School of Education

Assessing Effects of Inquiry-Based Learning in a Small Liberal Arts University General Education Mathematics Course

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Abstract

This mixed methods research study sought to determine the effects of inquiry-based learning on college students' understanding of key concepts in a general education mathematics, which students from all majors take during their first year of study at an American liberal arts college. Data was collected from 121 research participants over five semesters of study, including two semesters which were fully online, due to socially distancing regulations during the global COVID-19 pandemic.

Each semester, one course (the treatment) was taught using inquiry-based learning methods, while the other course (the control) was taught using traditional interactive lectures. Throughout the term, scores from classroom assessments were collected, assessing students' understanding of key concepts in the course; these included quizzes, exams and a comprehensive final exam. Additionally, Quantitative data were collected at the start and end of each term using the Attitudes Towards Mathematics Survey (ATMI), to assess the change in student attitudes towards mathematics over the semester in four categories: self-confidence, value, enjoyment, and motivation. Qualitative data were collected though semi-structured interviews with research participants and instructor reflections.

Previous studies showed significant differences in understanding of key concepts and student attitude towards mathematics favoring students who had taken an inquiry-based learning course. However, in this study there was improvement in the assessment scores for participants enrolled in the interactive lecture course, and higher reported changes of student perception of the value of mathematics, their motivation towards mathematics, and their enjoyment of mathematics. In the face-to-face courses, there were improvements in the final exam scores, overall grades, and improved perception of the value of mathematics for students who were enrolled in the interactive lecture course. However, in the online courses, no significant difference between the interactive lecture group and the inquiry-based learning groups were found.

This study complements other research, which were performed in courses for STEM majors only, at large public universities. The research demonstrates that using inquiry-based learning in a college mathematics classroom does change the learning environment for students. Qualitative data confirmed the relationships between the teaching method of the course, and student learning outcomes and attitudes towards mathematics. Implications for future research in this field, as well as recommendations for practice, are included.

Declaration

To the best of my knowledge and belief, this thesis contains no material previously published by any other person except where due acknowledgement has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

The research presented and reported in this thesis was conducted in accordance with the National Health and Medical Research Council National Statement on Ethical Conduct in Human Research (2007) – updated March 2014. The proposed research study received human research ethics approval from Curtin University Human Research Ethics Committee (EC00262), Approval Number #HRE 2019-0523 (14.08.2019), and the University Institutional Review Board (IRB, 22.08.2019).

Signature: ________________________________

Date: 14.10.2024

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This would not be possible without the support of my family. My grandfather completed his PhD in Mathematics Education in 1957, when my mother was young. My grandfather's passion for education was instilled in my mother, who has always encouraged me to pursue my passions. Both my parents constantly encouraged me to pursue our passions and further my education. My father championed my two sisters and I through undergraduate degrees, my master's degree, and my sister's doctorate of veterinary medicine, and was excited to see me complete my doctorate degree. My own children have been incredibly patient as I have spent time writing and researching, with my focus on my research instead of with my family. And my deepest appreciation and gratitude to my husband – his encouragement and support to be able to complete this research has been invaluable. He supported me when I was frustrated, encouraged me when I was tired, and have allowed me time to tune the world out to sit, write, and complete my thesis. Thank you for helping me reach this goal – I truly appreciate it.

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1 Introduction

1.1 Background to the study

"The only way to learn mathematics is to do mathematics." --Paul Halmos

Paul Hamos believed that you must *do* mathematics to really learn it. Though this quote has been around for decades, mathematics educators are still looking for ways to actively engage students in learning mathematics by *doing mathematics* (Fielding-Wells & Makar, 2008; Laursen & Rasmussen, 2019; Lenz, 2015). While primary and secondary students are typically engaged in classroom activities, university students tend to be less actively involved in the process of learning mathematics (Jaschik, 2018; Schrage, 2018). This research study will address the effects of inquiry-based learning (a process to *do mathematics*) on students at a liberal arts university.

1.2 Liberal arts university

A liberal arts university focuses on a collection of subjects known in Latin as *artes liberals* ("proper to free persons"). This definition stems from Ancient Greece, where it was believed a well-rounded, educated individual must study seven subjects – arithmetic, geometry, astronomy, music (the quadrivium) and grammar, rhetoric, logic (the trivium) (Roche, 2010). In ancient times, the aims of studying these liberal arts subjects were to produce individuals who could be "free" in civic life (public debate, jury service, military service); in modern times, the aim of a liberal arts education is to produce articulate critical thinkers, who are knowledgeable in multiple subjects, with skills transferable to a variety of fields. The seven liberal arts subjects continued to be the focus through medieval European universities. Modern liberal arts education is developed from Socratic seminars, where students are engaged in meaningful discussions, asking questions, and are actively engaged in the learning process (Roche, 2010).

A modern-day definition of a liberal arts education is one in which students study a broad variety of diverse subjects, often called a "core curriculum," as well as courses in a specific major in an arts and sciences discipline such as biology, chemistry, mathematics, history, psychology; the liberal arts education contrasts a professional degree program in an applied field such as criminal justice, agriculture, or zoology. The breadth and depth of subjects in the liberal arts “core curriculum,” which includes courses in a variety of subjects, from science and math to social sciences and sociology to music and art, aims to produce graduates who have “interests and capabilities that will enrich both the individual learner and future communities” (Hawkins, 1999, p.23). The “core curriculum” allows students to cultivate intellectual virtues, to think deeply and critically about a variety of subjects, and to work independently, while the academic major provides depth and focus in a specific field of study, preparing students for their future vocations (Roche, 2010). A liberal arts university is designed to “provide students with the knowledge and abilities to be successful, productive members of a free society. It provides them the opportunity to practice free-thinking,” (Strauss, 2015) or *artes liberals* (liberal arts).

A “small liberal arts college” is a distinction specific to the universities in the United States and Europe. In the US in particular, liberal arts colleges are usually residential four-year universities, typically with low faculty-student ratios and small class sizes. Students study a particular subject in depth (their major, which leads to a Bachelor of Science or a Bachelor of Arts degree in subjects such as biology, psychology, art, mathematics, or history) as well as a breadth of other classes in a variety of fields (social sciences, humanities, natural sciences, which include quantitative reasoning, oral and written communication, history, literature and foreign language requirements, usually offered as part of a core curriculum) (Roche, 2010). Faculty at liberal arts universities tend to teach full-time, instead of focusing on research, as is often the case at larger universities.

1.3 Challenges to a general education liberal arts mathematics course

General education (GE) courses at a liberal arts university encompass students from a wide variety of majors, who are fulfilling graduation requirements in a breadth of liberal arts subjects. For many of these students, mathematics is not their major field of study, and they often lack basic mathematical skills (Brady, 2014). In particular, students pursuing non-STEM majors often have a negative attitude towards mathematics, and a GE mathematics course is the last exposure to a formal mathematics course they will have (Clinkenbeard, 2015). These first-year university students often believe mathematics is memorizing formulas, and lack both mathematical reasoning skills and the ability to do mathematics without a calculator (Brady, 2014). A study by the Association of American Colleges and Universities found that the outcomes from general education courses were ranked as one of the least important outcomes of their college experience (Thompson et al., 2015).

In many general education classes, students are given limited opportunities to think deeply about rich ideas or to develop critical thinking skills (Kim et al., 2012). However, to truly develop their "*artes liberals*" habits of mind, liberal arts mathematics students should engage in questions using mathematics by developing deep critical thinking, involving creativity, precision and abstraction (Bremser, 2014; Su, 2020). According to mathematician and author Francis Su, "the proper practice of mathematics cultivates virtues that help people flourish" (2020, p. 10). Students can learn transferable skills such as reasoning and judgment, as well as the relevance of mathematics, its application in daily life, and the joy of mathematical thinking in the liberal arts undergraduate mathematics classroom; this is often one of the last academic opportunities in which students can learn those skills (Clinkenbeard, 2015). Additionally, a liberal arts mathematics course should engage students, through making mistakes and perseverance, in authentic exercises (Bremser, 2014).

The Mathematics Association of Two Year Colleges (MATYC) published *Beyond Crossroads* in 2006, filled with recommendations for implementing effective general education mathematics in either a two-year college or the first two years of a four-year college experience; one recommendation in the MAYTC report is to challenge educators and colleges to create an optimal mathematical learning environment which challenges all students. Instead of focusing on lower-level cognitive tasks, including memorization and rote learning, introductory classes should give students opportunities "to engage in higher cognitive tasks" which develop critical thinking skills applicable to real-life situations (Kim et al., 2013). Mathematics education should help prepare students to think deeply and communicate effectively about mathematical concepts which are applicable in work and everyday situations (Gravemeijer et al., 2017). College general education mathematics courses are the culminating class for students, and equip students with "critical thinking and problem-solving skills to be engaged citizens and to attain high-paying, in-demand jobs" (Gates Foundation, 2022, para. 2). These mathematics courses should give students the opportunity to develop their *artes liberals* habits of mind.

Students in liberal arts general education mathematics classes include a wide range of mathematical abilities, as compared to courses for STEM majors. In a course for a STEM major, students have typically completed the same pre-requisite coursework (Pre-Calculus for Calculus 1, Calculus 1 for Calculus 2, etc.), so their mathematical abilities are tightly aligned. However, general education liberal arts mathematics students often have a variety of diverse majors, including music, philosophy, theology, history and psychology, or a STEM major such as biology, chemistry, engineering or mathematics. These general education liberal arts students can have a variety of mathematics backgrounds which typically depend on the courses which were required for their high school graduation. This wide range of mathematics abilities, attitudes and aptitudes of general education liberal arts mathematics students, who are placed together in a classroom, provides a unique opportunity to study the effects of different teaching methods on those students. This study aims to research the impact of an active constructivist teaching method, inquiry-based learning, on student understanding of key concepts, mathematical self-concept and attitude towards mathematics in a liberal arts general education mathematics classroom.

1.4 Context for Research Study

The educator who authored this study has taught mathematics at a small, religious-based liberal arts university in Southern California for the past 17 years. She has seen many students over the years, who come into the classroom with little confidence in their mathematical abilities, and who have been told they cannot – or should not – "do math." Beyond teaching them the content of the class, in which they can understand, apply and use mathematics for problem solving, a personal goal of hers has always been to help boost the self-confidence of students in the classroom, not only in their mathematical thinking, but also in their understanding of the role of mathematics in their daily life.

A goal in her teaching has always been to keep students actively engaged in the process of learning – asking them questions about the new material, giving them time to think or discuss ideas with a partner, and to pose follow-up questions or ideas. Her goal has always been to allow students time to process information, think through new ideas and discuss what they understood (or did not understand).

She attended a local conference hosted by Academy of Inquiry-Based Learning, and funded by the National Science Foundation, in June 2017. The ideas of IBL had similarities to the way she was already teaching, but was more structured, and would allow students more time to think deeply on the material, engage with their peers, and develop skills which would be applicable in their daily lives. She became interested in learning more about this teaching method, and to see if it would benefit the students in her general education mathematics classroom. A colleague asked, “Who is doing the thinking in your classroom – the students, or you?” This further challenged the educator-researcher to consider the impact of teaching with IBL at the small liberal arts university. While there was research on the impact of IBL in courses for math or STEM majors, there was no research available which showed the impact to a diverse group of students in a general education course. For this research study, the overall student achievement on course assignments, as well as the change in student attitudes towards mathematics will be considered. These will be used to evaluate the impact of IBL teaching in a general education mathematics classroom, as compared to an interactive lecture classroom.

1.5 Definitions of key terms

Constructivism. Constructivism is an approach to learning and teaching that is guided by four ideals, including the belief that learning is an active process, that new knowledge is not innate but instead is constructed by individual learners, that learning is an active process, and that learning is a personal yet social activity (McLeod, 2019).

Interactive lecturing. Interactive lecturing is a content delivery method which involves “two-way interaction between the presenter and the participants” (Steinert & Snell, 1999, p. 37). This teaching method includes the teacher delivering material while giving the students time to work on engaging activities or questions, which allows students to practice the most important material of the day, allowing students to be more active and alert participants in class (Middendorf & Kalish, 1996).

Inquiry-based learning. IBL is a teaching method guided by four teaching ideals, including “deeply engaging students, providing students with opportunities to collaborate with their peers, instructor focus on student thinking, and instructor focus on equity” (Academy of Inquiry Based Learning, 2020, para. 4). For a student in a course, inquiry-based learning involves working on practice exercises, making generalizations, or proving conjectures; this scaffolded work is typically done during class in a small group where ideas are shared (Al Mamun et al., 2020). Students present these new ideas to others in the group or to the others in the class (Dawkins et al., 2019). To support the in-class group learning, individual students solve similar practice exercises on their own out of class.

Online learning. Online learning is a teaching method which is used "to deliver instruction to students who are separated from the instructor and to support regular and substantive interaction between the student and the instructor synchronously or asynchronously" (National Center for Education Statistics, 2023, para. 3). Online learning was used in particular to deliver instruction and content to students during the COVID-19 pandemic. This research study's online learning includes synchronous live Zoom sessions with students, during which students worked with the instructor or in breakout rooms.

Mathematical self-concept. Mathematical self-concept is a person's rating of their mathematical ability, skills, interest and enjoyment of mathematics, and is considered an important component in teaching and learning mathematics (Erdogan & Sengul, 2014, p. 596).

Student participants. The student participants were students enrolled in the general education mathematics class, who consented to be part of the research study. Each student participant enrolled in the general education course through the Registrar's Office; once they were enrolled in the course, the study was explained and they were asked to volunteer to be research participants.

Instructor/educator The instructor/educator for this study is the researcher. The instructor/educator has taught collegiate mathematics for 17 years, and has a master's degree in mathematics. The instructor/educator helped develop the curriculum for "The Nature of Mathematics" course, and has taught the course since its inception at the University in 2008.

1.5.1 **Constructivism**

Constructivism is an educational theory which says that learners must actively construct or make their own knowledge, based on previous experiences and knowledge, instead of passively learning new information (McLeod, 2019). Constructivism is based on five basic principles: learning is connecting what a person knows and what he or she still needs to know, learning is social, learning is a situational, learning is metacognitive, learning relates students' activity and autonomy (Bognar et al., 2015). Constructivism in a mathematics classroom leads students to think critically and imaginatively about the material (McLeod, 2019); this cognitive constructivism can lead to deeper knowledge and understanding and relates to two of the principles from above (connecting what a person knows and what he or she still needs to know, metacognitive). Allowing students the opportunity to construct knowledge may benefit their long-term knowledge and understanding, as well as to develop their creativity and critical thinking skills. Social constructivism, in which students "construct" knowledge through collaboration with others, allows students the opportunity to develop their social skills, adaptability to different situations, and autonomy (McLeod, 2019). Thus, teaching using an instructional method that allows them the opportunity to learn and practice both social and cognitive constructivism is a powerful tool in mathematics education (Noddings, 1990). Inquiry-based learning is one method by which students can develop both social and cognitive constructivism, through engaging activities and assignments, and working on these "in communities of practice through social interaction" (Vrasidas, 2000, p. 350).

1.5.2 **The Inquiry-Based Learning model**

A potential solution to improve mathematics students' understanding of mathematical topics, perception of mathematics and mathematical self-concept is to implement inquiry-based learning in general education mathematics classes. Inquiry-based learning (IBL) is an active learning technique that has been implemented more frequently across mathematics and STEM classrooms in the US as a way to engage students in mathematics, draw on student experiences and ideas, and allow them to develop their own mathematical ideas (Laursen & Rasmussen, 2019). Inquiry-based teaching is defined by the Academy of Inquiry-Based Learning as

> the use of a wide range of teaching methods in mathematics courses consistent with courses where students are deeply engaged in rich mathematical tasks [including writing proofs, problem solving and understanding of mathematical theorems or postulates], and have ample opportunities to collaborate with peers (where collaboration is defined broadly) "Supporting Instructors," 2018, para. 2

Inquiry-based learning can include different pedagogical techniques in the classroom (Keys & Bryan, 2001), including structured inquiry, problem-based inquiry, guided inquiry, and open inquiry. Structured inquiry allows students to work in small groups to investigate a question posed by the teacher, with step-by-step guidelines supporting students to a predetermined result (Zion & Mendelovici, 2012). Problem-based learning, however, allows students to solve real-world problems while working with peers to determine what knowledge is needed, and allowing the teacher to guide their work to a final result (Hmelo-Silver, 2004). Guided inquiry allows students to investigate questions posted by the teacher, but the students must develop the processes and guidelines to solve the question, while open inquiry allows students to research and answer open-ended questions through their own investigative process (Zion & Mendelovici, 2012).

While these pedagogical techniques have different challenges and expectations, in each of these types of inquiry-based learning, students investigate and explore questions, and are expected to do, think, and talk about mathematics in the classroom (Caswell, 2017; Laursen & Rasmussen, 2019). Educators agree that IBL classrooms should focus on problem solving, and that students should collaborate in classroom activities that are student-centered yet teacher-directed, with the instructor acting as a coach (Hotchkiss & Fleron, 2014). Educators should provide questions that help students actively engage in their learning, while guiding them through their discoveries to making their own conclusions (Kirschner et al., 2006). Both structured and guided inquiry allow students to learn content and master cognitive skills, while minimizing wasted in-class time and failures (Zion & Mendelovici, 2012).

