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A GRAVIMETRIC GEOID OF TASMANIA USING THE 1-D FFT WITH A MODIFIED INTEGRATION KERNEL

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ABSTRACT

A new gravimetric estimate of the separation between the GRS80 ellipsoid and the geoid over Tasmania and surrounding seas has been made. The geoid model utilised a combination of the EGM96 global geopotential model, the 1992 release of the Australian gravity database, and a nine-second digital elevation model. The computational technique used was the one-dimensional fast Fourier transform (1D-FFT), which has been refined so as to include deterministically modified Stokes integration kernels. Comparisons were made among Global Positioning System heights, optical levelling on the Australian Height Datum (Tasmania) and several gravimetric geoid solutions at 14 points. This showed that the Vanicek and Kleusberg (1987) modification for a degree 20 spheroid and cap radius of 15’ gave the smallest standard deviation of 186mm, which is a slight improvement upon the 232mm achieved when using AUSGEOID93.

1. INTRODUCTION

The geoid is the equipotential surface of the Earth's gravity field that corresponds most closely with the undisturbed mean sea level. It is commonly realised by tide gauge measurements of mean sea level, and is commonly used as the vertical datum via geodetic levelling from these tide gauges. Many geodesists have attempted to model the undulations in the geoid with respect to the reference ellipsoid using gravity data in the classical Stokes formula. This has been driven principally by the need to convert Global Positioning System (GPS) derived ellipsoidal heights to heights referred to the geoid. The gravimetric geoid model currently used for this purpose throughout Australia is called AUSGEOID93 (Steed and Holtznagel, 1994).

Since the computation of AUSGEOID93, improved fast Fourier transform (FFT) techniques have become available, the EGM96 global geopotential model (Lemoine et al., 1997) has been released, a further 250,000 terrestrial observations have been added to the Australian gravity database, and a new digital elevation model of Australia has been released. These additional data are currently being combined to produce an updated improvement upon AUSGEOID93 (Featherstone *et al*., 1997).

In this paper, however, an analysis of utilising modified integration kernels in the 1D-FFT geoid determination technique (Haagmans et al., 1993) will be investigated for the island of Tasmania. Many regional gravimetric geoids in other parts of the world have convolved the whole regional gravity dataset with an unmodified Stokes kernel via the 1D-FFT (eg. Sideris and She, 1995). However, this approach has been found to be inappropriate for the Australian gravimetric geoid (eg. Zhang, 1997, Forsberg and Featherstone, 1998). This is most probably due to the error characteristics of the Australian data and fit of the geopotential model, but distortions in the Australian Height Datum (AHD) cannot be ruled out (eg. Featherstone and Stewart, 1998). The "W curves" reported by Kearsley (1988) were also suspected to be caused by the Australian gravity data and fit of the geopotential model.

Therefore, the computation of the Australian gravimetric geoid must rely upon a Stokesian integration over a spherical cap of limited extent. Associated with this approach, however, is an increase in the size of the truncation error that results from the neglect of gravity data outside the limited integration domain. A large proportion of this truncation error can be eliminated using a high-degree global geopotential model. Nevertheless, the truncation error can be minimised yet further by an appropriate modification of Stokes's integration kernel. In addition, some of the modified kernels have been shown to behave as high-pass filters (Vanicek and Featherstone, 1998), thus removing a proportion of the low-frequency errors present in the terrestrial gravity data.

Featherstone and Sideris (1998) have presented preliminary results of using modified integration kernels in the 1D-FFT technique over Western Australia. The account on which this paper is based (Vella, 1997) is an application of this approach in a different region of Australia. This will determine whether the improvements made by Featherstone and Sideris (1998) are indeed consistent.

