construction': 0.191820082960894,
 'Education and Training': 0.0064708056121726,
 'Electricity, Gas, Water and Waste Services': 0.352236214965289,
 'Financial and Insurance Services': 0.0853811159781399,
 'Health Care and Social Assistance': 0.00018211683314419,
 'Information Media and Telecommunications': 0.0196053454992096,
 'Manufacturing': 0.0703855384760807,
```

```
 'Mining': 0.0975795346413856,
 'Other Services': 0.0268001844808829,
 'Public Administration and Safety': 0.00687400132497192,
 'Rental, Hiring and Real Estate Services': 0.0558534148309428,
 'Retail Trade': 0.0133966035076745,
 'Transport, Postal and Warehousing': 0.0281797726167772,
 'Wholesale Trade': 0.0340666427830476}
------------------------------
F_10 Electricity, Gas, Water and Waste Services 200000
F_11 Electricity, Gas, Water and Waste Services 200000
F_12 Electricity, Gas, Water and Waste Services 200000
------------------------------
{'Accommodation and Food Services': 0.0021789267168682,
 'Agriculture, Forestry and Fishing': 0.000832812755281722,
 'Arts and Recreation Services': 0.00028719146669492,
 'CONST': 1.0,
 'Construction': 0.553112310120101,
 'Education and Training': 0.00066129675810337,
 'Electricity, Gas, Water and Waste Services': 0.015460100910082,
 'Financial and Insurance Services': 0.0372956015963131,
 'Health Care and Social Assistance': 3.84554011107699e-05,
 'Information Media and Telecommunications': 0.0275592140123526,
 'Manufacturing': 0.159913521823421,
 'Mining': 0.00520570829609078,
 'Other Services': 0.0153655317810777,
 'Public Administration and Safety': 0.00694163135655167,
 'Rental, Hiring and Real Estate Services': 0.11717330416672,
 'Retail Trade': 0.00999864771046635,
 'Transport, Postal and Warehousing': 0.0236098067366893,
 'Wholesale Trade': 0.0243659383920748}
------------------------------
F_13 Construction 200000
F_14 Construction 200000
F_15 Construction 200000
------------------------------
{'Accommodation and Food Services': 0.02583761463345,
 'Agriculture, Forestry and Fishing': 0.000869927855045042,
 'Arts and Recreation Services': 0.00118166821360502,
 'CONST': 1.0,
 'Construction': 0.0651382962298471,
```

```
 'Education and Training': 0.00176143704927589,
 'Electricity, Gas, Water and Waste Services': 0.0163958603465452,
 'Financial and Insurance Services': 0.063727304406697,
 'Health Care and Social Assistance': 0.0001492617051173,
 'Information Media and Telecommunications': 0.280411541160886,
 'Manufacturing': 0.0501822931726048,
 'Mining': 0.0359152693528605,
 'Other Services': 0.0370415258619944,
 'Public Administration and Safety': 0.00898780113127804,
 'Rental, Hiring and Real Estate Services': 0.276480300431904,
 'Retail Trade': 0.0304367306683823,
 'Transport, Postal and Warehousing': 0.0421984021126843,
 'Wholesale Trade': 0.0632847656678229}
------------------------------
F_16 Wholesale Trade 200000
F_17 Wholesale Trade 200000
F_18 Wholesale Trade 200000
------------------------------
{'Accommodation and Food Services': 0.0190742618596676,
 'Agriculture, Forestry and Fishing': 0.00295054166161616,
 'Arts and Recreation Services': 0.00244683094841006,
 'CONST': 1.0,
 'Construction': 0.0390766751251188,
 'Education and Training': 0.00229365996934899,
 'Electricity, Gas, Water and Waste Services': 0.0204181727917409,
 'Financial and Insurance Services': 0.0585538620372086,
 'Health Care and Social Assistance': 0.000348337844534815,
 'Information Media and Telecommunications': 0.128545828038474,
 'Manufacturing': 0.091377194153372,
 'Mining': 0.00345372470578366,
 'Other Services': 0.0743319421716394,
 'Public Administration and Safety': 0.0125311244090705,
 'Rental, Hiring and Real Estate Services': 0.40351326706016,
 'Retail Trade': 0.0562855783552239,
 'Transport, Postal and Warehousing': 0.0279341926212245,
 'Wholesale Trade': 0.0568648062474057}
------------------------------
F_19 Retail Trade 200000
F_20 Retail Trade 200000
F_21 Retail Trade 200000
```

```
------------------------------
{'Accommodation and Food Services': 0.00755556427857266,
 'Agriculture, Forestry and Fishing': 0.0189693009600124,
 'Arts and Recreation Services': 0.00410851820221092,
 'CONST': 1.0,
 'Construction': 0.0763853368032077,
 'Education and Training': 0.00358199752282578,
 'Electricity, Gas, Water and Waste Services': 0.0643954999866141,
 'Financial and Insurance Services': 0.0604112402084754,
 'Health Care and Social Assistance': 0.000331420357417859,
 'Information Media and Telecommunications': 0.0751055403644336,
 'Manufacturing': 0.123338920640577,
 'Mining': 0.00662630663907087,
 'Other Services': 0.0188973730299016,
 'Public Administration and Safety': 0.00592092356759872,
 'Rental, Hiring and Real Estate Services': 0.306242212002158,
 'Retail Trade': 0.056170490229055,
 'Transport, Postal and Warehousing': 0.0613236853984491,
 'Wholesale Trade': 0.11063566980942}
------------------------------
F_22 Accommodation and Food Services 200000
F_23 Accommodation and Food Services 200000
F_24 Accommodation and Food Services 200000
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{'Accommodation and Food Services': 0.0114164887537751,
 'Agriculture, Forestry and Fishing': 0.000691585953366941,
 'Arts and Recreation Services': 0.00106885936994862,
 'CONST': 1.0,
 'Construction': 0.0427756289443071,
 'Education and Training': 0.00216920279419933,
 'Electricity, Gas, Water and Waste Services': 0.015954938078627,
 'Financial and Insurance Services': 0.0359953785741256,
 'Health Care and Social Assistance': 3.06282630409848e-05,
 'Information Media and Telecommunications': 0.113757661633658,
 'Manufacturing': 0.159392999529834,
 'Mining': 0.0130207562401894,
 'Other Services': 0.159081453599349,
 'Public Administration and Safety': 0.0232419330418141,
 'Rental, Hiring and Real Estate Services': 0.14437461532934,
 'Retail Trade': 0.0408912717552295,
```