Inquiry-based learning effects have been studied among mathematics and other STEM majors, and in particular, in upper-division mathematics courses (Kogan & Laursen, 2014; Laursen et al., 2011). Research in courses for STEM majors at large public universities, has assessed learning gains in IBL courses, as compared to lecture courses (Kogan & Laursen, 2014). Research has also shown that inquiry-based learning improves students' persistence and understanding of key concepts (Laursen, 2013). Mathematics majors, taught in courses using inquiry-based learning, have been shown to improve in "mathematical thinking, persistence and collaboration" (Laursen et al., 2011, p. viii); additionally, their confidence and attitude towards upper-division mathematics courses improve. One research study found that mathematics and science majors in an inquiry-based Ordinary Differential Equations course outperformed their peers in a lecture-based ODE course (Rasmussen & Kwon, 2007). Johnson et al. found that male mathematics majors outperformed their peers in an inquiry-based abstract algebra course (Johnson et al., 2020). Freeman et al. (2013) analyzed 225 science, mathematics, engineering and STEM courses, finding that there was an increase in student performance on examinations for students who had been taught through inquiry-based learning. Each of these studies has shown that inquiry-based learning benefits students majoring in mathematics and STEM majors in ways that increase persistence and interest in mathematics (Laursen & Rasmussen, 2019, p. 130).

As these studies have shown increased student performance outcomes for STEM majors, implementing IBL in classes in upper-division mathematics classes has grown in popularity. However, researchers have yet to study or understand the impact of IBL classes in a general education mathematics classroom. Additionally, the effect of inquiry-based learning in a first-year general education mathematics course has not yet been studied (Laursen et al., 2011), and researchers have yet to determine the impact that the method of instruction has on students' achievement in a general education mathematics course.

1.6 Research objectives

Studying the effects of inquiry-based learning in a first-year general education (GE) mathematics context will further the current research on the impact of inquiry-based learning. Students in a GE mathematics classroom have a diverse range of mathematical abilities and experiences, and often have different attitudes towards mathematics and mathematical self-concept. Studying the effect of inquiry-based learning in a GE mathematics classroom will help determine if inquiry-based learning has a similar effect on student understanding and attitude towards mathematics, as research has shown for STEM majors in upper-division courses within their specific field of study.

The impact of inquiry-based learning has been studied in some contexts in higher education, but Laursen et al. (2011) write that while their findings are concrete for mathematics majors, investigation of student outcomes must be researched further, including "in other types of courses, especially lower-level and general education courses, and in other types of institutions, including … liberal arts [universities]" (p.175). Bruder and Prescott argue that first year college students are an ideal population to engage in such a course because "IBL [is] most effective when students have the cognitive ability to think critically but have not yet previously been encouraged to think that way" (Bruder & Prescott, 2017, p. 5). Dr. Phil Hotchkiss implemented some inquiry-based learning in his liberal arts course at Westfield State University, and shares anecdotal evidence that his students benefited from inquiry-based learning throughout the course (Hotchkiss, 2018), but he did not formally study the impact of the impact IBL had on his students.

This study furthers current research, as the research subjects all attend a small liberal arts university and are enrolled in a general education mathematics course. These first-year students have a wide range of abilities, aptitudes and attitudes towards mathematics, yet their varying abilities are placed together in a general education course. The effect of inquiry-based learning for these students may differ significantly from those in previous studies.

Additionally, studying the impact of student understanding of key concepts and attitudes towards mathematics, when learning through a completely online format, has not yet been studied. During the COVID-19 pandemic in 2020, university courses in the US moved to a fully online format. Colleges were required to send students off campus, and learn online, using online video communications tools such as Zoom. There were few studies of the impact that online learning had on college students in 2020-2021; since the COVID-19 pandemic, more articles have been published, but according to Google Scholar, only a handful of those articles focus on collegiate general education mathematics courses. One case study in Indonesian high schools found that the average learning outcomes and students' positive responses were greater before fully online learning (Ariyanti & Santoso, 2020, p.10).

This research will study and compare the effect of inquiry-based learning to those taught with interactive lecturing in a general education mathematics class at a liberal arts university, taught both on campus in a face-to-face classroom environment, and also synchronously online during the global COVID-19 pandemic.

1.7 Problem statement

Studies have previously shown that inquiry-based learning can positively impact student learning and affect in undergraduate mathematics classrooms for STEM majors (Freeman et al., 2013; Johnson et al., 2020; Kogan & Laursen, 2014; Laursen & Rasmussen, 2019; Rasmussen & Kwon, 2007). However, it is unknown how inquiry-based learning will impact an undergraduate general education mathematics classroom, due to the content and diverse set of students enrolled in such a classroom. Researchers have called for studies on inquiry-based learning in a general education mathematics classroom to determine the impact on student learning and their attitudes towards mathematics (Laursen et al., 2011). Educators are also looking for ways to improve student perceptions and attitude towards mathematics in a liberal arts mathematics classroom (Clinkenbeard, 2015); inquiry-based learning is a possible solution to this issue. The specific problem examined here is how inquiry-based learning impacts the cognitive and affective domains of students in a general education mathematics course, specifically at a liberal arts university.

1.8 Purpose statement

The problem described above has developed from the author's teaching experiences, attendances different conferences and anecdotal observations of teaching general education mathematics and inquiry-based learning. The purpose of this mixed methods study is to explore how the implementation of inquiry-based learning impacts the attitudes, perceptions, and understanding of key concepts of mathematics students enrolled in a general education mathematics course at a small liberal arts university.

1.9 Research questions

As described in the purpose statement above, this research aims to investigate the impact of inquiry-based learning in a general education mathematics course on student attitudes, perceptions, and understanding of key concepts of mathematics. Specifically, these questions will guide this sequential mixed methods research.

1. How does an inquiry-based learning GE mathematics course for first year students differ from a traditional interactive lecture course in terms of
   a. Students' understanding of key topics?
   b. Students' attitudes towards mathematics?
   c. Students' mathematical self-concept?
2. What impact does inquiry-based learning have on students' achievement in a face-to-face GE mathematics course at a liberal arts university, as compared to learning by interactive lecture?

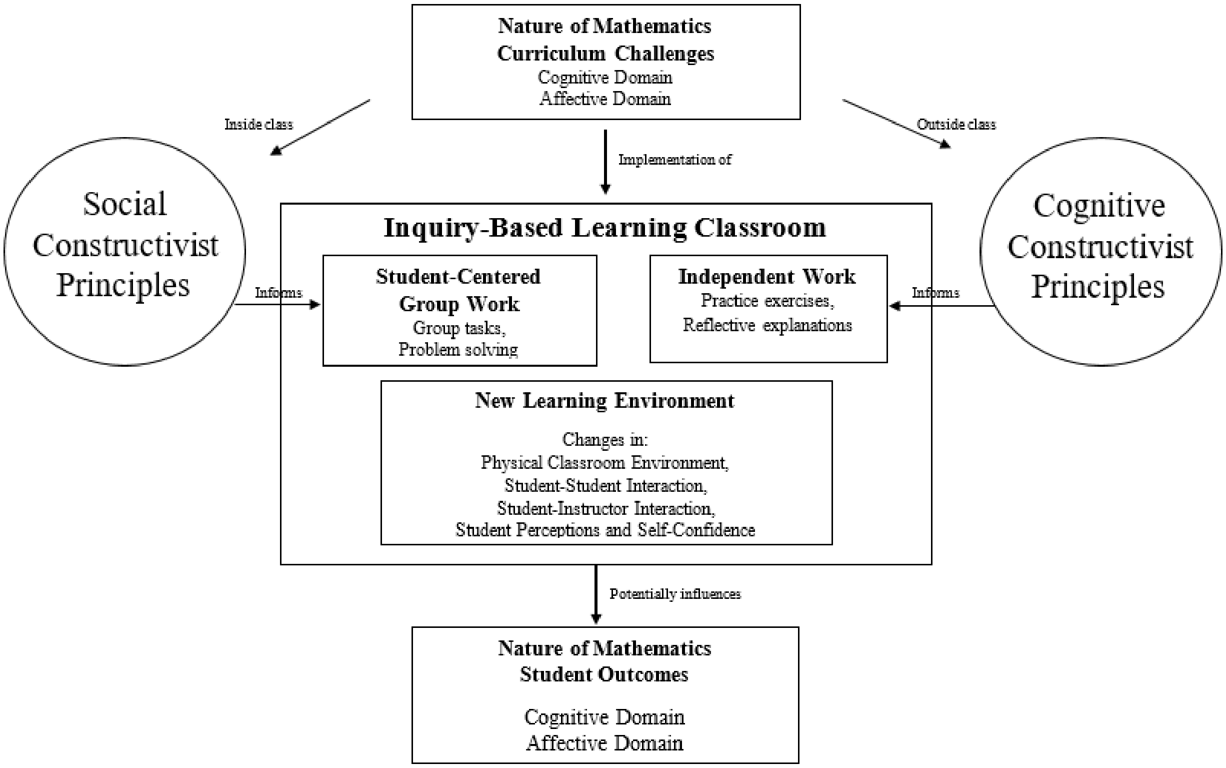
3. How do the student cognitive and affective results in an inquiry-based learning GE mathematics course differ from an interactive lecture course, in a fully online classroom?

The primary research question will be addressed using quantitative and qualitative data. The study will identify differences between student understanding and attitude of those who have participated in an inquiry-based learning course, as opposed to those who participated in an interactive lecture course. Quantitative data will be collected through assessment scores from the student participants to assess the impact of inquiry-based learning, as opposed to traditional interactive lecture, on understanding of key topics, including quizzes, exams, and the final exam. Additionally, students will be surveyed using the Attitude Towards Mathematics Inventory (Appendix A) to assess the impact of their attitudes towards mathematics and mathematical self-concept. Following the quantitative data collection, qualitative data will be collected to further explain the quantitative results. Qualitative data will be collected through group interviews of the research participants to understand and explain further the quantitative results. Using this mixed methods approach, the researcher will compare the findings of student understanding and attitude in the two groups.

1.10 Conceptual framework

Figure 1-1 is a visual representation of the conceptual framework for this research. It shows how the implementation of inquiry-based learning, using student-centered activities both in (group work, problem solving) and out (practice exercises, reflective explanations) of class affect both the cognitive and affective domains of students. These activities create a new learning environment for students and the instructor. The rationale for this conceptual framework will be further detailed in Sections 2.8 and 3.2.

Figure 1-1
Conceptual framework for this study



1.11 Significance

Efforts to improve students' mathematics learning should be informed by evidence, and the effectiveness should be evaluated systematically; such efforts should be coordinated, continual, and cumulative (Kilpatrick et al, 2001). The effects of these efforts can impact collegiate general education mathematics courses on several levels. Determining the impact of inquiry-based learning on a liberal arts general education course has potential to change similar courses specifically at other liberal arts universities; there are over 200 liberal arts universities in the United States (U.S. News & World Report, 2022). Additionally, determining the impact of inquiry-based learning could help guide curriculum and courses at other universities or schools with courses which also encompass a variety of mathematics ability in the same classroom.

Utilizing constructivist ideals, including active student-centered learning, will give students ownership of their own learning and encourage student inquiry (McLeod, 2019), allowing students an "active meaningful experience" (Bremser, 2014, para. 8), a central pillar for a liberal arts education. This study can also impact mathematics students and the perceptions they have of their own ability to do mathematics.

Determining the most efficient teaching strategy can help students become successful in their first-year general education courses. Success in first-year courses has been shown to improve student retention (Zepke, Leach & Prebble, 2006). The results of this study may contribute to a positive impact in retention rates of first year students at liberal arts universities.

Determining the effect of inquiry-based learning as compared to interactive lecturing in a first-year general education course will have significance in mathematics education. These results can benefit college mathematics educators as they develop future general education mathematics courses, and can allow others to see the effect of inquiry-based learning in a general education course. These outcomes and results can help guide more efficient GE mathematics coursework at similar institutions.

Finally, the results of this study can help further guide mathematics education in an online format. Some students in this study learned synchronously online during the worldwide COVID-19 pandemic, both through inquiry-based learning via Zoom breakout rooms and via interactive lecture. Assessing the impact of these different teaching methodologies in a fully online setting can help guide mathematics educators as they plan future online general education coursework. At the time that the researcher started collecting data for this study, there were very few studies which considered the impact of inquiry-based learning in a general education course, and in particular, a general education course which includes students from all majors and all mathematical backgrounds in the same classroom. There are many studies comparing the impact of inquiry-based learning for STEM students, for mathematics majors, or for non-STEM students; fewer studies have considered the impact for students of all majors (STEM and non-STEM) learning the same mathematics topics in the same classroom at the University level. Additionally, the data for this study was collected starting in Fall 2019, so is uniquely positioned as a study which includes student data before, during and, after the 2020 COVID pandemic.

1.12 Scope of the study

This research is to focus on the effects of inquiry-based learning in a general education mathematics course. Results are based on quantitative and qualitative data from 121 research participants, collected over five consecutive terms, in 13 different sections of a general education course at a small liberal arts university. The research was intended to be conducted in in a traditional face-to-face modality. However, during the worldwide COVID-19 pandemic in 2020-2021, two semesters were taught fully online and remotely.

The research participants were enrolled in multiple sections of the same course, and were not aware of which teaching method their course was using. The survey, Attitude Towards Mathematics Inventory (Tapia & Marsh, 2004; Appendix A), was used to assess student perceptions of their self-confidence and motivation in mathematics, as well as their perceived enjoyment and value of mathematics. The data collected from the Attitude Towards Mathematics Inventory is assumed to be truthful from each research participant, but is dependent on participants' individual introspection to assess themselves accurately. This data will be analyzed to show the impact that inquiry-based learning had on each of the research participants as he or she learned mathematics.

1.12.1 **Assumptions of the study**

There are several assumptions that were made for this research. First, it is assumed that students were randomly assigned into the different sections, and that student abilities and majors were distributed evenly among the different sections each term, and over the different terms of data collection. The research participants were all registered in the multiple sections of a course taught by a single professor over five terms, and it will be assumed that the professor taught the courses similarly each term.

Additionally, the following assumptions were made in the data collection. First, the quantitative data collected for each research participant is assumed to be an accurate measure of student understanding of the key concepts of the course. Second, the research participants who consented to participate in the research is assumed to be a representative sample of all students enrolled in the course. Additionally, it is also assumed that the research participants filled the mathematical attitudes survey accurately and honestly. Finally, it is assumed that each research participant gives truthful answers in their group interview at the conclusion of each term.

1.12.2 Study limitations

While the researcher aimed to minimize these limitations, several limitations to this research exist, and will be defined here. First, of the 283 potential research participants, 121 consented to be a subject in this study. This produced a relatively small sample size from which to make conclusions, and is potentially a biased sample of the participants, as they self-selected participation. Second, the research participants self-reported their data on the attitudes survey at both the start and end of the semester. This data cannot be verified but is assumed to be accurate. Third, quantitative data was collected throughout the semester, as the professor-researcher recorded quiz, exam and final exam scores. However, since the interviews for qualitative data were collected 5-6 weeks after the semester ended, many of the participants did not participate, even though multiple email reminders were sent, as well as offers for remuneration for their time. Despite these attempts to encourage participation, there is limited qualitative data due to low participation. Additionally, data for this research was collected from Fall 2019 until Fall 2021, which included two semesters (Fall 2020 and Spring 2021) which were taught fully online and remote due to the global COVID-19 pandemic. Teaching online was an unplanned teaching modality, and an additional limitation of this research study.

1.12.3 Study delimitations

This study is limited to the self-selected research participants over five consecutive terms, starting in Fall 2019, who were enrolled in liberal arts university. The general education mathematics course included students from all majors, and covered abstract liberal arts mathematics topics, focusing on the value and centrality of mathematics in everyday life, but not focusing on algebraic skills for future mathematics courses. Research variables not considered include students’ academic majors as well as their academic background. This study does not consider the impact the inquiry-based learning has in students’ future mathematics coursework.

1.13 Organization of the thesis

In this chapter, a context for this research of general education mathematics courses at a small liberal arts university is provided, as this study will investigate the impact of inquiry-based learning as compared to interactive lecture, and the purpose of this study is defined. In the following chapters, existing research will be discussed in Chapter 2, and a review of existing scholarly literature around inquiry-based learning will be explored. In Chapter 3, the method of this study, the research design, data collection and statistical analysis methods will be described. In Chapter 4, a summary of the results of the findings will be presented. Finally, in Chapter 5, the answers to the research questions will be discussed, as well as the implications of this research on future students, courses and educators.

2 Literature Review

This chapter examines the current research literature surrounding best practice of instruction for a general education mathematics course at a four-year university. Current research outlining constructivism as an engaging instructional tool is presented. Two constructivist methods of instruction, a more traditional interactive lecture style and an inquiry-based learning (IBL) style, are compared using current research in mathematics and other scientific classrooms. Next, research on student mathematical self-concept and attitude toward mathematics, based on different instructional methods, are compared. Finally, the impact of remote online learning is reviewed. This literature review presents the findings of the current research, and explains the research gaps of evidence regarding the impact of inquiry-based learning in undergraduate general education mathematics courses.