2. THE MATHEMATICAL MODELS

2.1 Stokes's integral by the 1D-FFT

As the name suggests, gravimetric geoid determination employs gravity data to estimate the separation between the geoid and reference ellipsoid. The most widely used relationship between the geoid height (N) and the gravity anomalies (Δg) is provided by Stokes's formula (eg. Heiskanen and Moritz, 1967), which may be expressed in geodetic coordinates as

$$N(\phi,\lambda) = \frac{R}{4\pi G} \int_{\lambda'=0}^{2\pi} \int_{\phi'=-\frac{\pi}{2}}^{\frac{\pi}{2}} \Delta g(\phi',\lambda') S(\psi) \cos\phi' \, d\phi' \, d\lambda' \qquad (1)$$

where γ is normal gravity on the reference ellipsoid, R is the radius of a spherical Earth, (ϕ,λ) are the geodetic coordinates of the computation point, (ϕ',λ') are the geodetic coordinates of each remote gravity anomaly used in the convolution, ψ is the spherical distance between these two points, and $S(\psi)$ is the spherical Stokes function given by

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} p_n(\cos\psi) \qquad (2)$$

where Pn (cos ψ) are the Legendre polynomials.

A quadrature-based numerical integration of Eq. (1) can be extremely time-consuming, which is especially problematic for a continent-wide geoid computation. The drawback is that the summation that is used to mimic the surface integral has to be repeated for every point at which the geoid is desired. As such, the fast Fourier transform (FFT) technique offers an attractive alternative, purely because of its computational speed. In fact, the FFT technique offers a very powerful tool for the efficient evaluation of all gravity field convolution integrals (Schwarz *et al*., 1990). Several FFT-based techniques for the evaluation of Stokes's integral have been presented over the last decade, with the best approach being the 1D FFT (Haagmans *et al*., 1993). The 1D-FFT makes use of the property that the geoid height is exact (i.e. identical to that produced by numerical integration) for all the computation points along each parallel. As such, the 1D-FFT is considered to be the 'best' FFT approach because it overcomes the kernel approximations that were necessary to use 2D-FFT methods. Therefore, Stokes's integral (Eq. 1) can be evaluated by the following 1D-FFT formula

$$N_{\Delta g}^{\varphi p} = \frac{R\Delta\varphi\Delta\lambda}{4\pi\gamma} \sum \mathbf{F}_1^{-1} \left\{ \sum_{\phi=\varphi_1}^{\varphi M} F_1 \left\{ S^{\varphi p \varphi} \right\} F_1 \left\{ \Delta g^{\varphi} \cos\varphi \right\} \right\} \qquad (3)$$

where $\mathbf{F_1}$ and $\mathbf{F}_1^{-1}$ denote the 1D Fourier transform operator and its inverse.

Another advantage of the 1D-FFT approach over the 2D-FFT approach is that it only has to deal with a single 1D complex array for each parallel. Also, the zero padding required to eliminate the effects of cyclic convolution and windowing (eg. Sideris and Li, 1994) need only be applied to the parallels. Both of these aspects result in a saving of computer memory. However, using FFT techniques can also have its drawbacks. For example, the FFT requires the input data to be gridded, but most gravity data have not been collected according to such a formation.

Therefore, the question arises as to how accurately can the observed gravity data be predicted at the unobserved locations of the grid nodes. This aspect will be considered later in the paper.

2.2 The remove restore approach using the geopotential model

In modern, regional, gravimetric geoid computations, the remove-restore approach is routinely used where a global geopotential model provides the long-wavelength component of the geoid. In this scheme, the terrestrial gravity data have the gravity anomalies implied by the global geopotential model removed as per the following spherical harmonic expression.

$$\Delta g_{GGM} = \frac{GM}{r^2} \sum_{n=2}^{M_{\max}} \left(\frac{a}{r}\right)^n (n-1) \sum \left(\delta \overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right) \overline{P}_{nm}(\cos\theta) \qquad (4)$$