```
 'Transport, Postal and Warehousing': 0.154807294975458,
 'Wholesale Trade': 0.0813293031637368}
------------------------------
F_25 Transport, Postal and Warehousing 200000
F_26 Transport, Postal and Warehousing 200000
F_27 Transport, Postal and Warehousing 200000
------------------------------
{'Accommodation and Food Services': 0.0243055206985149,
 'Agriculture, Forestry and Fishing': 0.000692217954087986,
 'Arts and Recreation Services': 0.014287198348753,
 'CONST': 1.0,
 'Construction': 0.0736222896385562,
 'Education and Training': 0.0044810808965996,
 'Electricity, Gas, Water and Waste Services': 0.0347532901555907,
 'Financial and Insurance Services': 0.0796347421141102,
 'Health Care and Social Assistance': 0.000815225468450243,
 'Information Media and Telecommunications': 0.214370027246627,
 'Manufacturing': 0.0627810022074138,
 'Mining': 0.00330882242741827,
 'Other Services': 0.0652940366989893,
 'Public Administration and Safety': 0.030740665310464,
 'Rental, Hiring and Real Estate Services': 0.261916315910187,
 'Retail Trade': 0.0263239525479323,
 'Transport, Postal and Warehousing': 0.0438111484863037,
 'Wholesale Trade': 0.0588624638900018}
------------------------------
F_28 Information Media and Telecommunications 200000
F_29 Information Media and Telecommunications 200000
F_30 Information Media and Telecommunications 200000
------------------------------
{'Accommodation and Food Services': 0.0160408203492468,
 'Agriculture, Forestry and Fishing': 0.000203519673764042,
 'Arts and Recreation Services': 0.00196990148686293,
 'CONST': 1.0,
 'Construction': 0.0232267270911627,
 'Education and Training': 0.0133592370735613,
 'Electricity, Gas, Water and Waste Services': 0.0164142556850247,
 'Financial and Insurance Services': 0.439522123631468,
 'Health Care and Social Assistance': 0.005095467985519,
 'Information Media and Telecommunications': 0.0622772168872966,
```

```
 'Manufacturing': 0.0156963302374304,
 'Mining': 0.000901905669546721,
 'Other Services': 0.0234443703138866,
 'Public Administration and Safety': 0.011673773558991,
 'Rental, Hiring and Real Estate Services': 0.326552625195399,
 'Retail Trade': 0.0138628589383724,
 'Transport, Postal and Warehousing': 0.0100023650582406,
 'Wholesale Trade': 0.0197565011642269}
------------------------------
F_31 Financial and Insurance Services 200000
F_32 Financial and Insurance Services 200000
F_33 Financial and Insurance Services 200000
------------------------------
{'Accommodation and Food Services': 0.0305749712451719,
 'Agriculture, Forestry and Fishing': 0.000341855728907573,
 'Arts and Recreation Services': 0.00764799763148891,
 'CONST': 1.0,
 'Construction': 0.0591182142517541,
 'Education and Training': 0.0136909679429718,
 'Electricity, Gas, Water and Waste Services': 0.0373349537333264,
 'Financial and Insurance Services': 0.0786209626121395,
 'Health Care and Social Assistance': 0.00067683799191184,
 'Information Media and Telecommunications': 0.110252257004042,
 'Manufacturing': 0.020736126748093,
 'Mining': 0.00391143673298421,
 'Other Services': 0.0293355405100632,
 'Public Administration and Safety': 0.0226745402684704,
 'Rental, Hiring and Real Estate Services': 0.52163041940968,
 'Retail Trade': 0.0166244684316301,
 'Transport, Postal and Warehousing': 0.017725452056916,
 'Wholesale Trade': 0.0291029977004496}
------------------------------
F_34 Rental, Hiring and Real Estate Services 200000
F_35 Rental, Hiring and Real Estate Services 200000
F_36 Rental, Hiring and Real Estate Services 200000
------------------------------
{'Accommodation and Food Services': 0.040375918530543,
 'Agriculture, Forestry and Fishing': 0.000411913190911201,
 'Arts and Recreation Services': 0.0107248812890148,
 'CONST': 1.0,
```

```
 'Construction': 0.0390146740652874,
 'Education and Training': 0.0180931914532944,
 'Electricity, Gas, Water and Waste Services': 0.0417801398421247,
 'Financial and Insurance Services': 0.069130348464283,
 'Health Care and Social Assistance': 0.000896466468233921,
 'Information Media and Telecommunications': 0.134872719203959,
 'Manufacturing': 0.0249189078349628,
 'Mining': 0.00488502980180255,
 'Other Services': 0.0359185643294742,
 'Public Administration and Safety': 0.0282104429839195,
 'Rental, Hiring and Real Estate Services': 0.472803395246137,
 'Retail Trade': 0.0185913752153185,
 'Transport, Postal and Warehousing': 0.0238658159479209,
 'Wholesale Trade': 0.0355062161328129}
------------------------------
F_37 Public Administration and Safety 200000
F_38 Public Administration and Safety 200000
F_39 Public Administration and Safety 200000
------------------------------
{'Accommodation and Food Services': 0.0364375011041681,
 'Agriculture, Forestry and Fishing': 0.00257189934684023,
 'Arts and Recreation Services': 0.00393877332191383,
 'CONST': 1.0,
 'Construction': 0.141284392596018,
 'Education and Training': 0.0153800866292119,
 'Electricity, Gas, Water and Waste Services': 0.0352687497614555,
 'Financial and Insurance Services': 0.0894497220846579,
 'Health Care and Social Assistance': 0.00312002381942319,
 'Information Media and Telecommunications': 0.126417831467386,
 'Manufacturing': 0.0439232485268889,
 'Mining': 0.00635135518134948,
 'Other Services': 0.0279896175940199,
 'Public Administration and Safety': 0.141981499509452,
 'Rental, Hiring and Real Estate Services': 0.239845309650254,
 'Retail Trade': 0.0139516582604833,
 'Transport, Postal and Warehousing': 0.0409041114901034,
 'Wholesale Trade': 0.0311842196563748}
------------------------------
F_40 Education and Training 200000
F_41 Education and Training 200000
```