2.1 Common issues in a general education mathematics classroom

General education courses at a liberal arts university typically account for approximately 30% of a student’s coursework at the university (Brint et al., 2009). Liberal arts universities often require at least one general education mathematics class, and the most common courses to fulfill the requirement are college algebra, geometry, trigonometry, calculus, or a quantitative analysis course. Many students see these courses as a “roadblock” to other fields (Shakerdge, 2016), and about 50% of college students do not earn a C, to pass College Algebra (Saxe & Braddy, 2015). Students find mathematics to be difficult, boring, and irrelevant, without application to their further studies or careers (Fielding & Makar, 2008). These courses are often taught via direct lecture where students become passive learners and are more likely to fail than in other courses (Bajak, 2014). A traditional college algebra or trigonometry is prerequisite for students continuing into Calculus, but fewer than 10% of students enrolled in college algebra or trigonometry enroll in Calculus 1 (Small, 2009). The Mathematical Association of America has called for educators and universities to implement mathematics courses in the first two years of collegiate study which allow students to communicate their mathematical ideas orally and in writing, and to increase student quantitative and logical reasoning skills (Saxe & Braddy, 2015). In many liberal arts mathematics courses, students typically investigate topics which allow them to understand and appreciate the application of mathematics to real-world situations, and how mathematics connects to other subjects (Cook & Garneau, 2017). The appreciation of mathematics, in particular, can be difficult to instill in college students, as they often have previous misconceptions about mathematics, including that mathematics is irrelevant to the real world, that mathematics problems have one correct method to solution, and that solving mathematics problems is solitary (Szydlik, 2013; Cook & Garneau, 2017).

2.2 Constructivism

Constructivism is a theory of knowledge and how individuals learn; constructivism does not define any one teaching method. While it is considered a relative new theory of knowledge and thinking, some may argue that the idea of constructivism began with Socrates. Socrates posed questions for his students to answer through a series of questions and dialogue, allowing them to construct their own knowledge. However, the modern day concept of constructivism was influenced by several constructivists, including Jean Piaget and Lev Vygotsky. Piaget and Vygotsky are considered two of the “fathers” of constructivist theory (Obi et al., 2019).

Jean Piaget (1896-1980) was a Swiss psychologist and biologist, whose focus was studying where learning and knowledge are constructed and developed (Sjoberg, 2010). He performed many experiments with children, connecting their previous experiences and knowledge, to the learning of new, advanced skills (Alhabib, 2021). Piaget’s theory of learning is “cognitive constructivism,” in which a student develops new knowledge through asking questions, analyzing, and interpreting results and patterns (Khan, 2019, p. 12457). He believed that people use their own current knowledge to develop new ideas and construct new knowledge. Piaget believed children were operational thinkers, who process their learning through three stages of intellectual development: preoperational, concrete and formal operational (Matthews, 2003). All children must go through these same three stages to fully develop and learn (McLeod, 2023a).

However, Russian psychologist Lev Vygotsky (1896-1934) had a different viewpoint. Vygotsky believed that social learning precedes one’s development (McLeod, 2023a). Vygotsky’s research focuses on social constructivism, in which a student’s new knowledge comes through social interactions with their peers and their teacher (Khan, 2019). He believed that social collectivity has a central role in individual learning and development (Lui & Matthews, 2005, p.389), and that speech and reasoning in social interactions helped children learn, and that the “social environment is ingrained with the child’s learning” (McLeod, 2023a, para. 12). Vygotsky’s theory of learning is known as “social constructivism”; he believed that children were influenced by the social environment in which they grew up, and that language and speech are a fundamental part of the learning process. Vygotsky also believed that students learn best in their zone of proximal development, defined as the “distance between the actual development level … and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers” (Vygotsky, 1978, p. 86).

Both of these constructivists believe that “learning is an active process” (Tam, 2000, p. 50), and that knowledge is “actively constructed by learners based on their existing cognitive structures” (Teaching Guide for GSIs, 2016, p. 5). Learning should be driven by internal motivation from the students, and not depend on rewards or punishments from the teacher (Matthews, 2003). Constructivism theory aims to “cultivate the learners’ thinking and knowledge construction skills” (Charalambos, 2000, p. 352). Student thinking and knowledge requires “rich learning environments that help translate the philosophy of constructivism into actual practice” (Tam, 2000, p. 54). Constructivist approaches to teaching and learning are based on ideas of shared cognition, discovery learning, collaborative learning and problem-based learning (Duffy & Cunningham, 1996). Piaget’s cognitive constructivism depends on students’ previous knowledge and asking questions, thinking deeply about concepts, and developing new ideas, while Vygotsky’s social constructivism requires interaction with peers and opportunities to talk with others when developing new ideas.

A classroom based on constructivism requires teachers and students to share knowledge and authority in the classroom; the teacher acts as a facilitator, and small groups act as a learning community (McLeod, 2019). Class time should be filled with broad questions and ideas which are tied together through “tasks, activities, and assignments that students are engaged in” (Charalambos, 2000, p. 353). Students should engage in learning experiences which allows them not only to actively construct knowledge through their own activity (cognitive constructivism), but also to share their ideas with others (social constructivism) (Serafin et al., 2015). Engaging activities, in which students are interacting with their peers as they develop new ideas, uses Vygotsky’s theory of social constructivism and allows students to develop intellectually while developing new skills and ideas (Obi et al., 2019). Experimentation, through activities with physical objects and focused questions, utilizes Piaget's theory of cognitive constructivism and allows students to build new knowledge from their previous ideas and experiences (Alhabib, 2021).

An ideal constructivist learning environment to increase student achievement should encompass both of these viewpoints – Piaget's cognitive constructivism and Vygotsky's social constructivism (Blake & Pope, 2008). Students will be challenged in a cognitive and social constructivism environment in which they can internally generate then ask questions, actively participate in the classroom, engage in dialogue with peers, and think and reflect on the new ideas and concepts (Khan, 2019). Two different cognitive-social constructivist methods, interactive lecturing and inquiry-based learning, will be compared in this study.

2.3 Constructivist instructional methods

Students must be engaged in an active learning process, where they are constructing knowledge, and making conclusions from new experiences and of how the world works (McLeod, 2019; Theobald et al., 2020). Active learning is grounded in constructivist theory, which holds that students "learn by actively using new information and experiences" to understand and explain how the world around them works (Theobald et al., 2020, p. 6474). Active learning has been shown to actively engage STEM students with mathematical ideas in the classroom, which increases student learning, attitudes towards mathematics, retention of mathematical concepts, and their persistence in mathematics (Yoshinobu et al., 2023). This study will focus on a traditional "interactive lecturing" teaching style which blends traditional and constructivist classroom ideals, and a student-centered "inquiry-based learning" as an active learning style. Both interactive lecturing and inquiry-based learning are considered active learning techniques, involving students not only actively working on tasks, but also thinking about what they are doing (McLeod, 2019).

2.3.1 Interactive lecturing

Lecture has been used in higher mathematics classrooms for centuries (Bajak, 2014) and recent studies have shown that "this approach still dominates undergraduate STEM courses in North America" (Theobald et al., 2020, p. 6475), but more interactive instructional techniques have been implemented in the last decades, including inquiry learning and constructivist learning to teach collegiate mathematics (Prince, 2004). Saxe and Brady have called for instructors to "move away from the use of traditional lecture as the sole instructional delivery method in undergraduate mathematics courses" (2015, p. 19). Interactive lecturing requires interaction between the instructor and students and allows for "learning beyond the recall of facts [because] students must be attentive and motivated" (Steinert & Snell, 1999, p. 37). Interactive lectures are classes in which the educator directly presents material to students, but in which there are opportunities or breaks in which students are able to interact with the educator or peers in the classroom. These opportunities or breaks allow the students time to work or reflect on the material presented.

In an interactive lecture course, an educator presents information to students, building new information on material known to the students. However, the educator will allow the students frequent opportunities to ask questions, and to think about or discuss the new concepts. These breaks can come through a variety of different methods; common techniques in an interactive lecture course, which have shown to improve student performance and interaction, include using pairs or small groups, frequently questioning the audience, then using those responses to guide instruction, and using written materials and handouts (Steinert & Snell, 1999). First, the educator can stop and give the students a few minutes to reflect on the material presented, then form and pose any questions they have about the new material. The educator could also use pairs or small groups; in this technique, the instructor would pause instruction for a few minutes, and allow students to summarize the new material presented. An educator can also frequently question the audience – asking questions every few minutes to the class, which keeps students focused on the material and ready to answer questions to ensure understanding. In each of these techniques, the educator gives students time to reflect, recall, review and discuss new concepts in small segments throughout a course period.

Students in an interactive lecture course are more engaged than in a traditional lecture course. As the educator provides breaks in the presentation of new material, a student can have a few moments to process the new material, and to think of any questions that have arisen. Having time to process the new material, then discuss or review it with a peer gives a student time to ensure they have understood the topic, and can restate the information in their own words.

Having moments to process the new information allows the student opportunity for cognitive constructivism; having time to discuss the new material with others, verbalize their responses, and pose questions gives the student opportunity for social constructivism (McLeod, 2019; Sjoberg, 2010).

Interactive lecturing has been found by students to be more engaging than traditional lecturing (Doherty, 2007), and this student-centered technique allows educators to “attempt to incorporate many of the factors associated with both enhancements of clarity and generation of interest” for students (Short & Martin, 2011, p. 72). Interactive lecturing allows the instructor to teach information via mini-lecture, providing time for students to ask questions to the teacher and interact with peers, and encourages students to work on related exercises during class, either individually or in small groups (Rodger, 1995). Students in a collegiate physiology course taught with interactive lecture were shown to be more engaged, interacted more in the classroom, and scored higher than those in didactic lectures (Ernst & Colthorpe, 2007). Additionally, Short & Martin (2011) found that interactive lectures in tertiary psychology courses generated higher interest from students, encouraged better attendance and resulted in higher course grades. In a similar study, Bajak (2014) found that undergraduate STEM students, taught with active learning strategies, engage highly in their coursework and have a high understanding and strong performance in the course. Overall, interactive lecturing provides students an interactive learning environment, which has a positive impact for students in STEM fields (Braun et al., 2017).

2.3.2 **Inquiry-Based Learning**

A more “ambitious active learning environment” for constructivism learning (Braun et al., 2017, p. 126) is an inquiry-based classroom; inquiry-based instruction includes “a wide range of strategies used to promote learning through students’ active, and increasingly independent investigation of questions” (Arsal, 2017, p. 1326). Inquiry-based learning focuses on exercises which require “critical and creative thinking so students can develop their abilities to ask questions, design investigations, interpret evidence, form explanations and arguments, and communicate findings” (Australian Government Department of Education, 2023, para. 2) and is considered a more equitable form of mathematics instruction (Kuster et al., 2018). Additional studies of the impact of inquiry-based learning supports active learning as a “mechanism for both improving student performance and supporting women in undergraduate mathematics" (Reinholz et al., 2021, p. 3).

The structure of an inquiry-based classroom differs from a traditional classroom environment, where the instructor is typically in the front, center of the classroom. Instead, students are actively engaged in the classroom working together in small groups "making meaning of the new mathematical ideas they encounter" (Starbird, 2018, para. 4). In a well-constructed IBL classroom, students should be engaged "in problem solving, explaining, and critiquing as they learn key disciplinary concepts and approaches" (Yoshinobu et al., 2023, p. 330). Student engagement is prioritized through an authentic, engaging mathematical exercises, with a decentralized classroom authority (Johnson et al., 2018). This allows the instructor to utilize a student-centered approach, based on cognitive and social constructivist principles, where students work together, searching for conclusions and patterns and to use their understanding to construct knowledge (Calleja, 2016).

Students in an inquiry-based learning course are actively engaged with their peers in small group problem solving. They work collectively on mathematical tasks, which have been designed by the instructor to be accessible to the student, with careful questioning by the instructor to allow students to use previous knowledge to solve problems, make conclusions, and ask questions on their own (Calleja, 2016). These mathematical tasks are typically one of the following: to introduce a new topic, to make a conjecture, to prove a theorem, to recognize and develop patterns, to practice routine or non-routine exercises, or to develop deeper understanding about a known concept (Ernst et al., 2017). Ideally, students work in groups of 3-4, allowing each individual the opportunity to generate new ideas, participate in the group discussion, and listen to the ideas of others.

The role of an educator is different in an inquiry-based learning class. An educator becomes more of a "guide on the side," as opposed to a "sage on the stage" as in a typical classroom (White-Clark et al., 2008, p. 40). The educator still prepares the lessons for each class, with specific learning outcomes and goals for the class period, and typically prepares course notes, consisting of directed questions and guided prompts. The educator gives the students the course notes, and allows them to work in their small groups, sharing ideas; while the groups are discussing new ideas and making conclusions, the educator is listening to the groups and roving the classroom, supporting groups who need guidance by giving them nudges or asking questions. The educator listens to the students' ideas and redirects their thoughts when necessary. This classroom looks different than a typical classroom, as the educator is not the focus of the classroom; instead, the discussions occurring among the student-led groups are the focal point.

"Inquiry-based learning" has not widely been used in the teaching and learning of college mathematics, but the practice has been shown to improve students' achievement and persistence (Laursen, 2013) and has been continually modified and adapted to meet the needs of different classrooms and environments. Inquiry-based learning also requires students to communicate with one another what they are working on and thinking; these "conversations are powerful in clarifying, solidifying, and elaborating learners' ideas" (Laursen & Rasmussen, 2019, p. 130). The IBL method of teaching has also been shown to improve learning outcomes and, ultimately, student retention in college mathematics (Laursen, 2014) in all levels of mathematics. Advocates of IBL believe learning with inquiry-based learning in the first-year of university will allow students to experience a strong student-centered learning environment, and that this experience will help students develop skills they can use throughout their university study (Spronken-Smith, 2008). The IBL method of teaching has been shown to positively impact students' higher order thinking skills at a polytechnic university in Africa, and improved students' conceptual understanding of mathematical topics (Abdurrahman et al., 2021). Overall, IBL has been shown to have a positive impact in STEM classes at large universities.

2.4 Inquiry-based learning classroom environment

In a traditional inquiry-based learning (IBL) classroom, students actively participate in contributing their mathematical ideas to solve problems, rather than applying teacher-demonstrated techniques to similar exercises (Yoshinobu, 2012). Inquiry-based learning has been used for years in advanced mathematics courses which rely on rigorous argumentation and proofs (Ernst et al., 2017). Activities which pique student curiosity and material relevance should be included, "connecting concepts and procedures, motivating students, building open inquiry, combining different forms of reasoning, developing initiative, independence and student leadership" (Handayani et al., 2018, p. 2). Current research suggests that students studying mathematics (either as a mathematics major or a major in a related field) benefit from inquiry-based learning, as they are asked to think deeply about mathematics problems while collaborating to determine a solution; students are asked to explain their reasoning and are challenged to think critically about each problem presented (Hassi et al., 2011). Inquiry-based learning encourages students to “take responsibility for directing the lesson with the teacher guiding the student’s mathematical activities in the classroom” (Handayani et al., 2018, p. 2). Students become responsible for presenting new ideas to their peers, developing new ideas and procedures, determining the completeness and validity of a solution, asking questions, and making conjectures (Ernst et al., 2017).

Additionally, active participation with the material allows students to use what they have learned, leading to deeper understanding (University of Copenhagen, 2021). Engaging students with the material in upper-division economics courses improves the educational outcomes of the courses and has “positive student outcomes” (Flaherty, 2020, para. 12). Additional studies have shown that students who have taken an inquiry-based mathematics course are more likely to enroll in future mathematics classes, and have a larger increase in grades from pre-IBL course to post-IBL course (Kogan & Laursen, 2014). Research in medical and science majors has demonstrated “that students learn most when they actively engage with the material” (University of Copenhagen, 2021, para. 8) by asking questions or pairing with a classmate. These learning outcomes have been studied extensively, particularly at large public universities that house “IBL Centers.”

Researchers have studied the effects of IBL classrooms in a variety of different courses and at a variety of universities, and have made conclusions about the differences in student attitude toward mathematics, grades (in both the IBL course and future courses), and differences in achievement by gender. Some studies have suggested that inquiry-based learning helps foster success across STEM majors, and fosters positive attitudes about mathematics (Ernst et al., 2017). These studies have been completed at large, public research universities in courses within a mathematics major or related STEM field. Findings from these studies, including the IBL Project and data from IBL research centers, have shown a benefit to student learning, retention and attitude towards mathematics (Kogan & Laursen, 2014).

2.4.1 Comparison of classroom learning environments

Table 2-1 compares the two classroom environments that were utilized in the research. The traditional classroom is more teacher-centered, with the teacher disseminating information to students, and the fixed curriculum is paced for the whole class. However, the active constructivist classroom is more student-centered, with small groups of students working through the new material, and the curriculum is paced based on the group's previous knowledge and new discoveries (McLeod, 2019).