Accordingly, the geoid heights of the corresponding degree (*M*) of the same global geopotential model are then restored to the Stokes solution using

$$N_{GGM} = \frac{GM}{r\gamma} \{ \sum_{n=2}^{n_{MAX}} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n \left[\left(\overline{C}_{nm} \cos .m\lambda + \overline{S}_{nm} \sin .m\lambda\right)\right] \overline{P}_{nm}(\sin\phi) \qquad (5)$$

where in Eqs (4) and (5), *GM* is the product of the Newtonian gravitational constant and the mass of the solid Earth, oceans and atmosphere, (r,θ,λ) are the spherical polar coordinates of the computation point, *a* is the length of the semi-major axis of the reference ellipsoid, *Cnm* and *Snm* are the fully normalised gravity potential coefficients of degree *n* and order *m* for which the even zonal harmonics have been reduced by those generating the normal gravity field, and *Pnm* (cos θ) are the fully normalised associated Legendre functions.

In the remove-restore scheme, Eq. (1) then becomes

$$N = N_{GGM} + \frac{R}{4\pi\gamma} \int_\sigma S(\psi)\left(\Delta g - \Delta g_{GGM}\right) d\sigma \qquad (6)$$

which can also be evaluated by the 1D-FFT according to

$$N = N_{GGM} + \frac{R\Delta\phi\Delta\lambda}{4\pi\gamma} \mathbf{F}^{-1}\left[\left\{\sum \mathbf{F}\{S(\psi)\}\mathbf{F}\{(\Delta g - \Delta g_{GGM})\cos\phi\}\right\}\right] \qquad (7)$$

2.3 Gravimetric terrain corrections and primary indirect effects by FFT

Another consideration in the computation of the geoid is that the disturbing potential (and thus geoid and gravity anomalies) is not a harmonic function beneath the topography. This violates one of the major assumptions made in the derivation of Stokes's integral. In practice, this can be accounted for in an approximate way by reducing the surface gravity data using the second-order free-air reduction (eg. Featherstone, 1995), the gravimetric terrain correction (Moritz, 1968), and also applying a correction for the gravitational attraction of the atmosphere (Featherstone et al., 1997). The Moritz (1968) terrain correction is given by

$$C = \frac{G\rho R^2}{2} \int_\sigma \frac{(H' - H)^2}{l^3} d\sigma \tag{8}$$

which can also be evaluated by the 1D-FFT according to

$$C = \frac{G\rho R^2}{2} \begin{pmatrix} \mathbf{F}^{-1}\left[\sum \mathbf{F}\left\{ 1/l^3 \right\} \mathbf{F}\left\{ H'^2 \right\}\right] - 2H\mathbf{F}^{-1}\left[\sum \mathbf{F}\left\{ 1/l^3 \right\} \mathbf{F}\left\{ H' \right\}\right] .. \\ + H^2 \mathbf{F}^{-1}\left[\sum \mathbf{F}\left\{ 1/l^3 \right\} \mathbf{F}\left\{ 1 \right\}\right] \end{pmatrix} \tag{9}$$

The terrain correction is added to the residual gravity anomalies, which are now assumed to be on the geoid and termed Faye anomalies (eg. Heiskanen and Moritz, 1967). These are then used in Eqs. (4) and (6) to compute the residual geoid undulations, which are added to the result from Eq. (5).

However, this produces what is called a Faye co-geoid, which must be converted to the true geoid by adding the indirect effect corresponding to the free-air reduction and gravimetric terrain correction. The co-geoid is produced because the gravity reductions change the gravity potential of the Earth. The indirect effect is given by Wichiencharoen (1982) as the following convolution-type integral.

$$N_i = -\frac{\pi G\rho H^2}{\gamma} + \frac{G\rho R^2}{6\gamma} \int \left(\frac{H'^3 - H^3}{l^3} \right) d\sigma \tag{10}$$

which can also be evaluated using the FFT by

$$N_i = -\frac{\pi G\rho}{\gamma} H^2 + \frac{G\rho\Delta\phi\Delta\lambda}{6\gamma} (H^3 \mathbf{F}^{-1}[\{\sum \mathbf{F}\{ 1/l^3 \}\mathbf{F}\{1\}\}] + \mathbf{F}^{-1}\left[\left\{\sum \mathbf{F}\left\{1/l^3\right\}\mathbf{F}\left\{H'^3\right\}\right\}\right] \tag{11}$$