```
F_42 Education and Training 200000
------------------------------
{'Accommodation and Food Services': 0.0268940910088458,
 'Agriculture, Forestry and Fishing': 0.000145228158205452,
 'Arts and Recreation Services': 0.0457579333904271,
 'CONST': 1.0,
 'Construction': 0.0125597148784847,
 'Education and Training': 0.0895029078884532,
 'Electricity, Gas, Water and Waste Services': 0.0789209498028235,
 'Financial and Insurance Services': 0.046554133201512,
 'Health Care and Social Assistance': 0.00519323111935652,
 'Information Media and Telecommunications': 0.133683711741104,
 'Manufacturing': 0.0727348268612989,
 'Mining': 0.00330253710650188,
 'Other Services': 0.0297494486168925,
 'Public Administration and Safety': 0.0482356791328881,
 'Rental, Hiring and Real Estate Services': 0.230416307580735,
 'Retail Trade': 0.0410339211745885,
 'Transport, Postal and Warehousing': 0.0480931900485318,
 'Wholesale Trade': 0.0872221882893514}
------------------------------
F_43 Health Care and Social Assistance 200000
F_44 Health Care and Social Assistance 200000
F_45 Health Care and Social Assistance 200000
------------------------------
{'Accommodation and Food Services': 0.0101331668340351,
 'Agriculture, Forestry and Fishing': 0.000351989607885525,
 'Arts and Recreation Services': 0.00819928282308633,
 'CONST': 1.0,
 'Construction': 0.0217015850426396,
 'Education and Training': 0.012353117853278,
 'Electricity, Gas, Water and Waste Services': 0.0521846285041282,
 'Financial and Insurance Services': 0.0961867383258083,
 'Health Care and Social Assistance': 0.049474679863831,
 'Information Media and Telecommunications': 0.0639258164131893,
 'Manufacturing': 0.0861552180873291,
 'Mining': 0.00523126984395809,
 'Other Services': 0.103292483439507,
 'Public Administration and Safety': 0.0225780838203409,
 'Rental, Hiring and Real Estate Services': 0.218485203371308,
```

```
 'Retail Trade': 0.0633297640207557,
 'Transport, Postal and Warehousing': 0.0411152834767005,
 'Wholesale Trade': 0.14530168867222}
------------------------------
F_46 Arts and Recreation Services 200000
F_47 Arts and Recreation Services 200000
F_48 Arts and Recreation Services 200000
------------------------------
{'Accommodation and Food Services': 0.013098892798196,
 'Agriculture, Forestry and Fishing': 0.00106928164786323,
 'Arts and Recreation Services': 0.00356639626608657,
 'CONST': 1.0,
 'Construction': 0.0465476228938216,
 'Education and Training': 0.0132395974296014,
 'Electricity, Gas, Water and Waste Services': 0.0538256635168658,
 'Financial and Insurance Services': 0.153696473395593,
 'Health Care and Social Assistance': 0.00154163727753805,
 'Information Media and Telecommunications': 0.0859377069486106,
 'Manufacturing': 0.156168620173708,
 'Mining': 0.00303901581187192,
 'Other Services': 0.0312243679551186,
 'Public Administration and Safety': 0.0314006704282172,
 'Rental, Hiring and Real Estate Services': 0.188380126360205,
 'Retail Trade': 0.0430269574852689,
 'Transport, Postal and Warehousing': 0.0313872763299335,
 'Wholesale Trade': 0.142849693281501}
------------------------------
F_49 Other Services 200000
F_50 Other Services 200000
F_51 Other Services 200000
------------------------------
```

Local ownership percentages for different firms are calculated on each run, presented below is an example of one such random assignments according to the rules previously explained in the main text. The third column of this list represents the local ownership of the firm with label given on the first column, belonging to economic sector given on the second colum, in the particular instance run.

```
F_1 Agriculture, Forestry and Fishing 0.9315
F_2 Agriculture, Forestry and Fishing 0.8974
F_3 Agriculture, Forestry and Fishing 0.9007
F_4 Mining 0.3327
F_5 Mining 0.3236
F_6 Mining 0.3381
F_7 Manufacturing 0.697
F_8 Manufacturing 0.6887
F_9 Manufacturing 0.6699
F_10 Electricity, Gas, Water and Waste Services 0.3784
F_11 Electricity, Gas, Water and Waste Services 0.358
F_12 Electricity, Gas, Water and Waste Services 0.3712
F_13 Construction 0.3278
F_14 Construction 0.3376
F_15 Construction 0.3292
F_16 Wholesale Trade 0.5774
F_17 Wholesale Trade 0.5736
F_18 Wholesale Trade 0.5948
F_19 Retail Trade 0.1545
F_20 Retail Trade 0.1579
F_21 Retail Trade 0.1553
F_22 Accommodation and Food Services 0.7804
F_23 Accommodation and Food Services 0.8115
F_24 Accommodation and Food Services 0.7943
F_25 Transport, Postal and Warehousing 0.0938
F_26 Transport, Postal and Warehousing 0.1023
F_27 Transport, Postal and Warehousing 0.1015
F_28 Information Media and Telecommunications 0.4198
F_29 Information Media and Telecommunications 0.4289
F_30 Information Media and Telecommunications 0.4181
F_31 Financial and Insurance Services 0.1355
F_32 Financial and Insurance Services 0.1357
F_33 Financial and Insurance Services 0.1336
F_34 Rental, Hiring and Real Estate Services 0.6319
F_35 Rental, Hiring and Real Estate Services 0.5993
F_36 Rental, Hiring and Real Estate Services 0.6561
F_37 Public Administration and Safety 0.1126
F_38 Public Administration and Safety 0.1137
```

```
F_39 Public Administration and Safety 0.1082
F_40 Education and Training 0.2737
F_41 Education and Training 0.2702
F_42 Education and Training 0.2711
F_43 Health Care and Social Assistance 0.2127
F_44 Health Care and Social Assistance 0.2289
F_45 Health Care and Social Assistance 0.2183
F_46 Arts and Recreation Services 0.8682
F_47 Arts and Recreation Services 0.8186
F_48 Arts and Recreation Services 0.8607
F_49 Other Services 0.8598
F_50 Other Services 0.841
F_51 Other Services 0.9035
```

H.2 Households

100 households have been created and saved for repeated use and their main details are shown in the output below. First column is the household identifier, second column contains the weekly salary, third column contains the main contributing sector to that household, the fourth column shows the composition of the household and finally the fifth column indicates the dwelling type.