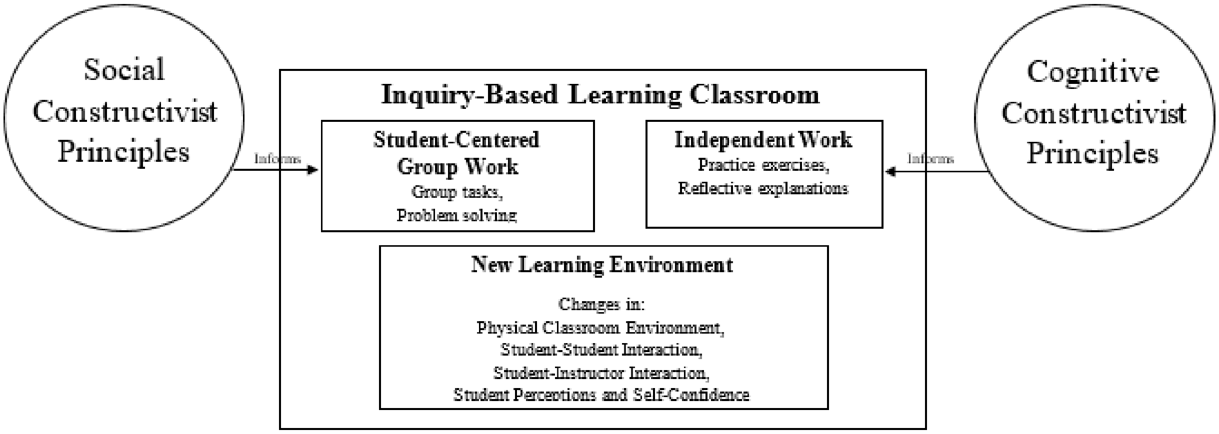
Table 2-1
Comparison of interactive lecture and IBL classroom learning environments

Traditional (Interactive lecture)	Active constructivist (Inquiry-based learning)
Teacher presents new concepts to the class through notetaking and examples.	Teacher prepares guided notes, facilitates discussions and student discovery.
Teachers uses interactive question techniques and allows time for student questions.	Teacher gives students tasks requiring them to experiment, conjecture, and explore mathematical concepts.
Students perform practice exercises in class, practicing after new concepts are presented.	Students guide group discussions, and explain solutions collaboratively.
Students sit in traditional rows.	Students sit in groups of 3-4 in classroom.

2.5 Application of Inquiry-Based Learning within the Conceptual Framework

This study implemented an inquiry-based learning environment in a first-year tertiary mathematics course, as presented in Section 1.10. This section will discuss basic principles of inquiry-based learning as it relates to the conceptual framework of this study. The principles used in this study and their relationship to social and cognitive constructivist principles, are shown in Figure 2-1.

Figure 2-1
Inquiry-based learning classroom basic structure



2.5.1 **Student-centered group work (during class)**

While there are different levels of inquiry, this study focused on structured inquiry, in which the students followed the educators' direction and prompts, to make conclusions and find patterns (Rooney, 2012). For this research, course note packets were given to students in the inquiry-based learning classroom (experimental group) each class period, which outlined the mathematical tasks for each day. These course notes included guiding questions, which allowed students to work in small groups (of 3-5 students) to make generalizations and conclusions about the material. This scaffolding was built on Vygotsky's zone of proximal development, allowing students to build upon their previous knowledge, while working with peers (McLeod, 2023b). Working with capable peers on challenging tasks allowed students to build both social and cognitive constructivist principles. While these course notes were educator-created, the notes provided questions, guided inquiry, and guided tasks for the students, leading the students to "discover" (or rediscover) important mathematical ideas (Kuster et al., 2018; Reinholtz et al., 2021). These educator-created notes provided scaffolded exercises to address specific student questions and understanding, gave students opportunity for repetition of key ideas and for immediate feedback on the tasks (Theobald et al., 2020). Engaging students with engaging tasks in which they both argue for and justify their thought process and try to understand the reasoning of others, improves student understanding (Kuster et al., 2018).

These course notes covered the same material as was covered by PowerPoint in the interactive lecture classroom (control group). For the terms which were taught synchronously online, these course notes were posted in the content management system (Blackboard) for students ahead of time, and the students worked in small groups (3-5 students) in Zoom breakout rooms.

2.5.2 Student-centered independent work (outside of class)

After making conclusions and working in groups during class time, students had individual work to complete at home. Having already been introduced to the material in class, these homework exercises allowed students to further engage with the material, making more conclusions, working additional exercises, and engaging in higher-order thinking about the mathematical ideas. Students were given unlimited time to work these at-home exercises, and were supported by university tutors and the instructor, if they needed guidance. Students who needed additional support and guidance to complete homework exercises and prepare for upcoming assessments were given the opportunity to sign up for weekly small group tutoring with a university tutor. These small groups, which included 1-3 students with one tutor, were 30-minute time slots in which students could ask questions about the material covered in class, work homework exercises, and prepare for upcoming exams. All students in the class (both the participants and non-participants in this study) were given the same access to these tutors to assist them in achieving the course objectives.

2.5.3 New learning environment paradigm

The inquiry-based learning classroom was a new learning environment for the majority of students enrolled in the class. The students were required to adjust their expectations of themselves and the classroom. The classroom was arranged so students focused on one another instead of on the instructor, changing the physical learning environment. While many students had always taken copious notes and repeated an algorithm presented by the instructor, in this environment they were required to engage with their peers and build new ideas from their previous experiences through interactions with their group. They worked in small groups of 3-5 students, following teacher-generated prompts and questions in their course notes, in order to generate new ideas and formulas on their own and in a small group. Some of the results of improvement in student understanding and achievement can be explained by the intensity of an inquiry-based classroom, in which there is a higher percentage of time in which students are actively engaged in classroom activities (Theobald et al., 2020, p. 6478).

Figure 2-2
New environment created by inquiry-based learning

New Learning Environment

Changes in:
Physical Classroom Environment,
Student-Student Interaction,
Student-Instructor Interaction,
Student Perceptions and Self-Confidence

2.5.4 Student outcomes

Inquiry-based learning is intended to engage students with rich mathematical tasks by allowing students to build ideas based on their previous knowledge and experiences. Students not only learn the content, but can also learn higher levels of content application, collaboration with their peers, creative thinking, and problem solving skills (Love et al., 2015, 759). This was done through completed course notes, which they completed in class through small group discussions and discovery. In this way, inquiry-based learning may influence student outcomes in both the cognitive and affective domains.

Figure 2-3
Potential influences in mathematics course, taught by inquiry-based learning

Nature of Mathematics
Student Outcomes

Cognitive Domain
Affective Domain

2.6 Mathematical self-concept

Academic self-concept is described as a “mental representation of one’s abilities in academic domains and school subjects” (Brunner et al., 2010, p. 964) and is regarded as an influential factor which affects one’s behaviors, cognitive outcomes, academic achievement, and happiness (Marsh & Martin, 2011). More specifically, mathematical self-concept describes "how sure a person is of being able to learn new topics in mathematics, perform well in mathematics class, and do well on mathematics tests" (Reyes, 1984, p. 560). Mathematical self-concept is based on student perceptions about themselves, their abilities, and their competence in mathematics (Parker et al., 2014; Passiatore et al., 2023), but is likely intertwined with their achievement in mathematics classes (Marsh & Martin, 2011). For example, students may have a better mathematical self-concept if they perceive that they have stronger mathematics skills than their peers, and they may have a lower mathematical self-concept if they perceive their mathematics skills are weaker than their peers (Marsh et al., 1991). Mathematical self-concept can be shaped "with the social and physical environment over time" and is based on a variety of factors including mathematics achievements, quality and quantity of instruction, classroom environment and mathematics attitude (Erdogan & Sengal, 2014).

Mathematics is considered to be one of the most challenging subjects that students study, and researchers have found that a positive attitude towards the learning of mathematics is essential (Felix et al., 2022). This relationship between self-concept, abilities, and achievements in mathematics is frequently studied in educational psychology (Parker et al., 2014). Passiatore et al. (2023) have found that students with a low mathematical self-concept "may be less motivated to perform, less willing to make efforts and accomplish tasks, and will tend to avoid mathematical situations" (p. 1146), while students with a high mathematical self-concept have less anxiety and more enjoyment in mathematics.

However, educators have been trying to determine the connection between a positive attitude and self-concept, but the correlation has not yet been determined (Ma & Kishor, 1997). There is empirical evidence that there is a correlation between a positive mathematical self-concept and learning achievement (Passiatore, 2023). A research study of 8th graders in Indonesia found that mathematical self-concept has a significant impact on academic achievement; a higher mathematical self-concept results in higher academic and learning achievement (Agustina, 2024).

Mathematical self-concept is part of the affective domain, and will be measured using Martha Tapia and George Marsh's research instrument, Attitude towards Mathematics Inventory (Tapia, 1996), which measures student attitudes in mathematics based on four factors: self-confidence, value, enjoyment and motivation. The Attitude towards Mathematics Inventory will be given to research participants at the start of the semester, and at the end of the semester, to consider any change in mathematical self-concept over the 15-week term.

2.7 Online learning

Learning remotely online became a global necessity in March 2020, and researchers quickly studied the impact of quickly changing to this teaching method. Higher education classes implemented emergency online learning in early 2020, as mandated by federal, state, and local guidelines. This rapid move to online learning was necessitated by the global pandemic, but presented some unusual situations for colleges and college students. First, the move to online was rapid and unplanned; colleges moved to online learning in the middle of a semester. Both educators and students had to quickly change modalities to teach and learn online. College students were sent home from dorms, which isolated many students from others. Both of these were new situations which may have impacted student learning.

Universities and colleges made this sudden switch in March 2020 to distance, remote, and online learning for all students. Students and educators both adapted quickly to new learning modalities and settings, with little advanced notice (Aliyyah et al., 2020; Weiner et al., 2021). These new educational settings due to safety, health, and hygiene included online learning, social distancing, limitations with interactions with peers and classmates, limited access to traditional classroom settings and materials, and isolation (Moliner et al., 2021). Students and faculty experienced greater challenges teaching and learning online during the COVID-19 pandemic than in traditional face-to-face classrooms (Capinding, 2022). Research at a large California university found the greatest challenges faced by students included disconnectedness and isolation, lack of engagement, perceived achievement in courses, and technological issues (Bonsangue & Clinkenbeard, 2021). A study of high school students in Spain found that the sudden change to online, as well as monotony, isolation, and lack of responsibility presented challenges to mathematics students (Moliner et al., 2021). Achievement of high school students taught online during the COVID-19 pandemic was impacted by individual student motivation, attitude, anxiety, and interest (Capinding, 2022).

The University of Copenhagen studied the impact of online and blended learning for 282 lectures in the science and medical fields early in the global pandemic (University of

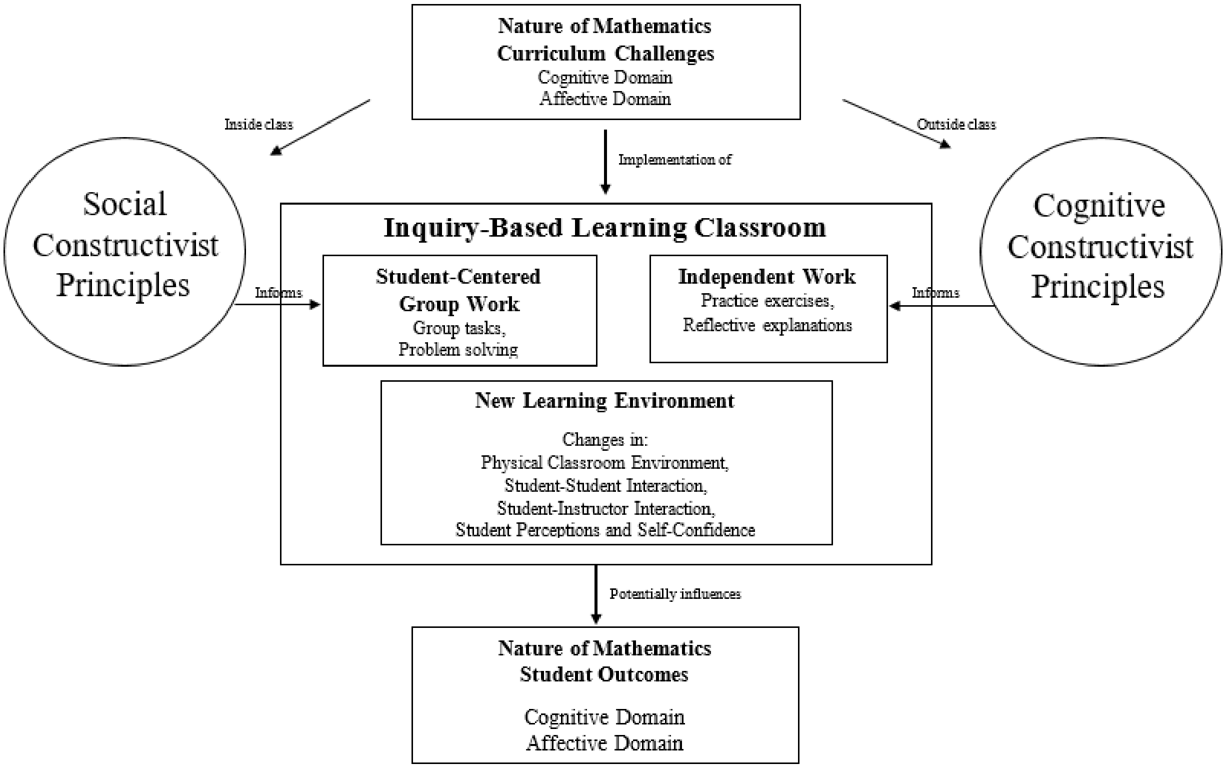
Copenhagen, 2021); results showed that online students were very active in class participation, and found that students in lecture courses were often more engaged, as they were able to ask questions in the chat feature instead of "in a physical space, where students don't want to be pegged as the 'he or she that asked a dumb question'" (University of Copenhagen, 2021, para. 5). However, at a study in the Philippines, online learners were found to have negative self-concept about their mathematical abilities and had trouble problem solving (Bringula, et al., 2021). Over 40% of students felt that their understanding of key concepts and overall grade would be lower in the online setting, as compared to a face-to-face setting (Bringula et al., 2021). A similar study of the impact of online learning in Indonesia found that students' understanding of key concepts was lower through online learning, and students' self-concept towards mathematics was also lower through online learning (Ariyanti & Santoso, 2020).

The research for the impact of the remote online learning during the COVID-19 global pandemic is limited, particularly for student achievement at the university level (Moliner et al., 2022; Znidarsic et al, 2022). For the research in this thesis, the impact of student understanding of the key mathematical concepts, as well as their attitude towards mathematics and mathematical self-concept for participants who attended class fully online will be compared to those who learned in a face-to-face classroom environment.

2.8 Conceptual framework

The conceptual framework in Figure 2-4 is based on constructivist and cognitivist principles, giving students experiences which will guide their learning both inside and outside the mathematics classroom. The framework design was based on existing data and theories, and teaching experience of the researcher. Each component of this framework will be discussed in further detail throughout the chapter.

Figure 2-4
Conceptual framework for this research study



Data will be collected both in the cognitive and affective domain, measuring not only what students have learned and demonstrated in the classroom on different assessments, but also their perceptions of their mathematical self-concept and ability to solve problems. These will be measured before and after the students take a one-semester undergraduate mathematics course taught with inquiry learning, and compared to students who took the same course taught by interactive lecturing. Two different modalities were used in this study – three semesters were taught face-to-face, and two semesters were taught fully online during the COVID-19 pandemic.

2.9 Chapter summary

Inquiry-based learning has been shown to improve student understanding of mathematical topics, persistence in mathematics and improved student confidence and attitude towards upper division mathematics courses (Laursen & Rasmussen, 2019, Laursen, 2013, Laursen et al., 2011). Students enrolled in previous research studies were enrolled in upper division mathematics classes and were pursing STEM majors; however, the impact of inquiry-based learning on students in a general education mathematics class have not been studied. In a general education mathematics class, students of all majors and disciplines, and with varying skills and backgrounds, are brought together in a single classroom.

While previous studies for STEM students have shown to improve student performance, attitude towards mathematics and self-perception of students who have learned mathematics by inquiry-based learning in face-to-face learning environments, studies in different environments have not been studied. This study will further that research, comparing students in a general education mathematics class in different instructional methods, also comparing differences between face-to-face and fully online learning. This study aimed to fill the gap in research literature by studying a diverse group of students which had not been studied before in this context, and in both online and face-to-face modalities.

This research seeks to improve the effectiveness of general education mathematics instruction by studying the cognitive and affective differences in students in different learning environments. Specifically, this study aims to answer the following research questions:

RQ1: How does an inquiry-based learning GE mathematics course for first-year students differ from a traditional interactive lecture in terms of students' understanding of key concepts, attitudes towards mathematics and mathematical self-concept?

RQ2: What impact does inquiry-based learning have on student's learning in a face-to-face GE mathematics course at a liberal arts university, as compared to learning by interactive lecture?