2.4 Introduction of modified Stokes integration kernels

There remains one major limitation of using Stokes's formula as defined in Eq. (4). This is because it is practically impossible to use due to the need to have access to gravity data over the entire surface of the Earth. This has driven the computation of the geoid to rely upon an approximation of Eq. (6) where the integration is performed over a limited domain. When the integral is truncated at a spherical radius of ψ_0, a truncation error is introduced because the effect of gravity data outside the integration cap on the geoid are ignored. Under this approximation, Stokes's integral (Eq. 6) is now written in the form

$$N_i = \frac{r}{4\pi\gamma} \int_{\sigma_0} S_2(\psi) \Delta g d\sigma + \delta N_i \tag{12}$$

where the corresponding truncation error

$$\delta N_i = \frac{r}{4\pi\gamma} \int_{\sigma-\sigma_0} S_2(\psi) \Delta g d\sigma \tag{13}$$

Using the truncated integration in Eq. (12) will only be successful, in an ideal but somewhat unrealistic situation, if the truncation error is reduced to zero. Molodensky can be thought as the pioneer of introducing modified integration kernels (eg. Molodensky et al., 1962), where the modification aims to minimise the truncation error. Many other authors have since discussed the topic of kernel modification. Four deterministic modifications proposed by Wong and Gore (1969), Meissl (1971), Vanicek and Kleusberg (1987) and Featherstone et al., (1998) will be tested in this study as these were implemented in the 1D-FFT for the study by Featherstone and Sideris (1998). The statistical modifications (eg. Vanicek and Sjoberg) have not been used since accurate estimates of the error spectra of the Tasmanian gravity data are not currently available.

2.4.1 Wong and Gore's kernel modification

Wong and Gore (1969) proposed to remove the low-degree Legendre polynomials from the spherical Stokes integration kernel (Eq. 2). This produces a modified kernel that is higher than second-degree, and is defined as follows:

$$S^M(\psi) = S(\psi) - \sum_{n=2}^{M-1} \frac{2n+1}{n-1} P_n(\cos\psi) \qquad (14)$$

2.4.2 Meissl's kernel modification

Meissl (1971) illustrated that the spectral series expansion of the truncation error converges to zero faster when the integration kernel is zero at the truncation radius ψ_0. This is achieved by simply subtracting the numerical value of the spherical Stokes kernel at the truncation radius (ie. $S(\psi_0)$) from the original kernel inside the cap. Therefore, the Meissl modified kernel is defined as

$$S_{me}(\psi) = S(\psi) - S(\psi_0) \qquad 0 \le \psi \le \psi_0 \qquad (15)$$

2.4.3 Vanicek and Kleusberg's kernel modification

Vanicek and Kleusberg (1987) extended Molodensky's theory to the spheroidal Stokes kernel (Eq. 14). This modification strategy minimises the upper bound of the truncation error in a least squares sense. It is achieved by subtracting a series of modifying coefficients from the spheroidal integration kernel. This gives the modified kernel as

$$S_{vk}^M(\psi) = S^M(\psi) - \sum_{k=2}^{M-1} \frac{2k+1}{2} t_k(\psi_0) P_k(\cos\psi) \qquad (16)$$

where the modifying coefficients are computed from the solution of the following set of M-1 linear equations.

$$\sum_{k=2}^{M-1} \frac{2k+1}{2} t_k(\psi_0) e_{nk}(\psi_0) = Q_n(\psi_0) - \sum_{k=2}^{M-1} \frac{2k+1}{k-1} e_{nk}(\psi_0) \qquad (17)$$

where

$$e_{nk}(\psi_0) = \int_{\psi_0}^{\pi} P_k(\cos\psi) P_n(\cos\psi) \sin\psi \, d\psi$$