```
H_0 2903.6 Construction couple_nc rented
H_1 3814.8 Mining couple_nc owned
H_2 11248.9 Construction couple_nc rented
H_3 3024.0 Health Care and Social Assistance couple_wc mortgage
H_4 1418.4 Education and Training couple_wc mortgage
H_5 4131.6 Transport, Postal and Warehousing couple_wc owned
H_6 3020.9 Mining couple_nc rented
H_7 5372.0 Other Services couple_nc mortgage
H_8 1145.8 Wholesale Trade couple_nc mortgage
H_9 3389.0 Electricity, Gas, Water and Waste Services one_pf rented
H_10 1414.4 Retail Trade couple_nc rented
H_11 1252.0 Construction one_pf rented
```

```
H_12 2867.9 Mining couple_nc rented
H_13 997.2 Agriculture, Forestry and Fishing couple_wc owned
H_14 148.8 Health Care and Social Assistance couple_wc mortgage
H_15 2320.0 Health Care and Social Assistance one_pf rented
H_16 3692.4 Financial and Insurance Services couple_wc mortgage
H_17 2158.8 Other Services couple_wc rented
H_18 4149.7 Mining couple_nc rented
H_19 3336.0 Mining couple_wc owned
H_20 2936.4 Mining couple_wc rented
H_21 12496.7 Manufacturing couple_nc owned
H_22 3148.0 Retail Trade one_pf rented
H_23 8403.6 Retail Trade couple_wc owned
H_24 4601.9 Mining couple_nc mortgage
H_25 1693.2 Mining couple_wc rented
H_26 2749.2 Mining couple_wc mortgage
H_27 4307.8 Manufacturing couple_nc mortgage
H_28 3574.8 Other Services couple_wc owned
H_29 1613.3 Mining couple_nc rented
H_30 1470.5 Mining couple_nc rented
H_31 3818.2 Other Services couple_nc owned
H_32 3236.4 Manufacturing couple_wc owned
H_33 1417.8 Education and Training couple_nc mortgage
H_34 2433.6 Accommodation and Food Services couple_wc mortgage
H_35 5025.2 Mining couple_nc mortgage
H_36 5763.0 Manufacturing one_pf mortgage
H_37 2035.2 Mining couple_wc mortgage
H_38 890.4 Construction couple_wc mortgage
H_39 5251.3 Accommodation and Food Services couple_nc mortgage
H_40 517.2 Mining couple_wc mortgage
H_41 2209.0 Education and Training one_pf rented
H_42 3031.2 Health Care and Social Assistance couple_wc mortgage
H_43 4588.3 Mining couple_nc rented
H_44 1451.0 Transport, Postal and Warehousing one_pf rented
H_45 469.2 Education and Training couple_wc mortgage
H_46 2238.0 Retail Trade couple_wc rented
H_47 6060.5 Mining couple_nc rented
H_48 2198.0 Mining one_pf rented
H_49 5144.2 Other Services couple_nc mortgage
H_50 727.6 Other Services couple_nc mortgage
H_51 1485.6 Retail Trade couple_wc mortgage
```

```
H_52 2868.0 Mining couple_wc mortgage
H_53 8299.2 Accommodation and Food Services couple_wc rented
H_54 1864.9 Retail Trade couple_nc owned
H_55 8078.4 Other Services couple_nc rented
H_56 723.6 Public Administration and Safety  couple_wc mortgage
H_57 6784.9 Other Services other_f mortgage
H_58 1790.0 Retail Trade one_pf rented
H_59 5890.5 Transport, Postal and Warehousing couple_nc mortgage
H_60 1294.0 Other Services one_pf rented
H_61 1366.8 Mining couple_wc mortgage
H_62 861.0 Mining one_pf owned
H_63 4238.1 Retail Trade couple_nc rented
H_64 1366.8 Other Services couple_nc rented
H_65 2470.1 Education and Training couple_nc rented
H_66 983.0 Construction one_pf rented
H_67 2247.6 Transport, Postal and Warehousing couple_wc mortgage
H_68 550.0 Transport, Postal and Warehousing one_pf mortgage
H_69 1883.6 Health Care and Social Assistance couple_nc mortgage
H_70 1283.5 Transport, Postal and Warehousing couple_nc owned
H_71 536.4 Other Services couple_wc rented
H_72 483.6 Health Care and Social Assistance couple_wc owned
H_73 782.4 Manufacturing couple_wc mortgage
H_74 3867.5 Other Services couple_nc rented
H_75 1898.9 Public Administration and Safety  couple_nc rented
H_76 6607.9 Public Administration and Safety  couple_nc rented
H_77 859.0 Transport, Postal and Warehousing one_pf rented
H_78 6135.3 Construction couple_nc mortgage
H_79 2733.6 Health Care and Social Assistance couple_wc mortgage
H_80 544.8 Transport, Postal and Warehousing couple_wc mortgage
H_81 4080.0 Education and Training couple_wc owned
H_82 1591.0 Other Services one_pf mortgage
H_83 1920.0 Wholesale Trade one_pf owned
H_84 6600.6 Wholesale Trade other_f mortgage
H_85 6153.6 Retail Trade couple_wc mortgage
H_86 1299.6 Health Care and Social Assistance couple_wc owned
H_87 1533.0 Other Services one_pf rented
H_88 1098.0 Manufacturing couple_wc mortgage
H_89 4032.4 Transport, Postal and Warehousing couple_nc rented
H_90 865.2 Other Services couple_wc owned
H_91 2769.3 Retail Trade couple_nc mortgage
```