RQ3: How do the student cognitive and affective results in an inquiry-based learning GE mathematics course differ from an interactive lecture course, in a fully online classroom?

The goal of this study is to compare the effects that an inquiry-based general education mathematics course has on student understanding of key topics, attitude towards mathematics and mathematical self-concept as compared to an interactive lecture-based course which teaches the same topics. This research is designed to understand the impact of inquiry-based learning in a general education mathematics class in a diverse classroom at a liberal arts university. Quantitative and qualitative data will be used to determine the impact on students' understanding of key concepts, student attitudes towards mathematics and their mathematical self-concept. Thus, a mixed methods design will be used, incorporating both qualitative and quantitative research methods. The research design and methodology will be discussed in Chapter 3, then the data and results will be presented in Chapter 4. Finally, Chapter 5 will include conclusions and recommendations based on the data from this research study.

3 Research Design and Methodology

3.1 Purpose

As discussed in Chapter 2, this study aims to fill the gap in existing research literature by studying a general education mathematics classroom, which has students of mathematically diverse backgrounds. While several studies have been conducted in STEM-specific classrooms, a population of general education students has not been studied before in this context. Additionally, this research will include students who took the general education mathematics course in both online and face-to-face formats. In this chapter, the methodology and philosophical foundation on which this research is grounded will be described, followed by the methodological approach and the research design used in this study.

This study used a developed and validated research measurement, the "Attitudes Towards Mathematics Inventory" (Marsh, 2005; Tapia, 1996; Tapia & Marsh, 2004) which was not only designed to be a short, easy-to-complete survey, but also to assesses multiple factors of a participant's attitude toward mathematics. The inventory assesses student self-confidence, perception of the value of mathematics, enjoyment of and motivation in mathematics. The study also assessed student understanding of major concepts through routine and non-routine assessments in the class. Additionally, small group interviews of the participants helped the researcher understand student perceptions of the classroom environment. This chapter will introduce the research methodology for this mixed methods study to collect and analyze data, to determine the impact of inquiry-based learning on first year general education mathematics students. This approach provides information to understand which teaching method most impacts student learning, student self-concept and attitude towards mathematics. In the following sections, the specific research methods used in this study will be described, including the setting and sample used, the data collection and storage methods, and data analysis. This will include the validity of the data and the role of the researcher-instructor in this study.

3.2 Mixed methods research design

Inquiry-based learning has been shown to improve student understanding of mathematical topics, persistence in mathematics and improved student confidence and attitude towards upper division mathematics courses (Laursen et al., 2011; Laursen, 2013; Laursen &

Rasmussen, 2019). Students enrolled in previous research studies were enrolled in upper division mathematics classes and were pursing STEM majors; however, the impact of inquiry-based learning on students in a general education mathematics class has not been studied. In a general education mathematics class, students of all majors and disciplines, and with varying skills and backgrounds, are brought together in a single classroom. The goal of this study is to compare the effects that an inquiry-based general education mathematics course has on student understanding of key topics, attitude towards mathematics and mathematical self-concept as compared to an interactive lecture-based course which teaches the same topics. The course has traditionally been taught using interactive lecture (control group), but in this study, the course will also be taught using inquiry-based learning (experimental group). Quantitative and qualitative data was collected throughout the terms to determine the impact on students' understanding of key concepts, student attitudes towards mathematics and their mathematical self-concept. Thus, a mixed methods design will be used, incorporating both qualitative and quantitative research methods.

Mixed methods research requires a researcher to collect and analyze both quantitative and qualitative data. These two different data types allow the researcher to better understand the research problem, and the qualitative data can be used to help explain differences in results of the quantitative data (Croswell, 2015). The data for this study will be collected sequentially. Quantitative data will be collected before, during and after the term from the research participants, and qualitative data will be collected after the academic term has ended, through interviews. These interviews will help the researcher understand and explain any differences noted in the quantitative data which was collected during the academic term. Thus, a mixed methods design, specifically an explanatory sequential mixed methods design, will be used. The explanatory sequential design will incorporate quantitative data before and during the term of study, and will use qualitative data after the conclusion of the term to explain the quantitative results.

3.3 Research methods

The researcher-instructor for the course used in the study has used a consistent syllabus and content for the course since over 10 years. Students in both groups received identical course notes and information, and both courses covered identical content. Students in both courses were given the same assignments, quizzes, exam and final exam. However, the instructional method used during each class period throughout the term was different.

In the interactive lecture course (control group), the instructor presented material either through power point slides or by working examples on the whiteboard at the front of the class. The instructor worked practice exercises for the class and gave explanations to the whole class, while the students took notes and filled in their handouts at the same pace as the instructor. The instructor modeled each exercise and explained each theorem, then would give students short periods of time (30 seconds to 2 minutes) to explain to their neighbor or ask follow up questions to the instructor.

Students in the inquiry-based learning class (experimental group) worked in small groups, with 3-4 students per group, on inquiry activities. These inquiry-based activities were teacher-designed inquiry; that is, the instructors of the course had set goals and learning objectives for the students each class period, and asked scaffolded questions to the students. These students generated their own ideas, formed hypotheses and made conclusions on their own, without the instructor presenting material directly to them. The students managed their time throughout the class period; there were no time limits for discussing concepts within their groups; this contrasts the limited time (30 seconds – 2 minutes) that students in the interactive lecture courses had to share ideas with a neighbor.

The course goals and learning objectives were identical for both the control group and the experimental group. However, the students in the IBL sections worked daily in their small groups, working on handouts with course notes, which posed guided questions and key concepts to the students; this differs the presentations on the whiteboard or Powerpoint that were presented to students in the interactive lecture courses. Table 3-1 compares physical differences in the traditional classroom environment, as compared to an inquiry-based learning environment. Students in the IBL courses generated their own ideas, formed hypotheses and made conclusions on their own, without the instructor presenting material directly to them.

Table 3-1
Classroom Comparison

Interactive lecture classroom	Inquiry-based classroom
*Educator presents new concepts to the class through note-taking and examples *Students observe problem-solving process, then replicate with a similar example to learn new information *Students learn independently while watching the educator *Educator uses interactive question techniques and allows time for questions *Students perform practice exercises, in class, after new concepts are presented *Educator confirms correct solutions *Students sit in traditional rows *Educator is sole authority in classroom	*Educator facilitates discussions and discovery and redirects student questions *Students use prior knowledge and discovery process to learn new information *Students form conclusions, find pattern, and investigate problems collaboratively *Educator gives students tasks requiring them to experiment, conjecture and explore core concepts. *Students guide class discussions and explain solutions collaboratively *Students sit in groups or 3-4 in classroom *Students act as authority in classroom, presenting ideas or solutions to the class
**Online Interactive lecture also included: *Educator presented on Zoom with PPT *Educator would present new information, then allow students to work a similar concept *Educator used interactive question techniques, and responses such as “thumbs up” or Zoom chat to share solutions	**Online IBL also included: *Students worked in small groups in Zoom breakout rooms *Students shared a Google Jamboard to discuss, find patterns, investigate collaboratively

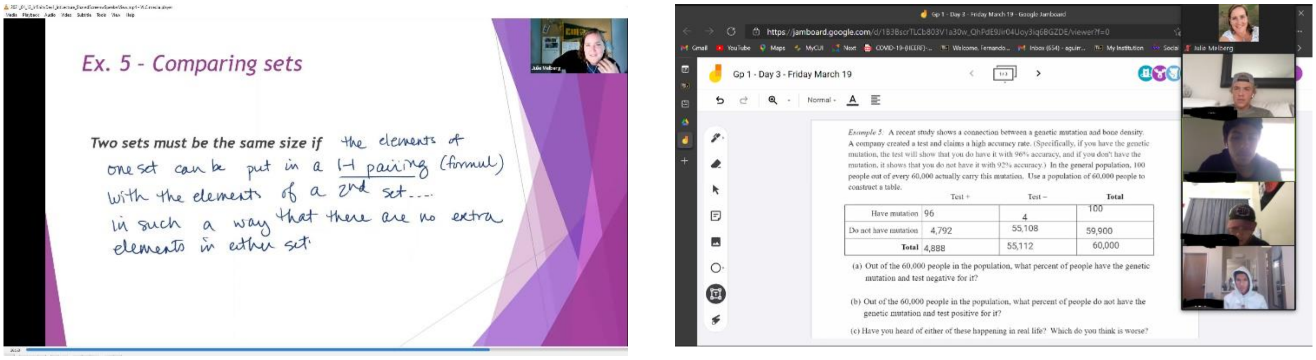
Figure 3-1 compares the different classroom environments for the face-to-face and online sections of this research study. The top two photos show differences in the face-to-face classrooms. The top left photo shows an interactive lecture classroom, where students are taught via direct instruction, and ask questions or interact with their peers at different times during class. The top right photograph shows an inquiry-based classroom, where students are working on instructor-developed course notes, and working collaboratively to explore core concepts and make conclusions about new material. The bottom two photos show differences in the online classrooms. The bottom left photo shows an online interactive lecture classroom, where students are taught using a PowerPoint presentation via direct instruction.

Figure 3-1

Photo comparison of Interactive Lecture and IBL Classrooms

Interactive lecture classroom configuration, face-to-face

Inquiry-based learning classroom configuration, face-to-face



Ex. 5 - Comparing sets

Two sets must be the same size if the elements of one set can be put in a 1-1 pairing (formul) with the elements of a 2nd set.... in such a way that there are no extra elements in either set

Example 5. A recent study shows a connection between a genetic mutation and bone density. A company created a test and claims a high accuracy rate. (Specifically, if you have the genetic mutation, the test will show that you do have it with 96% accuracy, and if you don't have the mutation, it shows that you do not have it with 92% accuracy.) In the general population, 100 people out of every 60,000 actually carry this mutation. Use a population of 60,000 people to construct a table.

	Test +	Test –	**Total**
Have mutation	96	4	100
Do not have mutation	4,792	55,108	59,900
Total	4,888	55,112	60,000

(a) Out of the 60,000 people in the population, what percent of people have the genetic mutation and test negative for it?

(b) Out of the 60,000 people in the population, what percent of people do not have the genetic mutation and test positive for it?

(c) Have you heard of either of these happening in real life? Which do you think is worse?

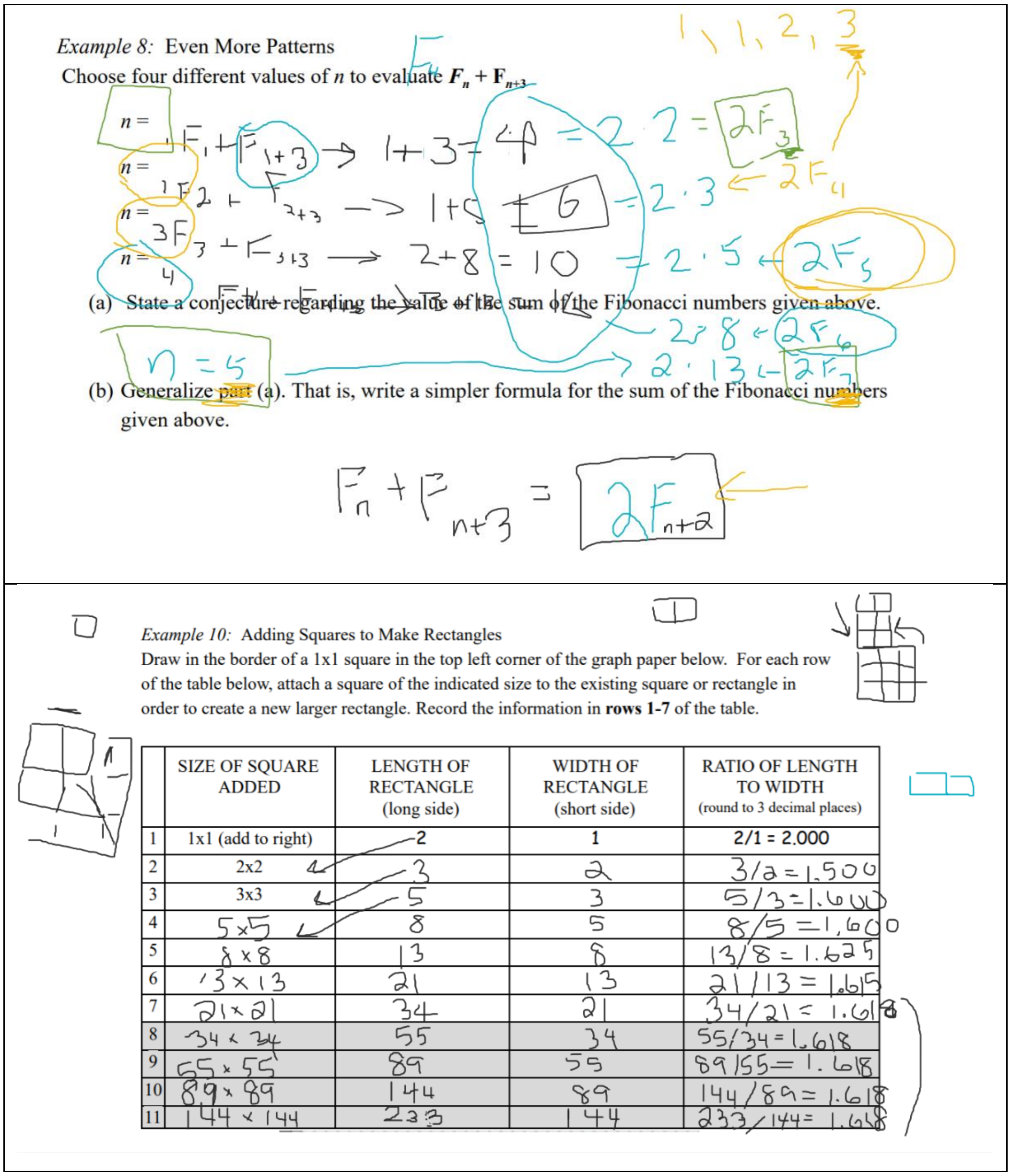
Interactive lecture classroom, online

Inquiry-based learning classroom, online

Students could interact with their instructor by asking questions publically or privately in the Zoom chat box, by showing “thumbs up,” “thumbs down,” or a numeric solution using their hands, by typing solutions in the Zoom chat box, or by having short breakout rooms (1-2 minutes) to share their ideas with their peers. The bottom right photo shows an online inquiry-based learning classroom. Students interacted with their peers in a Zoom breakout room, using a shared Google Jamboard. The Jamboard allowed students to each write on the document, so that they could see and hear what their peers thought. Students were encouraged to take their own notes on paper as well. The educator would monitor the Jamboards and Zoom breakout rooms, circulating between the rooms throughout the breakout time (35-45 minutes) to assist groups or listen to their conversations and conclusions. Work on sample Jamboards is shown in Table 3-2;

students had worked together online to make generalizations (different students wrote on the Jamboards in different colors).

Figure 3-2
Sample Google Jamboards used by IBL students in a fully online class



Example 8: Even More Patterns

Choose four different values of *n* to evaluate $F_n + F_{n+3}$

1, 1, 2, 3

$n = 1$: $F_1 + F_{1+3} \rightarrow 1+3 = 4 = 2 \cdot 2 = 2F_3$

$n = 2$: $F_2 + F_{2+3} \rightarrow 1+5 = 6 = 2 \cdot 3 \leftarrow 2F_4$

$n = 3$: $F_3 + F_{3+3} \rightarrow 2+8 = 10 = 2 \cdot 5 \leftarrow 2F_5$

$n = 4$

$2 \cdot 8 \leftarrow 2F_6$

$n = 5 \rightarrow 2 \cdot 13 \leftarrow 2F_7$

(a) State a conjecture regarding the value of the sum of the Fibonacci numbers given above.

(b) Generalize part (a). That is, write a simpler formula for the sum of the Fibonacci numbers given above.

$$F_n + F_{n+3} = \boxed{2F_{n+2}}$$

Example 10: Adding Squares to Make Rectangles

Draw in the border of a 1x1 square in the top left corner of the graph paper below. For each row of the table below, attach a square of the indicated size to the existing square or rectangle in order to create a new larger rectangle. Record the information in **rows 1-7** of the table.

	SIZE OF SQUARE ADDED	LENGTH OF RECTANGLE (long side)	WIDTH OF RECTANGLE (short side)	RATIO OF LENGTH TO WIDTH (round to 3 decimal places)
1	1x1 (add to right)	2	1	2/1 = 2.000
2	2x2	3	2	3/2 = 1.500
3	3x3	5	3	5/3 = 1.666
4	5x5	8	5	8/5 = 1,600
5	8x8	13	8	13/8 = 1.625
6	13x13	21	13	21/13 = 1.615
7	21x21	34	21	34/21 = 1.618
8	34x34	55	34	55/34 = 1.618
9	55x55	89	55	89/55 = 1.618
10	89x89	144	89	144/89 = 1.618
11	144x144	233	144	233/144 = 1.618

3.3.1 **Setting**

The setting for this research is in a general education mathematics class at a small Christian liberal arts university in southern California, USA, enrolling approximately 1500 undergraduate students. This general education course is one of 17 courses that meet the university general education requirements, and is taken by all first-year students, regardless of major or mathematics background. Students in the class come from a variety of socioeconomic, educational and cultural backgrounds, and students of varying levels of mathematics are placed together in the same classroom. These students encompass a variety of majors (humanities, fine arts, social sciences, natural sciences, and business) and have a wide range of mathematical abilities, as some have taken Calculus 1 (differential calculus) or Calculus 2 (integral calculus), while others have completed minimum high school graduation requirements (algebra, geometry or financial mathematics) before enrolling at the university.