2.4.4 Featherstone, Evans and Olliver's kernel modification

The kernel modification proposed by Featherstone *et al* (1998) is a hybrid of Eqs. (16) and (15). This makes combined use of the minimisation of the upper bound of the truncation error and ensures that its series expansion converges to zero more quickly. It is computed by setting the integration kernel to zero at the truncation radius by subtracting the value of Eq. (16) at ψ_0, as follows.

$$S_{mvk}^{M}(\psi) = S_{vk}^{M}(\psi) - S_{vk}^{M}(\psi_0) \qquad \text{for } 0 \le \psi \le \psi_0 \qquad (18)$$

3. THE DATA USED IN THE COMPUTATIONS

The region chosen for the computations of the Tasmanian gravimetric geoid is bound by 38°S <= ϕ <= 45°S, 143°E <= λ <= 150° and used the following data sources:

- The EGM96 global geopotential model (Lemoine et al., 1997), complete to spherical harmonic degree and order 360.
- The verified 1992 release of the Australian Geological Survey Organisation's land and marine gravity database. From this, atmospherically corrected, second-order free-air gravity anomalies were computed. The validation and reduction procedures have been described in more detail by Featherstone et al. (1997) and Featherstone (1995).
- The Australian digital elevation model (DEM), which was supplied by the Australian Surveying and Land Information Group, which is supplied on a regular 9' by 9' grid interval. The production of this DEM is described in more detail by (Carrol and Morse, 1996).

To test the gravimetric geoid solutions for the four kernel modification strategies with varying cap radius, a GPS network co-located with AHD(Tas) benchmarks was used. These data were supplied by A/Prof R Coleman of the University of Tasmania. The GPS network had used several hours of observations per baseline, and the data processed using SKI software with precise ephemerides produced by the International GPS Service for Geodynamics.

3.1 Gravity data gridding

As stated, the 1D-FFT approach to gravimetric geoid computation requires a grid of gravity and terrain data. The DEM data were already provided on a 9” grid, so were used directly for the terrain and indirect effect computations. However, the point gravity observations had to be interpolated onto a regular 5’ by 5’ grid, which is commensurate with their observation spacing. Firstly, the gravity anomalies implied by the EGM96 global geopotential model were subtracted point-by-point from all terrestrial anomalies according to Eq. 4. The resulting values are termed residual free-air gravity anomalies. This stage provides a band-width-limited gravity data set that avoids the interpolation algorithm having to cope with the low frequencies.

The surface fitting technique, based on the minimum tension spline algorithm of Smith and Wessel (1990), was used to interpolate the residual gravity data onto the 5’ by 5’ grid. Obviously the 9” grid took considerably longer to produce the corresponding terrain correction and indirect effect as the array size was 420 by 420 where as for the re-gridded DEM the array size was only 85 by 85. The following tables show the effect of re sampling these data sets onto five-minute grids, and also include the values of the de-trended gravity values. The data initially has the effect of EGM96 subtracted from it, then it undergoes a block-mean (BLM) after which a surface is fitted to it (XYZ). The BLM is the result of mean longitude and latitude values computed within a specified grid size and the XYZ is the tension spline surface used to interpolate the data onto a regular grid.

Table 1. The effects of re-sampling the AGSO land and marine data onto a 5’ by 5’ grid (units in mgal).

	LAND +MAR	LAND +MAR-EGM96	LAND +MAR-EGM96 (BLM)	LAND +MAR-EGM96 (XYZ)
# of data	49724	49724	3841	7225
Minimum	-105.643	-73.169	-61.499	-60.378
Maximum	135.114	128.143	118.343	107.062
Mean	14.298	-5.202	-1.063	1.412
std.	28.533	15.962	17.713	15.480
rms	31.916	16.788	17.745	15.544

Note that the standard deviation of the residual gravity anomalies is quite large when compared to the fit in other countries (eg. Forsberg and Featherstone, 1998, Table 4). This implies that the geopotential model is not as good as a fit in Tasmania as it is in other parts of the world.