```
H_92 6188.0 Manufacturing couple_nc mortgage
H_93 2317.1 Construction couple_nc mortgage
H_94 3135.6 Construction couple_wc mortgage
H_95 3401.7 Mining couple_nc other
H_96 1390.8 Education and Training couple_wc mortgage
H_97 3821.6 Education and Training couple_nc owned
H_98 363.8 Retail Trade couple_nc owned
H_99 813.6 Education and Training couple_wc owned
```

H.2.1 Joint Distribution of Sectors and Salaries

The outcome of using the iterative proportional fitting algorithm to create a joint distribution of sectors and salary categories is shown below. This has been moved from the main text to the appendix to avoid the exccesive distraction it may bring in reading the thesis.

	sec	cat	total
0	Agriculture, Forestry and Fishing	NIL	0.202667
1	Mining	NIL	35.316232
2	Manufacturing	NIL	4.470247
3	Electricity, Gas, Water and Waste Services	NIL	1.289208
4	Construction	NIL	9.415624
5	Wholesale Trade	NIL	3.711405
6	Retail Trade	NIL	2.593453
7	Accommodation and Food Services	NIL	8.595971
8	Transport, Postal and Warehousing	NIL	8.745353
9	Information Media and Telecommunications	NIL	0.505977
10	Financial and Insurance Services	NIL	0.890581
11	Rental, Hiring and Real Estate Services	NIL	1.166398
12	Public Administration and Safety	NIL	3.920370
13	Education and Training	NIL	3.493134
14	Health Care and Social Assistance	NIL	13.432919
15	Arts and Recreation Services	NIL	0.481455
16	Other Services	NIL	9.902922
17	Agriculture, Forestry and Fishing	$1-$149	0.179875
18	Mining	$1-$149	3.095749
19	Manufacturing	$1-$149	2.938895
20	Electricity, Gas, Water and Waste Services	$1-$149	0.791065
21	Construction	$1-$149	5.777481
22	Wholesale Trade	$1-$149	0.650668

23	Retail Trade	$1-$149	6.138094
24	Accommodation and Food Services	$1-$149	0.968792
25	Transport, Postal and Warehousing	$1-$149	2.956885
26	Information Media and Telecommunications	$1-$149	0.319341
27	Financial and Insurance Services	$1-$149	0.562079
28	Rental, Hiring and Real Estate Services	$1-$149	1.380295
29	Public Administration and Safety	$1-$149	3.092865
30	Education and Training	$1-$149	3.674412
31	Health Care and Social Assistance	$1-$149	10.597520
32	Arts and Recreation Services	$1-$149	1.012881
33	Other Services	$1-$149	8.928719
34	Agriculture, Forestry and Fishing	$150-$299	0.294621
35	Mining	$150-$299	32.596723
36	Manufacturing	$150-$299	4.951221
37	Electricity, Gas, Water and Waste Services	$150-$299	0.713960
38	Construction	$150-$299	12.166810
39	Wholesale Trade	$150-$299	0.685122
40	Retail Trade	$150-$299	19.389341
41	Accommodation and Food Services	$150-$299	10.200912
42	Transport, Postal and Warehousing	$150-$299	14.529459
43	Information Media and Telecommunications	$150-$299	1.345001
44	Financial and Insurance Services	$150-$299	0.887762
45	Rental, Hiring and Real Estate Services	$150-$299	1.937845
46	Public Administration and Safety	$150-$299	9.769907
47	Education and Training	$150-$299	13.541418
48	Health Care and Social Assistance	$150-$299	4.959409
49	Arts and Recreation Services	$150-$299	0.799886
50	Other Services	$150-$299	9.401505
51	Agriculture, Forestry and Fishing	$300-$399	0.536302
52	Mining	$300-$399	11.867251
53	Manufacturing	$300-$399	4.506387
54	Electricity, Gas, Water and Waste Services	$300-$399	2.599261
55	Construction	$300-$399	22.147407
56	Wholesale Trade	$300-$399	4.988547
57	Retail Trade	$300-$399	23.529785
58	Accommodation and Food Services	$300-$399	25.996400
59	Transport, Postal and Warehousing	$300-$399	15.113239
60	Information Media and Telecommunications	$300-$399	2.142283
61	Financial and Insurance Services	$300-$399	3.232010
62	Rental, Hiring and Real Estate Services	$300-$399	7.936841
63	Public Administration and Safety	$300-$399	20.748343
64	Education and Training	$300-$399	28.170999
65	Health Care and Social Assistance	$300-$399	18.055359
66	Arts and Recreation Services	$300-$399	3.397433
67	Other Services	$300-$399	4.278422
68	Agriculture, Forestry and Fishing	$400-$499	0.565259
69	Mining	$400-$499	109.445023
70	Manufacturing	$400-$499	12.467968
71	Electricity, Gas, Water and Waste Services	$400-$499	5.593358
72	Construction	$400-$499	23.343219
73	Wholesale Trade	$400-$499	2.300329

74	Retail Trade	$400-$499	43.400413
75	Accommodation and Food Services	$400-$499	13.700015
76	Transport, Postal and Warehousing	$400-$499	34.845241
77	Information Media and Telecommunications	$400-$499	2.257952
78	Financial and Insurance Services	$400-$499	6.954973
79	Rental, Hiring and Real Estate Services	$400-$499	13.012809
80	Public Administration and Safety	$400-$499	5.467154
81	Education and Training	$400-$499	12.990269
82	Health Care and Social Assistance	$400-$499	41.628623
83	Arts and Recreation Services	$400-$499	1.790436
84	Other Services	$400-$499	23.674496
85	Agriculture, Forestry and Fishing	$500-$649	0.538294
86	Mining	$500-$649	95.290603
87	Manufacturing	$500-$649	9.046248
88	Electricity, Gas, Water and Waste Services	$500-$649	0.434819
89	Construction	$500-$649	12.702664
90	Wholesale Trade	$500-$649	8.762380
91	Retail Trade	$500-$649	19.680979
92	Accommodation and Food Services	$500-$649	11.182693
93	Transport, Postal and Warehousing	$500-$649	15.169369
94	Information Media and Telecommunications	$500-$649	0.921531
95	Financial and Insurance Services	$500-$649	1.622007
96	Rental, Hiring and Real Estate Services	$500-$649	5.310879
97	Public Administration and Safety	$500-$649	17.850344
98	Education and Training	$500-$649	24.741173
99	Health Care and Social Assistance	$500-$649	4.530604
100	Arts and Recreation Services	$500-$649	2.922901
101	Other Services	$500-$649	8.588623
102	Agriculture, Forestry and Fishing	$650-$799	0.875919
103	Mining	$650-$799	135.675857
104	Manufacturing	$650-$799	17.173536
105	Electricity, Gas, Water and Waste Services	$650-$799	2.751558
106	Construction	$650-$799	16.076622
107	Wholesale Trade	$650-$799	9.505506
108	Retail Trade	$650-$799	29.890137