Topics for this course were determined by the mathematics faculty at the university at the inception of the course (and the University's core curriculum program) in 2008. The faculty wanted to expose students to topics which allow them to understand and appreciate the interdisciplinary nature of mathematics, and how mathematics is in the world around them (Cook & Garneau, 2017), so based the curriculum on topics presented by Michael Starbird and Edward Burger's textbook *The Heart of Mathematics* and their book *Coincidences, Chaos and All That Math Jazz*. The course covers both computational and theoretical mathematics topics, including the Fibonacci numbers and the golden ratio, a study of Cantor's theorem and multiple sizes of infinity, voting theory, analyzing data, and Euclidean and non-Euclidean geometry. These topics are selected by the mathematics faculty as topics which are new to undergraduate students, who have typically focused their mathematics studies on algebra and calculus, and using formulas to solve a mathematics exercise. However, these topics are accessible to first-year students, as they can learn and make conclusions on these topics with limited algebra and geometry knowledge. These topics provide students the opportunity not only to learn mathematics, but also how to think and solve problems crucially and creatively. For example, students can generate patterns using logic and critical thinking to make conclusions about the Fibonacci numbers, and they can build upon their previous knowledge of Euclidean geometry, as they apply the axioms of geometry to spherical or hyperbolic surfaces.

Students who take this course meet 150 minutes per week, either twice a week (for 75-minute class periods) or three times a week (for 50-minute class periods). Students in two of the five semesters in this research were enrolled in an online section of course, due to the COVID-19 pandemic. As a result of the high number of cases and a high transmission rate in Southern California, the location of the University, the students learned remotely, synchronously online, during the Fall 2020 and Spring 2021 semesters. Additionally, for the final four weeks of the Spring 2020 semester, students learned remotely, as campus was closed due to local and state regulations.

For the online semesters, classes were conducted synchronously online using Zoom (Fall 2020 and Spring 2021). Students were expected to participate in class, either through the interactive lecture (using a PowerPoint presentation online) or through inquiry-based learning (using breakout rooms on Zoom to allow students to work in small groups. Specifications for class attendance and participation were given in the course syllabus at the start of the term:

> Students are expected to actively participate in synchronous class, including sitting with a web camera on from a private, quiet, well-lit place. Position your camera properly - in a stable position, focused at eye level, if possible. Best online practices for Zoom are expected (including displaying your first and last name, being appropriately clothed, having microphones muted, raising your hand and using the chat only for questions/answers related to course materials or content). (University Undergraduate Syllabus, 2020)

3.3.2 **Sample**

The target participants for this study were enrolled in a synchronous course, The Nature of Mathematics, which serves as the general education mathematics course at a 1500-student liberal arts university. The student participants in this research study were undergraduate first-year students (17-20 years old), who were enrolled in a general education mathematics course at a small liberal arts university in Southern California, USA. The course, titled “The Nature of Mathematics,” is one of 14 courses which are part of the university’s *Enduring Questions and Ideas* general education requirement for students of all majors at the university. The student participants in this research study included students from a variety of majors (biology, business, theology, Christian ministry, computer science, education, art, music, theater, kinesiology, chemistry, mathematics, psychology). The student participants, like most students at the university, live on campus and participate in academics as well as athletics and other extracurricular activities. After graduation, many students either continue to graduate school to pursue a Masters or Doctorate in their field of study, or find a job in a field related to their chosen major.

Where the research for this study occurred, all classes moved online in for the final 4 weeks of the semester during the Spring 2020 semester, and all students were sent off-campus. Online learning continued throughout the 2020-2021 academic year; while some students returned to campus, the course in this study was taught fully online during the Fall 2020 and Spring 2021 semesters; all online courses were taught synchronously through Zoom. During the Fall 2021 semester, students returned to campus and to the face-to-face classroom, but both students and instructors were required to be fully masked in the classroom at all times, and socially distanced when possible.

The size of the population in this research is 283 students, who were enrolled in 11 different sections of the general education mathematics course over five consecutive terms (Fall 2019, Spring 2020, Fall 2020, Spring 2021, Fall 2021). These participants and total enrolment, by term, are shown in Table 3-3.

Table 3-2
Research Participants by Term

Term	Modality	IBL Participants	Interactive lecture participants	Total enrolment
Fall 2019	Face-to-face	22	21	56
Spring 2020	Face-to-face	9	10	51
Fall 2020	Online	11	8	75
Spring 2021	Online	9	10	51
Fall 2021	Face-to-face	14	7	50
Total Participants		***65***	***56***	**283**

These students were enrolled in their first year of study at the liberal arts university, and each took one semester of mathematics to fulfill the university general education requirement. Research participants in the sample were invited to participate in the research during the first week of the semester. The research project was explained during class by a teaching assistant or fellow mathematics colleague while the researcher-instructor was not present in the classroom, then the target participants were given time to understand the research project and to volunteer as a research participant. Students were given information on the study, how data was to be collected and used in the study, using the four elements of informed consent (Cohen et al., 2011). Students were given the opportunity to participate without the researcher-instructor present to help address the power imbalance and so that students did not feel any pressure to participate in the study from the instructor.

During the face-to-face sections in this study, students were given physical copies of the *Participant Information Statement*. Students were given time to read and ask questions on the *Participant Information Statement*, then students gave consent to become a participant by signing and submitting the *Participant Consent Form.* These forms were given by the teaching assistant to a departmental colleague, who kept them in a secured location in her locked office throughout each term. During the online sections in this study, students were given an electronic copy of the *Participant Consent Form* in Blackboard, the university's content management system, and the project was explained by the teaching assistant to the students over Zoom, the university's online video conferencing platform, used for online classes. These online students were given the opportunity to consent to participate by emailing a departmental colleague through her official university email, secured by US Family Educational Rights and Privacy Act (FERPA) protocols. This departmental colleague kept records of which students were research participants through the term. After each term was finalized and final marks submitted, the departmental colleague gave the researcher access to names and coded numbers of each of the research participants. The departmental colleague was the only person who knew which students were participants in the research study throughout the term; this was done so students knew their grades were not influenced by their participation in the research study. Throughout the academic term, the researcher had no knowledge of which students in the class were participants in the study, to ensure integrity of the research design.

Over the course of five academic semesters, a total of 121 participants consented to participating in the research, out of 283 students enrolled in the courses. Approximately 15% of these students were from a traditional STEM field (Mathematics, Physics, Biology, Chemistry), while the others were from a variety of other liberal arts majors on campus (English, Theology, History and Political Thought, Art and Graphic Design, Music, Kinesiology). Each course was

15 weeks long, and included 150 minutes of synchronous instructional time per week, either in a classroom (face-to-face courses) or on Zoom (fully online courses).

3.4 Inquiry-Based Learning classroom

The following sections will explain the development of the inquiry-based learning content, based on both social and cognitive constructivism, which was used in both the face-to-face and classrooms throughout this study.

3.4.1 Development of Inquiry-Based Learning curriculum

The curriculum was developed in stages for the inquiry-based learning classroom. First, the overall curriculum for the course was developed by the mathematics faculty at the university. The topics are determined by the faculty, and all sections of the course are taught the same topics annually. For this research study, the inquiry-based learning and the interactive lecture sections of the course all learned the same topics. The interactive lecture students were taught by direct instruction from the educator, who would present information on a PowerPoint or write notes on the classroom whiteboard. Work was paced by the teacher leading the class through each example, and then students solved practice exercises paralleling those that the educator had previous worked.

In the IBL courses, however, work was self-paced by students and the group they were in, and was guided by course notes developed by the mathematics faculty at the university. The in-class course notes were designed for students to complete during class time, with minimal support needed from the educator. The course notes covered the same topics, but students were given time to think and work independently or in groups on each concept. This was achieved by asking scaffolded questions throughout each class period, which would lead students to make conclusions and generalizations in their small groups. Groups were encouraged to discuss what they had found individually, and to make conclusions together. If a group got stuck working through the exercises, the educator was available to assist students, asking additional leading questions to enable them to make conclusions.

The course notes were developed from a variety of resources, including from a textbook closely aligned with the content of the course (Edward Burger & Michael Starbird's *The Heart of*

Mathematics), The Art of Mathematics (www.artofmathematics.org), and other online resources and websites. Tables and images in the course notes were created by the researcher or a fellow colleague at the university. The course notes contained the main ideas for each of the topics covered, examples to be worked and fill-in-the-blanks for major definitions and theorems. The course notes were created in Word, and exported as a PDF for printing and posting on Blackboard (the course LMS).

3.5 Research design: quantitative and qualitative data

Throughout the semester of enrollment, quantitative data was collected not only through the grades in course assessments, but also through a survey which was administered at the start and at the end of the course. During the semester of enrollment, students completed the ATMI, or Attitude Towards Mathematics Inventory (Appendix A), to assess student self-confidence, motivation, enjoyment and perception of value of mathematics. This survey was completed electronically by the research participants during the first week of the semester, as well as during the final class period. Additionally, course quizzes and exams, which were part of the overall grading in the course, were collected and analyzed. Scores from exams and departmental final exams, covering 3-5 different mathematical topics, were used in the overall grade calculation and were collected as quantitative data as well. After the course had ended, qualitative data was collected, through semi-structured interviews. Figure 3-3 shows the sequential mixed methods data collection for this study. Each of the data collection instruments will be described further in this section.

Figure 3-3
Research questions and the mixed method data used for each

Research Questions	Quantitative results	Qualitative follow up
How does an IBL GE math course differ from a traditional interactive lecture in terms of student:		
1. Understanding of key concepts	Student achievement on quizzes, exams	Semi-structured interviews, instructor reflections
2. Attitude toward mathematics	ATMI value, enjoyment	Semi-structured interviews
3. Mathematical self-concept	ATMI self-confidence, motivation	Semi-structured interviews
What effect does IBL have on students' achievement, as compared to interactive lecture?	Student achievement on quizzes, exams, ATMI	Semi-structured interviews, instructor reflections

How does an IBL GE math course impact student learning in a fully online course?	Student achievement on quizzes, exams	Semi-structured interviews, instructor reflections

3.5.1 Criterion-referenced quizzes and exams

Course assessments (quizzes and exams) were used to measure knowledge of key concepts in the course. These quizzes and exams were criterion-referenced and measured on a standard 10-point grading scale (90-100 A, 80-89 B, 70-79 C, 60-69 D, 59 and below F). These were given every 2.5-3 weeks, and included content covered in the class notes/discussion, and variations of questions from homework assignments. Students were asked to solve questions similar to those covered in class, as well as new generalization exercises, which students had not worked before. The quizzes, which ranged in value from 25-35 points, assessed students on a single topic covered in class. These assessments were developed by the instructor in collaboration with the other mathematics educators, and was aligned to the content in the course curriculum. This was achieved by faculty working together to create and edit assessments, which were variations of the exercises students had worked in class or independently. The quizzes and exams varied in question type, including but not limited to, free response questions, multiple choice, fill-in-the-blank, and matching questions; these questions types were closely aligned to the content presented in class, and gave the students opportunities to answer calculation-based questions (through multiple choice and fill-in the blank), as well as open-ended generalizations through matching or free response questions. The Final Exam was also used to measure overall understanding of key concepts in the course, and was developed and written by the four-member mathematics department. The Final Exam included questions similar to previous quizzes and exams, as well as variations of free-response questions which allowed students to showcase different methods by which to solve a mathematics question. Varying question types on these assessments allowed students to demonstrate content knowledge, as well as problem solving, applied knowledge, and self-reflection on mathematical processes (National Research Council, 1993, p.72).

3.5.2 Attitude Toward Mathematics Inventory

The survey instrument, “Attitude Towards Mathematics Inventory” (ATMI) is a 40-question inventory, based on the 5-point Likert scale with 11 reverse coded questions. The

ATMI (Appendix A), which was developed by Martha Tapia and George Marsh (Tapia & Marsh, 2004) to measure student attitudes towards value, motivation, self-confidence and enjoyment towards high school mathematics, was used to collect quantitative data about students' attitudes towards mathematics. The ATMI has been tested for reliability (Cronbach alpha = 0.963) and validity for high school students (Tapia, 1996). Content validity of the ATMI was established by relating each of the 40 questions to the four variable subscales (self-confidence – 15 questions, value – 10 questions, enjoyment – 10 questions, motivation – 5 questions) (Tapia & Marsh, 2004). Sample questions from each of the four factors is given in Table 3-3; the whole ATMI is given in Appendix A.

Table 3-3
Sample "Attitude Towards Mathematics Inventory" questions, by factor

Self-confidence

9. Mathematics is one of my most dreaded subjects. (RC)
14. When I hear the word mathematics, I have a feeling of dislike. (RC)
22. I learn mathematics easily.

Value

5. Mathematics is important in everyday life.
36. I believe studying math helps me with problem solving in other areas.

Enjoyment

3. I get a great deal of satisfaction out of solving a mathematics problem.
25. Mathematics is dull and boring. (RC)

Motivation

23. I am confident I could learn advanced mathematics.
28. I would like to avoid using mathematics in college. (RC)

The ATMI has previously been tested on university students, using the same data and structure as this research study, and was found that the Cronbach alpha scores were 0.96 (self-confidence), 0.93 (value), 0.88 (enjoyment), and 0.87 (motivation) (Marsh, 2005). These scores confirmed a high reliability of the ATMI for university students. Students were also tested at four-month intervals, for re-test reliability; the Pearson correlation coefficient for the total scale was 0.89 (with individual subscale coefficients of self-confidence 0.88, value 0.70, enjoyment 0.84, and motivation 0.78) (Tapia & Marsh, 2004). Self-confidence was assessed with 15 questions, value and enjoyment were assessed with 10 questions each, and motivation was assessed with 5 questions. Of the 40 questions on the ATMI, eleven were reverse-coded.

Dr. Martha Tapia, one of the developers of the ATMI, granted permission for its use in this research (Appendix I). Dr. Tapia also revealed the coding (and reverse coding) of the ATMI with the researcher, and gave permission for the ATMI questions to be included in this thesis (Appendix J).

3.5.3 **Semi-structured interviews**

In order to give the research participants time to reflect on their experience, research participant interviews were conducted for qualitative data several weeks after the term concluded. These interviews were conducted by a fellow mathematics professor at the university, so that the participants could speak freely about their experience, and so they would not feel influenced by the researcher-instructor of the course. The professor who conducted the interviews who had great knowledge of the content of the course, as well as with which method the participants were taught; the teaching method did not influence the questions which were asked to individual students. A list of questions (Table 3-4) was given to the interviewer, who would also ask follow up questions based on student responses. Follow up questions were asked when students would give partial or short answers to the questions below, to encourage them to discuss details of how material was presented, their perception of the time working in groups or independently, themselves as problem solvers, or the overall structure of the course. The interviews were conducted and recorded in Zoom. While the original goal was to have groups of students being interviewed, nearly all of the students interviewed were questioned individually due to the low number or participants willing to be interviewed, and of those who were willing, individual scheduling constraints.

Table 3-4
Sample Questions from Post-Course Interviews

1. Thinking back to your Core Mathematics course, describe your overall experience in the course.
2. Did the course change your mind about how you think about mathematics, or yourself as a mathematics student? How did that happen?
3. Throughout the course, did your impression of yourself as a mathematics student change? If so, how did that happen?
4. Do you feel confident sharing your ideas and explanations about mathematical ideas? Have you always felt that way, or how has that changed?
5. How did lessons presented in class help or hinder your understanding of the topics presented?
6. What was your impression of mathematics before this term, and how, if at all, did it change by taking Core Mathematics?

These interviews were used to gather qualitative data, and to allow the research participants to explain in their own words, their experiences in the course. Each interview was between 10-20 minutes in length, and the interviewer allowed the research participants to explain their attitudes and opinions about the course and how the course affected them. Unfortunately, since the interviews were conducted several weeks after the term had ended (and final grades had been posted), very few participants responded or showed up to the interviews, despite multiple email reminders and the offer of remuneration for their time. Because these interviews were conducted several weeks after the terms had ended, the recollection of participants and specific experiences or memories from their course may have been affected. In all, only 13 students participated in the interviews. The interviews were transcribed using Zoom, and those transcripts were checked by a teaching assistant for accuracy.

3.5.4 Instructor reflections

Instructor reflections were written throughout the semesters of study, in both the inquiry-based learning and interactive lecture classroom. Reflections included not only the instructor’s perception of student attitude in the classroom, but also unsolicited feedback and comments from students enrolled in each course. These reflections were written by the instructor during each term, based on class discussions and individual feedback from students enrolled in the course. These notes included how the lessons went, challenges or struggles from each group of students, and comments overheard in each classroom. The instructor made every effort to cite evidence and avoid generalizations in these reflections.