It is visible from Table 1 in the BLM column that the number of points reduces very dramatically and then increases again in the (XYZ) surface column. This is due to the fact that the BLM or Block Mean has the effect of reducing the values in a specified area to one, with a mean value of latitude, longitude and gravity. The XYZ column or Surface then uses the BLM values to generate a surface of which it places a point at the specified interval, therefore increasing the value from one. This gives values in all of the 5 by 5 cells, even if there wasn’t an observation. For example, the extent of the data set is 7^o, so in the case of the above tables if $\Delta\phi$ and $\Delta\lambda$ are both 7^o and m is 5’, then the number of surface fitted points is (85×85) which is equal to 7225. The data from the surface fitted (XYZ) columns are the data used in the FFT to produce the residual Faye co-geoid undulations.

3.2 Computation of Terrain Corrections and The Indirect Effect

The indirect effect is computed in order to compensate for the assumption of displaced mass when the Faye co-geoid is computed. The indirect effect amounts to less than 1 mm. Using the global geopotential model, EGM96 geoidal undulations are derived over Tasmania. The results of HARMONICS are output on a 5 minute grid. This gives the long wavelength type of the geoid in the form of $N_{GGM.}$ Next the local gravity data is considered to provide the short wavelength geoid information is produced in the form of $N_{\Delta g}$ which is the result of FFTGEOID (1D-FFT routine from MGS)

These results can be added to produce the Faye co-geoid given by equation 2. Due to the fact that the geoid computed thus far is a co-geoid it is necessary to account for the change in gravity potential. This is done through the application of equation 5. The results of the indirect effect are in meters and the terrain corrections are in mgals. Therefore it is necessary to use FFTGEOID in order to produce N values that can be added to each of the separate solutions to provide the final geoid TASGEOID97. EGM96 was used to produce the longer wavelengths of the final geoid solution and was computed on a 5' grid that is commensurate with the gravity data on a point by point bases over Tasmania.

3.3 Results on using different kernel modifications

The final geoid solution for Tasmania, TASGEOID97 has been computed using different combinations of extra added in gravity data. This is done to investigate which combinations yield the smallest standard deviations. All TASGEOID97 solutions are compared to AUSGEOID93 too, as well to the GPS derived geoidal heights N_{GPS}.

The kernels used in this study are the original Stokes, Wong and Gore (1969), Meissl (1971), Vanicek and Kleusberg (1987) and Featherstone *et al* (1998). All have different approaches to the modification and the way the truncation error is minimised, and not all approaches yield the same results, as will be seen from the following.