109	Accommodation and Food Services	$650-$799	18.870574
110	Transport, Postal and Warehousing	$650-$799	38.397018
111	Information Media and Telecommunications	$650-$799	3.498898
112	Financial and Insurance Services	$650-$799	4.789934
113	Rental, Hiring and Real Estate Services	$650-$799	10.082254
114	Public Administration and Safety	$650-$799	3.765263
115	Education and Training	$650-$799	17.892961
116	Health Care and Social Assistance	$650-$799	51.605810
117	Arts and Recreation Services	$650-$799	4.315796
118	Other Services	$650-$799	16.304774
119	Agriculture, Forestry and Fishing	$800-$999	0.448400
120	Mining	$800-$999	52.091384
121	Manufacturing	$800-$999	3.296803
122	Electricity, Gas, Water and Waste Services	$800-$999	5.070879
123	Construction	$800-$999	27.776059
124	Wholesale Trade	$800-$999	3.649544

125	Retail Trade	$800-$999	40.166065
126	Accommodation and Food Services	$800-$999	21.735502
127	Transport, Postal and Warehousing	$800-$999	27.641531
128	Information Media and Telecommunications	$800-$999	4.030100
129	Financial and Insurance Services	$800-$999	7.093467
130	Rental, Hiring and Real Estate Services	$800-$999	11.612941
131	Public Administration and Safety	$800-$999	39.032144
132	Education and Training	$800-$999	46.371303
133	Health Care and Social Assistance	$800-$999	13.209022
134	Arts and Recreation Services	$800-$999	6.391310
135	Other Services	$800-$999	43.820381
136	Agriculture, Forestry and Fishing	$1,000-$1,249	1.036882
137	Mining	$1,000-$1,249	133.840186
138	Manufacturing	$1,000-$1,249	10.164709
139	Electricity, Gas, Water and Waste Services	$1,000-$1,249	5.862953
140	Construction	$1,000-$1,249	14.273197
141	Wholesale Trade	$1,000-$1,249	2.813069
142	Retail Trade	$1,000-$1,249	70.765750
143	Accommodation and Food Services	$1,000-$1,249	41.884331
144	Transport, Postal and Warehousing	$1,000-$1,249	25.567322
145	Information Media and Telecommunications	$1,000-$1,249	0.690312
146	Financial and Insurance Services	$1,000-$1,249	2.430065
147	Rental, Hiring and Real Estate Services	$1,000-$1,249	11.935011
148	Public Administration and Safety	$1,000-$1,249	46.800428
149	Education and Training	$1,000-$1,249	63.543139
150	Health Care and Social Assistance	$1,000-$1,249	71.270630
151	Arts and Recreation Services	$1,000-$1,249	5.473805
152	Other Services	$1,000-$1,249	19.301008
153	Agriculture, Forestry and Fishing	$1,250-$1,499	0.851816
154	Mining	$1,250-$1,499	153.932784
155	Manufacturing	$1,250-$1,499	33.401924
156	Electricity, Gas, Water and Waste Services	$1,250-$1,499	4.816515
157	Construction	$1,250-$1,499	52.765533
158	Wholesale Trade	$1,250-$1,499	16.176891
159	Retail Trade	$1,250-$1,499	14.533817
160	Accommodation and Food Services	$1,250-$1,499	48.172166
161	Transport, Postal and Warehousing	$1,250-$1,499	21.003989
162	Information Media and Telecommunications	$1,250-$1,499	5.103925
163	Financial and Insurance Services	$1,250-$1,499	5.989019
164	Rental, Hiring and Real Estate Services	$1,250-$1,499	3.268272
165	Public Administration and Safety	$1,250-$1,499	38.447347
166	Education and Training	$1,250-$1,499	58.726990
167	Health Care and Social Assistance	$1,250-$1,499	33.457162
168	Arts and Recreation Services	$1,250-$1,499	2.698094
169	Other Services	$1,250-$1,499	71.352478
170	Agriculture, Forestry and Fishing	$1,500-$1,749	1.447667
171	Mining	$1,500-$1,749	224.236948
172	Manufacturing	$1,500-$1,749	28.383394
173	Electricity, Gas, Water and Waste Services	$1,500-$1,749	5.457133
174	Construction	$1,500-$1,749	46.498339
175	Wholesale Trade	$1,500-$1,749	18.328488

176	Retail Trade	$1,500-$1,749	16.466878
177	Accommodation and Food Services	$1,500-$1,749	23.391117
178	Transport, Postal and Warehousing	$1,500-$1,749	31.730149
179	Information Media and Telecommunications	$1,500-$1,749	3.855180
180	Financial and Insurance Services	$1,500-$1,749	4.523723
181	Rental, Hiring and Real Estate Services	$1,500-$1,749	1.851483
182	Public Administration and Safety	$1,500-$1,749	37.338010
183	Education and Training	$1,500-$1,749	44.358624
184	Health Care and Social Assistance	$1,500-$1,749	18.953555
185	Arts and Recreation Services	$1,500-$1,749	2.037968
186	Other Services	$1,500-$1,749	71.860150
187	Agriculture, Forestry and Fishing	$1,750-$1,999	1.468079
188	Mining	$1,750-$1,999	75.799589
189	Manufacturing	$1,750-$1,999	14.391805
190	Electricity, Gas, Water and Waste Services	$1,750-$1,999	9.684640
191	Construction	$1,750-$1,999	10.104425
192	Wholesale Trade	$1,750-$1,999	31.863302
193	Retail Trade	$1,750-$1,999	62.621499
194	Accommodation and Food Services	$1,750-$1,999	35.581410
195	Transport, Postal and Warehousing	$1,750-$1,999	36.199749
196	Information Media and Telecommunications	$1,750-$1,999	8.796463
197	Financial and Insurance Services	$1,750-$1,999	10.321896
198	Rental, Hiring and Real Estate Services	$1,750-$1,999	22.531074
199	Public Administration and Safety	$1,750-$1,999	18.932245
200	Education and Training	$1,750-$1,999	78.722168
201	Health Care and Social Assistance	$1,750-$1,999	86.493632
202	Arts and Recreation Services	$1,750-$1,999	9.300170
203	Other Services	$1,750-$1,999	122.973868
204	Agriculture, Forestry and Fishing	$2,000-$2,499	1.265954
205	Mining	$2,000-$2,499	392.180927
206	Manufacturing	$2,000-$2,499	37.231016
207	Electricity, Gas, Water and Waste Services	$2,000-$2,499	14.316441
208	Construction	$2,000-$2,499	17.426491
209	Wholesale Trade	$2,000-$2,499	6.869089
210	Retail Trade	$2,000-$2,499	151.199327
211	Accommodation and Food Services	$2,000-$2,499	81.820148
212	Transport, Postal and Warehousing	$2,000-$2,499	124.863045
213	Information Media and Telecommunications	$2,000-$2,499	6.742546
214	Financial and Insurance Services	$2,000-$2,499	23.735402
215	Rental, Hiring and Real Estate Services	$2,000-$2,499	19.428992
216	Public Administration and Safety	$2,000-$2,499	48.976949
217	Education and Training	$2,000-$2,499	77.581355