3.5.5 **Research design: Analysis**

Data from the ATMI was collected during Week 1 and Week 15 (the first and last week) of the term, and those two data values were compared to determine any change in student self-confidence in, value of, enjoyment of and motivation in mathematics. Student gender, teaching method and instructional modality were noted, and the quantitative scores throughout the term were collected. These can be combined into a single numeric score for the course, or compared individually as exam, quiz, and final exam scores. Each of these variables is compared for the two different teaching methods – interactive lecture and inquiry-based learning, as well as the different teaching modalities – face-to-face and online learning.

For the first research question, quantitative data (exams, quizzes, final exams) from student scores were compared for the inquiry-based learning and interactive lecture courses. For the second research question, the results from the Attitude Toward Mathematics Survey results were compared, to determine if there was a difference in students taught by the two different instructional methods in their understanding of key concepts, their attitude towards mathematics and their mathematical self-concept. For the third research question, data was compared from the three terms that were fully face-to-face and the two terms which were online, to determine if there were any notable difference in data. For each of these research questions, the interviews, though limited in number, were used to try to understand why these differences may have occurred.

3.5.6 **Quantitative data analysis**

All quantitative data were entered into a Microsoft Excel^{TM} spreadsheet by a teaching assistant and saved as a password-protected file. After confirmation of accuracy of the records by a second teaching assistant, the data was imported into Statistical Package for Social Sciences (SPSS), a statistical package which allows for complex statistical analysis. As described above (Section 3.3.2) the sample consisted of two groups, a traditional lecture group (control) and an inquiry-based learning group (treatment). Additionally, because some of the data collection occurred during the COVID-19 global pandemic, the control and treatment groups also each included a face-to-face cohort, as well as an online cohort. These groups were compared using *t*-tests (both paired and independent sample), as well as ANOVAs and the Mann-Whitney U tests by analyzing overall course grade, quiz scores, and final exam scores. Additionally, further quantitative data were collected to assess student perceptions of their attitudes towards mathematics through the ATMI survey.

3.5.6.1 First research question

The ATMI was used to assess student attitudes towards mathematics in four scale factors. Thus, first, the ATMI inventory results were tested for reliability and validity using the Cronbach's alpha score. Cronbach's alpha scores were computed for each of the scale factors, and resulted in high Cronbach's alpha scores (self-confidence: 0.96, value: 0.88, enjoyment: 0.91, motivation: 0.85), indicating reliable survey results. However, the ATMI scores resulted in non-parametric data, common for data collected from a Likert scale; therefore, the non-parametric Wilcoxon sign rank test was performed to calculate significance and effect size (Triola, 2011). In class assessments were also tested, including final exam scores and overall grade using an ANOVA test, testing the differences in the variances in the scores. ANOVA tests were performed on the final exam and overall scores of the research participants to test for variation in the standard deviation of these scores. The Final Exam scores were found to have unequal variances, so further testing was not performed on those scores. However, overall grades in the course will be used, since the differences in the variances of the overall course grades was shown to be non-significant.

The first research question, *How does an IBL GE mathematics course for first-year students differ from a traditional interactive lecture course in terms of student understanding of key concepts, attitudes towards mathematics, and mathematics self-concept*, was considered using the quantitative data collected through the course. To do so, overall grades in the course were tested with an independent samples *t*-test, comparing the overall course scores for participants from the two different groups (interactive lecture and inquiry-based learning). Normality was established using the Shapiro-Wilk statistic, and Levine's test for equality was used to confirm that the variances of the two groups can be assumed equal. Next, similar tests were performed on the ATMI data from participants, including Shapiro-Wilk test, Levine's test and an independent samples *t*-test. These tests were performed in SPSS for each of the four scale factors of the ATMI. Additionally, a paired *t*-test was performed using a significance level of 0.05 for each survey question in the ATMI, by scale factor, comparing the pre- and post-course average results for the two different teaching methods.

Another factor that was considered was the effect of gender on teaching method. A 2x2 factorial between group ANOVA was performed in SPSS to test the effect of gender. Additionally, a one-way between groups ANOVA was performed to determine differences that teaching method and the modality (face-to-face and online) had on the overall course grades of the participants. The data was also considered by both teaching method and modality, utilizing four sub-groups (IBL face-to-face, interactive lecture face-to-face, IBL online, interactive lecture online). The Shapiro-Wilk statistic was calculated, confirming normality of the data and equal variances. An ANOVA was also performed, but showed that there were no significant differences in the data from the four sub-groups. The Pearson's coefficient was calculated to determine if there exists a correlation between the overall grades in the course and their change in self-confidence of the research participants. Lastly, the relationship between gender, and the four sub-groups of teaching methods and modalities was considered. A multiple regression analysis was performed to note any differences in the overall course grades by gender.

3.5.6.2 Second and third research questions

The second research question, *What effect does IBL have on students' achievement in a face-to-face GE mathematics course, as compared to learning by interactive lecture,* was then considered. Assessments, including both the final exam and overall course grades, were considered. The differences in the Final Exam scores for the face-to-face participants violated normality, so a non-parametric Mann Whitney *U* test was performed, while an independent samples *t*-test was used to compare the overall course grades for the face-to-face participants. Differences in the ATMI scores for the face-to-face participants was compared with an ANOVA test for each of the four ATMI scale factors. Similar tests were used to answer the third research question, *How do the student cognitive and affective results in an IBL GE mathematics course differ from an interactive lecture course, in a fully online classroom,* for the online participants.

3.5.7 **Qualitative data analysis**

The participant interviews were recorded in and transcribed by Zoom. The transcripts were reviewed by both the researcher and a research assistant for accuracy. There were 13 research participants who agreed to share their experiences by interview. The research participants were interviewed in groups of 1-3, for a total of 8 interviews. The interviews were transcribed verbatim, then were reviewed and coded for analysis. Instructor reflections were compiled, analyzed, and coded using the same process. Because of the small number of interview quotes and instructor reflections, the data was organized and coded by hand using the process called inductive content analysis.

The inductive content analysis used relied on three stages: content reduction, data grouping, and formation of themes (Kyngas, 2020). Using inductive content analysis allowed the researcher and research assistant to read the data for emerging sub-themes, then to build higher order themes from those by clustering like sub-themes (Elo & Kyngas, 2008; Vears & Gillam, 2022). The data was read carefully and "open coded" by noting similarities in the transcripts with "big picture" terms, then codes were developed which included the setting and context from the course and individual perspectives of the participants (Elo & Kyngas, 2008). These codes were then grouped into sub-categories and developed into refined themes which were interpreted to address the research questions and to support the data analysis (Vears & Gillam, 2022). Table 3-5

Illustration of grouping of quotations into themes illustrates the grouping of verbatim quotes into themes; these sub-categories were grouped into Theme 5, which explains student's positive experiences with inquiry-based learning and the IBL classroom environment.

Table 3-5
Illustration of grouping of quotations into themes

"I actually really enjoyed the group and being able to actually discuss with peers and get multiple ideas and just get everyone's thoughts." "Making us do group work like with different people and putting us in different groups kind of forced us to, you know, talk to other people." "The interactiveness of the class helped me stay engaged and learn more about the topics with the group work." "It helped when you were the one who understood it, and you could really teach it back to someone because it then definitely stuck in your brain a little more."	Views of interactive class and group work
"She just made it a comfortable and supportive setting where if you did say something, and it was maybe, wasn't right, you didn't feel bad about yourself, but it was a safe place to share those ideas." "I feel like in the Core Math class a lot of people who normally wouldn't ask questions feel free to ask questions without feeling like, embarrassed, or like, dumb." "I'm usually one of the people who doesn't like it when the class just sits in silence after a question so even if my answer is wrong, I say it with confidence."	Increased confidence in asking questions
"It was definitely tricky and probably frustrating sometimes when it was like, I don't think we know what we're doing. We're trying to learn something, but it was definitely a cool experience and you saw a lot of people step up." "I just see people who normally wouldn't speak in a math class,…more confident in sharing their answers even if they're wrong."	Engagement with active learning

The themes which emerged from the inductive content analysis of both the student interviews and the instructor reflections seemed to confirm that the relationships identified by the quantitative data should be further analyzed. The comments made by the research participants and through instructor reflections seem to show that an inquiry-based learning environment play a role in the mathematical self-concept and achievement of student in a mathematics class. These ideas will be further analyzed in Chapter 4.

3.5.8 **Data management**

Students who volunteered to be a research participant did so in one of two ways: during the face-to-face sections of the course, they filled out a *Participation Form*, which were stored in a sealed envelope in a locked filing cabinet for the duration of the term, while during the online only sections of the course, students emailed a department colleague, using their FERPA compliant emails, to volunteer. Students took the *Attitudes Towards Mathematics Inventory* during the first week of class, using a coded student number. The data was compiled into an Excel file, which was stored on a password-protected laptop. Course scores from criterion-referenced tests were stored in Blackboard, the university's content management system, then downloaded as an Excel file at the conclusion of the term.

All electronic data has been stored and backed up monthly. Data will also be stored on Curtin University's research drive (*R:* drive) for 7 years from the completion of the research. At the end of 7 years, the raw data will be destroyed using the Western Australia University Sector Disposal Authority. The researcher, colleagues of the researcher who maintained the list of research participants, and the thesis committee were the only individuals who had access to the data.

3.6 Validity and trustworthiness

This section will discuss the research instruments, and the controls that were in place in this study to ensure validity and trustworthiness of its results. The validity and reliability of the quantitative data will be discussed first, then the credibility and dependability of the qualitative data will be discussed.

3.6.1 Trustworthiness in quantitative data: Validity and reliability

For a research study to produce results which are trustworthy to others, the study itself must be considered both valid and reliable. The validity of a study refers to how well the findings generalize the greater population, while he reliability of a study refers to consistency of a study if it were repeated to produce similar results (Creswell, 2015). This research study included measures to ensure validity and reliability, and hence trustworthiness in its quantitative data collection.

3.6.1.1 Validity

To be considered valid, a research study must include both internal validity and external validity. Internal validity will allow others to be confident that any differences found in the study cannot be explained by other external or confounding factors, while external validity will allow the results to be generalized (Creswell, 2015).

While not all external factors can be predicted or controlled, every effort was made to ensure internal and external validity in this study. First, a control group (interactive lecture cohort) was used to compare to the experimental group (the inquiry-based learning cohort). Students were assigned to these groups without knowledge of how the course would be taught. An issue with an instructor-researcher is that the research participants will be concerned that their participation or non-participation in the research study will affect their overall grade. To prevent this, participation in research study was not known to the researcher until after the term was complete. Participants were recruited by a teaching assistant or a fellow faculty member. The ATMI was administered at the start and end of each term by a teaching assistant, and stored by coded number which were not available to the researcher until the term had ended. Additionally, the data were analyzed after each term, and the semi-structured interviews took place after the term had ended and final grades were submitted. To ensure that students felt comfortable sharing their experiences about the classroom, a fellow faculty member conducted the interviews.

Additionally, construct validity tells how well the research instruments measure the effects it was designed to evaluate, particularly when the variable being measured is not directly observable (Westen & Rosenthal, 2003, p.608). To ensure construct validity, the course assessments and instruments used to collect quantitative data must be reliable. For this study, quantitative data was collected through course assessments, including quizzes, exams, and a final exam. The Final Exam was a departmental final exam, developed by the four members of the mathematics department, who each have significant experience in mathematics education. The ATMI (Section 3.5.2) is a well-established survey instrument, developed in 1996, and has been validated and tested for reliability in multiple classrooms (Marsh, 2005).

3.6.1.2 Reliability

Reliability refers to the ability of a study to produce consistent results using the same methods and produce similar results under similar circumstances (Creswell, 2015). For this research, the reliability of the quantitative measures is related to its construct validity – the instruments used here (course assessments and ATMI) have been shown to be both credible and replicable. The ATMI has produced reliable results in assessing the attitude towards mathematics for secondary and tertiary students (Tapia & Marsh, 2004). The course assessments were developed by an experienced instructor, and similar assessments have been used for over 10 years at the university, showing credibility and replicability. Additionally, the study took place over the course of 5 terms, allowing the researcher to collect data from 121 research participants.

3.6.2 **Trustworthiness in qualitative data: Credibility & dependability**

The trustworthiness of qualitative data is measured differently than quantitative data. Instead of validity and reliability considered with quantitative data, the quality of qualitative data depends on its credibility and dependability (Creswell, 2015).

3.6.2.1 Credibility

Credibility measures the truth or accuracy of the qualitative data; credibility is extremely important when using qualitative data to interpret results (Creswell, 2015); credibility in qualitative data is equivalent to internal validity of quantitative data. This research study requires credibility to demonstrate confidence in the results (Cohen et al., 2011). In this case, the credibility is dependent on the participants, the interviewer, and researcher to accurately record and report their experiences.

One method by which to establish credibility is by prolonged engagement between the researcher and participants. This allows the researcher time to know each of the participants individually and collectively. In this research study, each cohort of students took the course over a 16-week term; the researcher-instructor met with the students 150-minutes per week (both in the face-to-face classes and in the synchronous online classes). This allowed the researcher-instructor sufficient time to get to know the participants well, and to take research notes throughout the course.

Another method by which this research established credibility is through triangulation. Cohen et al. define triangulation as “the use of two or more methods of data collection” (2011, p.195) and allows the researcher to have increased confidence in the data and results. This study included time triangulation, where data was collected at the start and end of the term, throughout each term, and several weeks after the term had ended, as well as methodological triangulation, where different data gathering methods (semi-structured interviews and instructor reflections) were used. However, only thirteen of the 121 research participants took part in the semi-structured interviews; these interviews took place several weeks after each term had ended. Conclusions and explanations will be cautiously made from the qualitative data because of the time lag in the collection of the interviews and the small number of participant interviews.

3.6.2.2 Dependability

Dependability in qualitative data is equivalent to reliability of quantitative data (Creswell, 2015). Dependability refers to the ability for another researcher to understand the methodological decisions made by the researcher. Throughout this study, the objectives and goals were outlined, the research design and results have been presented, and the techniques for analyzing the results are explained, to establish dependability and transparency in the data collection and interpretation of the results.

3.6.3 **Ethical considerations**

Research ethics were upheld throughout this study. First, permission was sought and granted from Curtin University Human Research Ethics Committee and the university Institutional Review Board before the study commenced. At the start of the term, students were given information about the study and how the data will be used so that they have full comprehension of the research project. All students were adult undergraduate students (aged 18-20 years old) and enrolled in the general education mathematics course for first-year students at a private liberal arts university in Southern California. A colleague in the department came to class during the first week, and recruited students (Appendix B); this colleague recruited participants so that students did not feel pressure to participate in the research study, and so they clearly understood their grades would not be impacted by their participation or non-participation in the research. For the face-to-face terms (Fall 2019, Spring 2020, Fall 2021), students were read and given a printed copy of the *Participant Information Statement* (Appendix C) and *Participant Consent Form* (Appendix D). All efforts to fulfill the four elements of informed consent were made (Cohen et al., 2011) and each student was asked to provide their informed consent to participate in this study by signing and returning the *Participant Consent Form* to a departmental colleague. The departmental colleague kept the *Participant Consent Forms* for each research participant for the entire term, to ensure confidentiality and privacy of the participants. The researcher-educator had no knowledge of which students in the class were participants in the study throughout each term; this was done to lessen the perceived power of the researcher-educator on the students to participate in the study.

Student filled out surveys electronically, using a coded number consisting of the first three letters of their mothers' first names, their eight-digit birth date (YYYYMMDD); only the students knew their coded numbers to protect the privacy of all the participants. Throughout the term, the researcher-instructor had no knowledge of which students were participating in the research, and could not identify individual student responses. Student participation in the research was kept confidential to avert any power imbalance; data was collected for all students enrolled in the course throughout the term, and then specific data for the research participants was compiled after the term was complete and final grades submitted. This included course grades (quizzes, exams, and final exam) as well as ATMI survey results. Thus, research participants could know that there was no impact on their course grade or how they were treated in class by their participation or non-participation in the research. All efforts to treat students with fairness, respect and confidentiality, and all efforts were made to teach both groups with fairness, dignity and respect, with equal days spent learning each topic in the different sections.

The project was approved by both the Curtin University Office of Research and Development (Appendix E) as well the researcher-educator's home institution Institutional Review Board (Appendix F) before data collection began in Fall 2019.

3.7 Chapter Summary

This chapter has outlined the methods used to examine the impact of inquiry-based learning on student understanding of key ideas, and attitude towards mathematics in a general education mathematics class at a liberal arts university. It reviewed the conceptual framework for this study and explained the rationale which led to the mixed-methods approach to the study. This chapter also detailed the population and sample which was used in this study, and how the inquiry-based learning classroom was developed and implemented.

This chapter explained how through the mixed-methods approach, this study incorporated quantitative data collected from course assessments as well as survey data from the Attitude

Towards Mathematics Inventory (Appendix A), as well as qualitative data collected from students semi-structured interviews and educator/researcher reflections. The methods of data analysis which were used throughout this study were also described. Additionally, the chapter discussed the methods used to ensure the validity and trustworthiness of the study, as well as the rigor and quality of the studied. The ethical considerations for this study were reviewed, as well as the techniques used by the educator/researcher to ensure that the participants' rights were protected throughout the study.