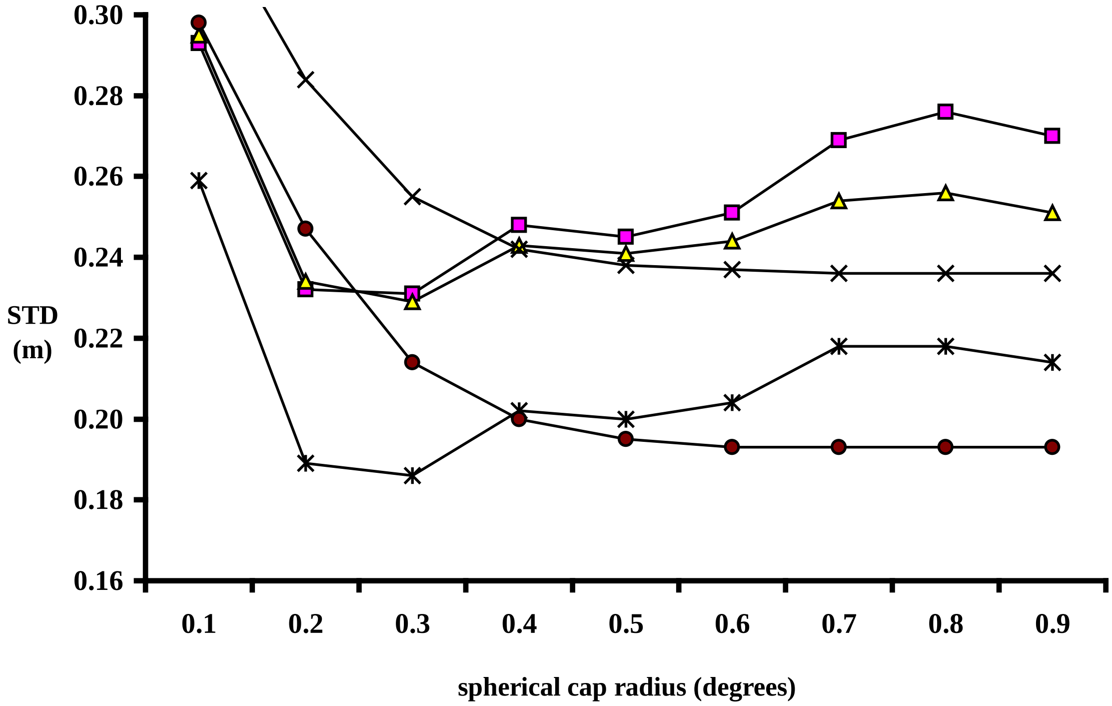

Figure 1. Different permutations of modifications of Stokes's integral with varying cap sizes using the land and marine data set

Figure 1 represents the tests done over Tasmania in order to find whether or not a modification is necessary and if so which modifications best suits the data area. The standard deviations are related to the fit between the 14 GPS control points and the respective data source used, for example the Ausgeoid93 is represented by a straight line in order to show the amount of deviation with respect to the other data used.

This is done in order to find what is the optimum capsize for the Tasmanian data. It is to be kept in mind that the idea of truncation is to use as small as possible an integration area to reduce computation time and maximise the influence of detailed local gravity coverage. The initial tests were only done for a total of 7^0 the extent of the test area, as this contains the maximum number of regularly gridded gravity data and exceeding this limit would only be a waist of time as there are no data outside the 7^0. The smallest standard deviation occurs at the 1^0 capsize, therefore in Figure 1, the interval was narrowed down to a limit of 1^0 with 0.1^0 intervals.

Figure 1 shows the final fit to N_{GPS} with respect to the three modifications used. As can be seen Vanicek and Kleusberg's modification achieves the lowest standard deviation, although the Featherstone *et al* (1998) modification yields slightly worse but more consistent results. Ausgeoid93 does not contain terrain corrections or corrections due to the indirect effect as discussed in previous chapters, whereas the geoid being computed for this research does contain all the above.

The final geoid solution contains only Land and Marine gravity anomalies combined with EGM96, terrain corrections and indirect effect. The Land and Marine data is processed using Vanicek and Kleusberg's modification upto n_{max20}. The data set with the smallest standard deviations is the Land and Marine set with Vanicek and Kleusberg's modification of Stokes integral. The standard deviation for the final solution using Vanicek and Kleusberg's modification and a spherical cap size of 0.25^0. The data set is the land and marine, the indirect effect is excluded here as it amounts to zero.

4. SUMMARY AND CONCLUSIONS

The final geoid solution is given, for the region encompassing Tasmania and is called TASGEOID97. The highlight of this research would, by no doubt, have to be the results with the modifications and the permutations thereof. It is clear that a modification of some kind must be incorporated. To suggest which modification would be foolish as it would depend on the coverage and ability of the data to model the gravity field in a particular region. For the test done in this research it is concluded that Vaniceck and Kleusberg's modification be used in conjunction with the Land and Marine data set, with a spherical cap size of 0.25^0. Also it is to be noted that Featherstone *et al* modification yields almost similar yet more consistent results.

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