218	Health Care and Social Assistance	$2,000-$2,499	198.893795
219	Arts and Recreation Services	$2,000-$2,499	2.673240
220	Other Services	$2,000-$2,499	188.520540
221	Agriculture, Forestry and Fishing	$2,500-$2,999	1.110017
222	Mining	$2,500-$2,999	309.485849
223	Manufacturing	$2,500-$2,999	52.232003
224	Electricity, Gas, Water and Waste Services	$2,500-$2,999	5.021193
225	Construction	$2,500-$2,999	82.511699
226	Wholesale Trade	$2,500-$2,999	3.613784

227	Retail Trade	$2,500-$2,999	11.363572
228	Accommodation and Food Services	$2,500-$2,999	86.090122
229	Transport, Postal and Warehousing	$2,500-$2,999	43.793104
230	Information Media and Telecommunications	$2,500-$2,999	5.320816
231	Financial and Insurance Services	$2,500-$2,999	1.560881
232	Rental, Hiring and Real Estate Services	$2,500-$2,999	17.887571
233	Public Administration and Safety	$2,500-$2,999	42.944104
234	Education and Training	$2,500-$2,999	10.203765
235	Health Care and Social Assistance	$2,500-$2,999	117.716360
236	Arts and Recreation Services	$2,500-$2,999	9.844623
237	Other Services	$2,500-$2,999	74.384596
238	Agriculture, Forestry and Fishing	$3,000-$3,499	1.204498
239	Mining	$3,000-$3,499	111.942756
240	Manufacturing	$3,000-$3,499	35.423637
241	Electricity, Gas, Water and Waste Services	$3,000-$3,499	1.362145
242	Construction	$3,000-$3,499	29.844941
243	Wholesale Trade	$3,000-$3,499	23.528266
244	Retail Trade	$3,000-$3,499	98.646420
245	Accommodation and Food Services	$3,000-$3,499	23.354458
246	Transport, Postal and Warehousing	$3,000-$3,499	35.640472
247	Information Media and Telecommunications	$3,000-$3,499	6.735991
248	Financial and Insurance Services	$3,000-$3,499	11.856163
249	Rental, Hiring and Real Estate Services	$3,000-$3,499	13.864359
250	Public Administration and Safety	$3,000-$3,499	55.919238
251	Education and Training	$3,000-$3,499	88.578205
252	Health Care and Social Assistance	$3,000-$3,499	42.578663
253	Arts and Recreation Services	$3,000-$3,499	7.630404
254	Other Services	$3,000-$3,499	80.715968
255	Agriculture, Forestry and Fishing	$3,500-$3,999	0.246147
256	Mining	$3,500-$3,999	190.635178
257	Manufacturing	$3,500-$3,999	57.912383
258	Electricity, Gas, Water and Waste Services	$3,500-$3,999	12.526339
259	Construction	$3,500-$3,999	60.990055
260	Wholesale Trade	$3,500-$3,999	20.033968
261	Retail Trade	$3,500-$3,999	75.596361
262	Accommodation and Food Services	$3,500-$3,999	35.794751
263	Transport, Postal and Warehousing	$3,500-$3,999	12.138932
264	Information Media and Telecommunications	$3,500-$3,999	0.983245
265	Financial and Insurance Services	$3,500-$3,999	12.114416
266	Rental, Hiring and Real Estate Services	$3,500-$3,999	8.499813
267	Public Administration and Safety	$3,500-$3,999	28.568639
268	Education and Training	$3,500-$3,999	33.940360
269	Health Care and Social Assistance	$3,500-$3,999	14.502039
270	Arts and Recreation Services	$3,500-$3,999	3.118644
271	Other Services	$3,500-$3,999	96.219822
272	Agriculture, Forestry and Fishing	$4,000 or more	1.779925
273	Mining	$4,000 or more	220.561844
274	Manufacturing	$4,000 or more	73.285295
275	Electricity, Gas, Water and Waste Services	$4,000 or more	6.038655
276	Construction	$4,000 or more	117.607526
277	Wholesale Trade	$4,000 or more	52.152705

278	Retail Trade	$4,000 or more	127.551342
279	Accommodation and Food Services	$4,000 or more	34.511627
280	Transport, Postal and Warehousing	$4,000 or more	17.555688
281	Information Media and Telecommunications	$4,000 or more	12.797985
282	Financial and Insurance Services	$4,000 or more	2.502890
283	Rental, Hiring and Real Estate Services	$4,000 or more	24.585367
284	Public Administration and Safety	$4,000 or more	96.405917
285	Education and Training	$4,000 or more	16.361855
286	Health Care and Social Assistance	$4,000 or more	41.946566
287	Arts and Recreation Services	$4,000 or more	20.296243
288	Other Services	$4,000 or more	119.276562
289	Agriculture, Forestry and Fishing	Partial	1.452482
290	Mining	Partial	269.979483
291	Manufacturing	Partial	34.173378
292	Electricity, Gas, Water and Waste Services	Partial	14.783274
293	Construction	Partial	107.968422
294	Wholesale Trade	Partial	23.643592
295	Retail Trade	Partial	14.869491
296	Accommodation and Food Services	Partial	70.406793
297	Transport, Postal and Warehousing	Partial	57.304266
298	Information Media and Telecommunications	Partial	1.160401
299	Financial and Insurance Services	Partial	2.042447
300	Rental, Hiring and Real Estate Services	Partial	10.031268
301	Public Administration and Safety	Partial	56.193330
302	Education and Training	Partial	106.814853
303	Health Care and Social Assistance	Partial	68.459785
304	Arts and Recreation Services	Partial	16.562458
305	Other Services	Partial	97.333923
306	Agriculture, Forestry and Fishing	Not Stated	0.580727
307	Mining	Not Stated	67.463920
308	Manufacturing	Not Stated	14.943997
309	Electricity, Gas, Water and Waste Services	Not Stated	3.694124
310	Construction	Not Stated	8.993243
311	Wholesale Trade	Not Stated	5.908187
312	Retail Trade	Not Stated	29.725345
313	Accommodation and Food Services	Not Stated	17.593627
314	Transport, Postal and Warehousing	Not Stated	21.479246
315	Information Media and Telecommunications	Not Stated	2.029771
316	Financial and Insurance Services	Not Stated	4.593400
317	Rental, Hiring and Real Estate Services	Not Stated	2.506667
318	Public Administration and Safety	Not Stated	2.808378
319	Education and Training	Not Stated	20.018601
320	Health Care and Social Assistance	Not Stated	12.830324
321	Arts and Recreation Services	Not Stated	2.759144
322	Other Services	Not Stated	28.376043

H.2.2 Output of the First Run of Baseline Case

The outcome of using the first serious run of the algorithm is shown below. This has been moved from the main text to the appendix to avoid the exccesive distraction it may bring when reading the thesis.

```
Economy Agent, Iteration number 0
Economy Agent, Iteration number 1
Economy Agent, Iteration number 2
Economy Agent, Iteration number 3
Economy Agent, Iteration number 4
Economy Agent, Iteration number 5
Economy Agent, Iteration number 6
Economy Agent, Iteration number 7
Economy Agent, Iteration number 8
Economy Agent, Iteration number 9
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 4: [77.85215645, 0.0],
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 6: [57.03040674, 0.0],
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 8: [43.50157793, 0.0],
 9: [43.53592769, 0.0]}
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```

```
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```

H.3 Code for Baseline Example

```python
import Economy as ec
import Company as co
import Household as ho
import Market as mk
import simpy
import random as rd
import sys
import pickle
if not sys.warnoptions:
    import warnings
    warnings.filterwarnings(action= 'ignore')
mcompanies = 3
nHouseholds = 100
nIters = 10
initBudget = 50000
nRepeat = 3
prefixDir = './tests/baseline1/'

prices = {'Agriculture, Forestry and Fishing': 30,
```

```
          'Mining': 50,
          'Manufacturing': 26,
          'Electricity, Gas, Water and Waste Services': 22,
          'Construction': 9,
          'Wholesale Trade': 11,
          'Retail Trade': 13,
          'Accommodation and Food Services': 14,
          'Transport, Postal and Warehousing': 5,
          'Information Media and Telecommunications': 8,
          'Financial and Insurance Services': 6,
          'Rental, Hiring and Real Estate Services': 23,
          'Public Administration and Safety': 14,
          'Education and Training': 4,
          'Health Care and Social Assistance': 19,
          'Arts and Recreation Services': 8,
          'Other Services': 20,
          'CONST': 1}

labels = ['Agriculture, Forestry and Fishing',
          'Mining',
          'Manufacturing',
          'Electricity, Gas, Water and Waste Services',
          'Construction',
          'Wholesale Trade',
          'Retail Trade',
          'Accommodation and Food Services',
          'Transport, Postal and Warehousing',
          'Information Media and Telecommunications',
          'Financial and Insurance Services',
          'Rental, Hiring and Real Estate Services',
          'Public Administration and Safety',
          'Education and Training',
          'Health Care and Social Assistance',
          'Arts and Recreation Services',
          'Other Services']

local_ownership = {'Agriculture, Forestry and Fishing': 89.9,
          'Mining': 33,
          'Manufacturing': 68.3,
          'Electricity, Gas, Water and Waste Services': 36.7,
          'Construction': 33.2,
          'Wholesale Trade': 58.3,
          'Retail Trade': 15.4,
          'Accommodation and Food Services': 78.9,
          'Transport, Postal and Warehousing': 9.9,
          'Information Media and Telecommunications': 42.3,
          'Financial and Insurance Services': 13.4,
          'Rental, Hiring and Real Estate Services': 63.2,
          'Public Administration and Safety': 11.2,
```

```
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            'Health Care and Social Assistance': 22,
            'Arts and Recreation Services': 83.5,
            'Other Services': 85.4}

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]

class IOMatrix(object):
    def __init__(self, name, labels = None, matobj = None):
        self.name = name
        self.labels = labels
        self.matobj = matobj
        self.values = self.create()

    def create(self):
        tmpDict = {}
        i = 0
        for label_i in self.labels:
            j = 0
            for label_j in labels:
                tmpDict[(label_i, label_j)] = self.matobj[j][i]
                j += 1
            i += 1
        return dict(sorted(tmpDict.items()))

    def calculaSumaRows(self):
        return [sum(x) for x in self.matobj]

    def calculaSumaCols(self):
        return [sum(x) for x in zip(*self.matobj)]


def main(env, printSector=False):
    sectorColl = ec.economy(env, print=False)
    # Creates Matrix with the labels already given and the normalised IO Matrix
    mat = IOMatrix("Baseline", labels=labels, matobj=IOMat)
    mat.create()

    # Creates the collections of sectors that belong to the Economy
    for label_i in mat.labels:
        tmpDict = {'avgprice': prices[label_i], 'print': printSector, 'name':
            label_i}
        tmpDict['env'] = env
        # Creates object named sector initialised with its parameters
        sectorColl.__add_val__(ec.sector(**tmpDict))
```

```
        sectorColl.collection[label_i].technology['CONST'] = 1.0
        for label_j in mat.labels:
             sectorColl.collection[label_i].technology[label_j] = mat.values[(
                 label_i, label_j)]
        # Every sector has teh labels put in order for the dictionary
        sectorColl.collection[label_i].technology = dict(sorted(
            sectorColl.collection[label_i].technology.items()))
    return sectorColl

def createHouseholds(env, filenames):
    pass

if __name__ == "__main__":
    for i in range(nRepeat):
        # Enironment is created
        envi = simpy.Environment()
        # Economy with sectors is created and populated
        econo = main(env = envi, printSector=False)
        # We get the list of sectors from the economy
        sectors = list(econo.returnSectorList())
        for sect in sectors:
            econo.localsalary[sect] = 0
        # We add the 'CONST' sector, needed in the Cobb-Douglas function
        sectors.insert(0, 'CONST')
        # Initialises an empty collection of firms
        firmColl = co.firms(envi, print=False)
        # Load the firms files and create them
        with open('firms.txt','rb') as f:
            Firms_JSON = pickle.load(f)
        for key, value in Firms_JSON.items():
            tmpDict = {}
            tmpDict['cdCoeffs'] = value['cdCoeffs']
            tmpDict['name'] = key
            tmpDict['env'] = envi
            tmpDict['economy'] = econo
            tmpDict['sector'] = value['sector']
            tmpDict['budget'] = initBudget # value['budget']
            tmpDict['print'] = False # value['print']
            tmpDict['localperc'] = round((local_ownership[value['sector']] +
                                       rd.normalvariate(0, local_ownership[value[
                                           'sector']]/40))/100,4)
            tmpFirm = co.firm(**tmpDict)
            firmColl.__add_val__(tmpFirm)
            econo.collection[value['sector']].firms[key] = tmpFirm
        # Initialise an empty collection of Households
        houseColl = ho.households(envi, print=False)
        # Reading Data From Household File
        with open('households.txt','rb') as f:
            Households = pickle.load(f)
```

```
        for key, value in Households.items():
            tmpDict = {}
            tmpDict['cdCoeffs'] = value['cdCoeffs']
            tmpDict['name'] = key
            tmpDict['env'] = envi
            tmpDict['economy'] = econo
            tmpDict['budget'] = value['budget']
            tmpDict['salary'] = value['salary']
            tmpDict['industry'] = value['industry']
            tmpDict['composition'] = value['composition']
            tmpDict['dwelling'] = value['dwelling']
            tmpDict['print'] = value['print']
            econo.globalsalary += value['salary']
            econo.localsalary[value['industry']] += value['salary']
            houseColl.__add_val__(ho.household(**tmpDict))
        mercado = mk.market(env=envi, economy=econo, households=houseColl,
                            firms=firmColl, print=False)

        envi.run(until=nIters)

        name1 = f'{prefixDir}gloMult{i}.pkl'
        name2 = f'{prefixDir}gloMultA{i}.pkl'
        name3 = f'{prefixDir}secMult{i}.pkl'
        name4 = f'{prefixDir}secMultA{i}.pkl'
        file1 = open(name1, 'wb')
        file2 = open(name2, 'wb')
        file3 = open(name3, 'wb')
        file4 = open(name4, 'wb')
        print(econo.globalmultiplier)
        print(econo.globalmultiplieralt)
        print(econo.sectmultiplier)
        print(econo.sectmultiplieralt)
        pickle.dump(econo.globalmultiplier, file1)
        pickle.dump(econo.globalmultiplieralt, file2)
        pickle.dump(econo.sectmultiplier, file3)
        pickle.dump(econo.sectmultiplieralt, file4)
        file1.close()
        file2.close()
        file3.close()
        file4.close()
```

Listing H.1: Code for Baseline Experiment