4 Data Analysis and Results

4.1 Introduction

This study used course assessments, as well as the Attitudes Towards Mathematics Inventory (ATMI) (Appendix A), to determine possible associations between the teaching method and modality. The course assessments measured understanding of key concepts in the course, while the ATMI was used to assess the change in student self-confidence, perception of value of mathematics, enjoyment of mathematics and motivation towards mathematics. The course assessments were given throughout the term, while the ATMI was administered at the start and end of each term. Other quantitative data were collected through course assignments and assessments. Qualitative data were obtained through interviews from a subsample of the participants completing these instruments, and from the researcher's notes and observation throughout the terms.

A statistical analysis was performed on the scores of the final exam, overall course grades, and change in scores on the 40-question ATMI for the four factors measured (self-confidence in mathematics, perception of the value of mathematics, enjoyment of mathematics and motivation towards mathematics) for the participants who were enrolled in the interactive lecture course, with those enrolled in the inquiry-based learning course. The change in score was determined by comparing the pre-course and post-course scores, as reported on the questionnaire at the start and end of the course.

This chapter will introduce and report the different analyses completed to answer the research questions of this study. Section 4.2 includes a description of the sample and research participants, including which instructional method (inquiry-based learning or interactive lecture) was used in their course section. Section 4.3 reviews the research questions, purpose, and conceptual framework for this research study. Section 4.4 compares the quantitative data from the inquiry-based learning and interactive lecture groups, comparing both results of the ATMI survey as well as data collected from in-class assessments. Data analysis compares the mean and standard deviation of the results from each group, and results are presented. The following sections (Section 4.5, Section 4.6, Section 4.7) discuss the analysis of the quantitative data from the study. Further analysis of these results, as explained by the qualitative data, is described in

Section 4.8, including both the semi-structured interviews as well as the instructor reflections. The chapter will be summarized in Section 4.9.

4.2 Description of the sample

As described in Chapter 3, the study took place in a first-year general education mathematics class at a small, private liberal arts university in Southern California. The researcher was the instructor for the courses sampled in this study; hence, the sample is a convenience sample. Each term, two to three other mathematics professors also taught the same course, but to limit confounding variables that would be introduced by including other educators, only students in the researcher-educator's sections of the course were considered as the population, or potential participants, in this research study. Students were placed in the sections of their courses by the Academic Advisors during their first semester at the university (each Fall), and they self-selected the section of the courses they take during the spring semester each year. Thus, the sample is not a random sample of the students enrolled at the university (as their choice of courses was influenced by other schedules – student work schedules, sports practices, preference of time of day for classes, etc.). Additionally, the research participants self-selected to take part in this study, which is not a random sample from the population of students in each class.

The sample included 121 participants in 11 sections of the course over 5 different terms, as shown below in Table 4-1. The original research design for this study was to explore only traditional face-to-face classes, but because of the COVID-19 global pandemic, three of the terms studied were taught face-to-face, while two were taught fully online during the height of the COVID-19 pandemic.

The COVID-19 outbreak did have an impact on this study. Students enrolled in the course in Spring 2020 were sent home, by state and federal mandate in the United States, after 12 weeks of face-to-face instruction. The final two weeks of the semester were conducted synchronously online using Blackboard Collaborate, but since the majority of the course was taught in person and on campus, the data from those students are included as face-to-face sections of the course. (Note – in subsequent semesters, where courses were taught fully online, the University switched to Zoom for all synchronous courses.)

Table 4-1
Participants in the research study, by term, teaching method and modality

Term		Enrolled		Participants	
Teaching method	Modality	Males	Females	Males	Females
Fall 2019					
Interactive lecture	Face-to-face	11	17	8	13
Inquiry-based learning	Face-to-face	11	17	9	13
Spring 2020					
Interactive lecture	Face-to-face	10	16	3	7
Inquiry-based learning	Face-to-face	15	10	6	3
Fall 2020					
Interactive lecture	Online	8	16	2	6
Inquiry-based learning	Online	6	19	2	7
Inquiry-based learning	Online	9	17	0	2
Spring 2021					
Interactive lecture	Online	10	15	1	9
Inquiry-based learning	Online	12	14	3	6
Fall 2021					
Interactive lecture	Face-to-face	16	8	4	3
Inquiry-based learning	Face-to-face	8	18	5	9
TOTAL		**116**	**167**	**43**	**78**

The highest level of participation in a class was during Fall 2019, the first term of study, in which 43 of the 56 students (76.7%) enrolled in the course consented to participate in the research study. The lowest level of participation was during Fall 2020, in which only 19 of the 75 students (25.3%) enrolled in the course consented to participation. The low participation during Fall 2020 was not surprising, as students enrolled that term were beginning their first year of college during a global pandemic, and were taking courses both remotely and fully online; hence, the low participation rate was expected. Of the 283 students enrolled in the course during the five terms studied, 41.0% were male and 59.0% were female, while 35.5% of the research participants were male and 64.5% were female.

4.3 Review of purpose, conceptual framework and research questions

As introduced in Chapter 1, the purpose of this explanatory mixed methods research study is to study the impact of inquiry-based learning in a general education mathematics classroom on students' understanding of core concepts and attitude towards mathematics. The purpose of this research is based on the conceptual framework outlined in Section 1.10, which seeks to determine how inquiry-based learning can create a new learning environment and experience for students, potentially influencing both the cognitive and affective learning domains.

To determine the impact of inquiry-based learning and to clearly define the goal of this research study, the primary research question was developed: *How does an inquiry-based learning GE mathematics course for first year students differ from a traditional interactive lecture course in terms of students' understanding of key topics, attitudes towards mathematics, and mathematical self-concept?*

After the primary research question was developed, secondary research questions were generated, which clearly define the data and measures needed for the research: *What impact does inquiry-based learning have on students' achievement in a face-to-face GE mathematics course at a liberal arts university, as compared to learning by interactive lecture?*

Additionally, because this research spanned the 2020-2021 global pandemic, where college campuses were shut and instruction was delivered remotely online, there was an opportunity to include an additional research question: *How do the cognitive and affective results in an inquiry-based learning GE mathematics course differ from an interactive lecture course, in a fully online classroom?*

As this is an explanatory mixed methods research study, quantitative data was initially collected, including administering the Attitudes Towards Mathematics Inventory (ATMI), and collecting achievement data (quizzes, exams) throughout the course. This was followed by semi-structured interviews and instructor reflections. The data was then triangulated to make conclusions about the impact that inquiry-based learning had on the students' achievement.

4.4 Quantitative data analysis: Comparing instructional groups

This section investigates the mean and standard deviation of quantitative data collected from the research participants. The ATMI was administered at the start and end of each term of study; descriptive statistics as well as the alpha reliability for the scores was calculated.

4.4.1 Attitude Towards Mathematics Inventory reliability

First, the reliability and validity of the ATMI was established. These measures attempted to identify a level of confidence that researchers can have in the results obtained from these instruments.

The Cronbach's alpha score for the ATMI was used to assess the internal consistency of the questionnaire. The questionnaire has four subscales, so each was assessed separately; the results of the Cronbach alpha reliability are shown in Table 4-2.

Table 4-2
Mean, standard deviation, internal consistency (Cronbach's alpha reliability) scores for the four subscales of the Attitudes Towards Mathematics Inventory

Scale	*No. of Items*	*Mean*	*Standard Deviation*	*Alpha reliability*
Self-Confidence	15	48.48	12.63	0.96
Value	10	37.66	5.87	0.88
Enjoyment	10	32.97	7.76	0.91
Motivation	5	14.50	4.43	0.86

N = 121 students, Classes = 9

Thus, with these Cronbach alpha scores of 0.86 and higher, the results from the ATMI were used to answer the research questions. The Cronbach alpha score indicates the consistency of the participant's responses over the questions answered, and measures the reliability of the survey. Because the Cronbach alpha is 0.86 ($\alpha > 0.70$), the researcher can have high confidence in the results obtained from this instrument.

In the table below, the scale factors from the ATMI were considered separately. The mean and standard deviation, were given for the pre- and post-tests. The Statistical Package for Social Sciences (SPSS) was used to conduct statistical testing of the data. Because the results were non-normal, the non-parametric Wilcoxon sign rank test was performed in SPSS. The z-scores and p-values for the Wilcoxon sign rank test are given in Table 4-3, and the effect size of this test ($r = \frac{z}{\sqrt{n}}$) are given.

Table 4-3
Effect sizes and p-values for differences between Week 1 survey and Week 15 survey, for interactive lecture and IBL groups, all teaching modalities, for the ATMI

ATMI Scale Factor	Group	Week 1 Mean and Standard Deviation ($\bar{x} \pm s$)		Week 1 Median ($\tilde{x}$)	Week 15 Mean and Standard Deviation ($\bar{x} \pm s$)		Week 15 Median ($\tilde{x}$)	Wilcoxon Sign Rank p-value	Effect size ($r = Z/\sqrt{n}$
Self-Confidence	Lecture	3.27±0.80		3.43	3.49±0.78		3.67	$z = -1.91$ $p = 0.06$	$r = -0.31$
	IBL	3.03±0.87		3.07	3.19±0.86		3.20	$z = -2.44$ $p = 0.15$	$r = -0.36$
	Mann-Whitney U p-value		0.17			0.05			
	Effect size		-0.15			-0.19			
Value	Lecture	3.67±0.58		3.70	3.92±0.67		3.90	$z = -3.34$ $p < 0.001$	$r = -0.57$
	IBL	3.72±0.48		3.70	3.76±0.58		3.70	$z = -0.57$ $p = 0.57$	$r = -0.09$
	Mann-Whitney U p-value		0.86			0.08			
	Effect size		-0.02			-0.17			
Enjoyment	Lecture	3.30±0.74		3.30	3.54±0.75		3.50	$z = -2.62$ $p = 0.01$	$r = -0.44$
	IBL	3.12±0.73		3.20	3.27±0.83		3.35	$z = -1.93$ $p = 0.05$	$r = -0.28$
	Mann-Whitney U p-value		0.19			0.08			
	Effect size		-0.13			-0.17			
Motivation	Lecture	2.90±0.84		2.70	3.02±0.95		3.00	$z = -0.32$ $p = 0.75$	$r = -0.06$
	IBL	2.85±0.81		2.80	2.85±0.95		3.00	$z = -0.37$ $p = 0.72$	$r = -0.06$
	Mann-Whitney U p-value		0.89			0.46			
	Effect size		-0.01			0.07			
Effect sizes:		Small effect: <0.30		Medium effect: 0.30<0.50		Large effect: >=0.50			$p < 0.05$

From Table 4-3, the change in self-confidence from Week 1 to Week 15 is significant, with p = 0.05, with a small effect ($r = -0.31$ for interactive lecture participants and $r = -0.36$ for IBL students). The interactive lecture students showed significance in their view of the value of mathematics ($z = -3.34$, $p < 0.001$, with a large effect size of $r = -0.57$). Similarly, both the interactive lecture and inquiry-based learning groups showed significant results in their perception of the enjoyment of mathematics (highlighted in green); while the inquiry-based learning group showed a small effect, the interactive lecture showed a medium effect size.

4.4.2 In-class assessments results

The mean scores and standard deviation were found for each cohort of students, by teaching method (interactive lecture and inquiry-based learning) and teaching modality.

Table 4-4
Comparison of means and standard deviations of Final Exam and Overall Course scores for the Interactive Lecture and Inquiry-Based Learning participants

		Interactive lecture	Inquiry-based learning
	N	56	65
Final exam scores	Mean	84.98	79.87
	Standard deviation	11.80	15.75
Overall grade	Mean	82.10	76.25
	Standard deviation	11.56	13.14

An analysis of variance was conducted in SPSS to compare the standard deviation of the final exam scores for the two teaching methods. The difference in final exam score standard deviations yielded an *F* ratio of $F(55,64) = 0.56$, $p = 0.03 < \alpha = 0.05$, indicating that the inquiry-based learning participants ($M = 84.98$, $SD = 11.80$) has more variation in their final exam scores than the interactive lecture participants ($M = 79.87$, $SD = 15.75$). Similarly, an analysis of variance was conducted to compare the standard deviation of the overall grades for the interactive lecture participants and inquiry-based learning participants. The difference in overall grades scores was non-significant, with an *F* ratio of $F(55,64) = 0.77$, $p = 0.33$. There is not sufficient evidence to suggest the inquiry-based learning group varies more than the interactive lecture group. Hence, in further testing, the overall grades will be used, as the variances are assumed to be equal.

4.5 Research Question One

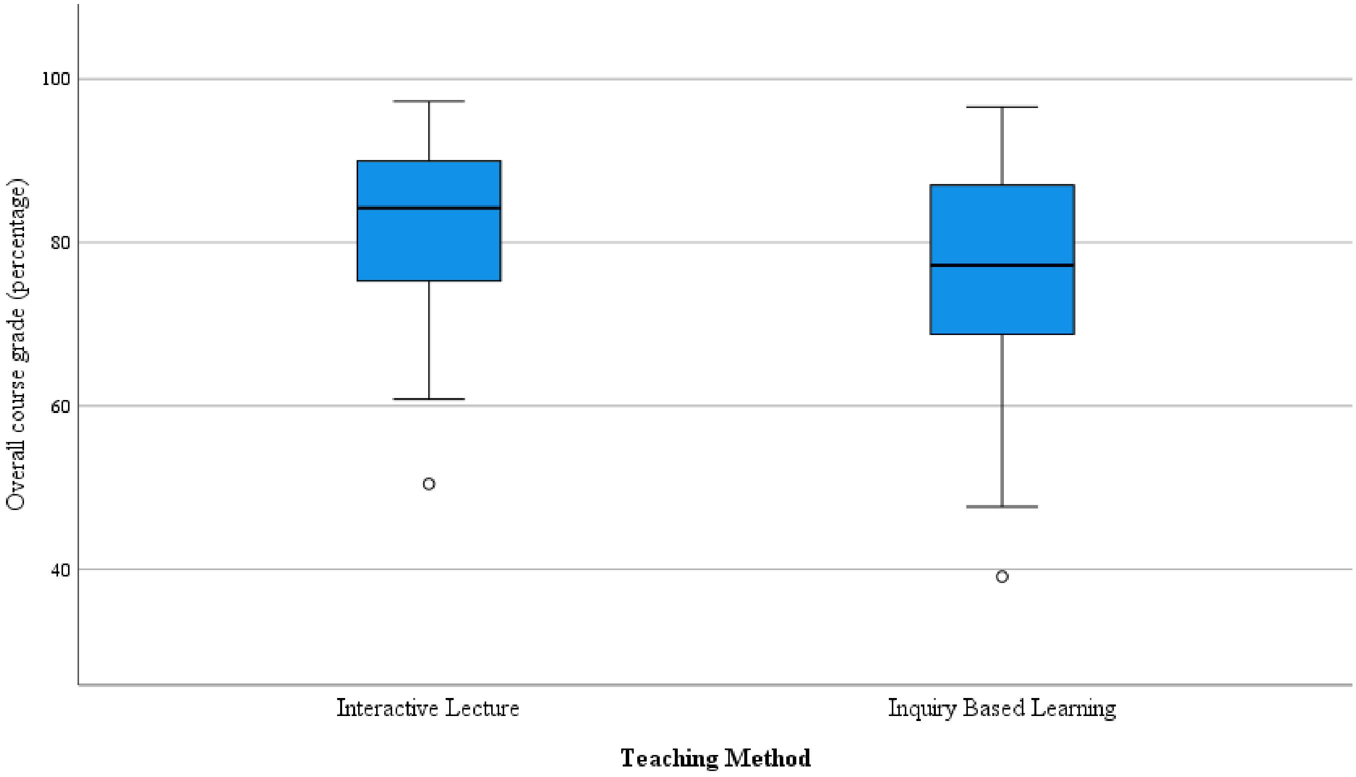
How does an inquiry-based learning GE mathematics course for first-year students differ from a traditional interactive lecture in terms of students’ understanding of key concepts, attitudes towards mathematics and mathematical self-concept?

4.5.1 Impact of IBL on students' understanding of key concepts

4.5.1.1 Assessments

To assess student understanding of core content, the average grades for all in-class assessments were compared for students who were enrolled in the interactive lecture course and those enrolled in the inquiry-based learning course. These assessments included in-class quizzes, exams and the final exam, and were determined by the total points earned by the participant divided by the total number of points possible for these assessments in the class. Homework assignments and other assignments completed in unproctored environments and in which students could have accessed additional resources (internet, tutors, etc.) were not included in their overall assessment score. Figure 4-1 shows the comparison of these overall scores. An independent samples *t*-test was run in SPSS to compare the overall course scores for the participants enrolled in the interactive lecture course (*n* = 56) with the mean overall course scores for the participants enrolled in the inquiry-based learning course (*n* = 65).

Figure 4-1
Comparison of overall scores on all assessments achieved by the participants in Interactive Lecture and Inquiry-Based Learning courses



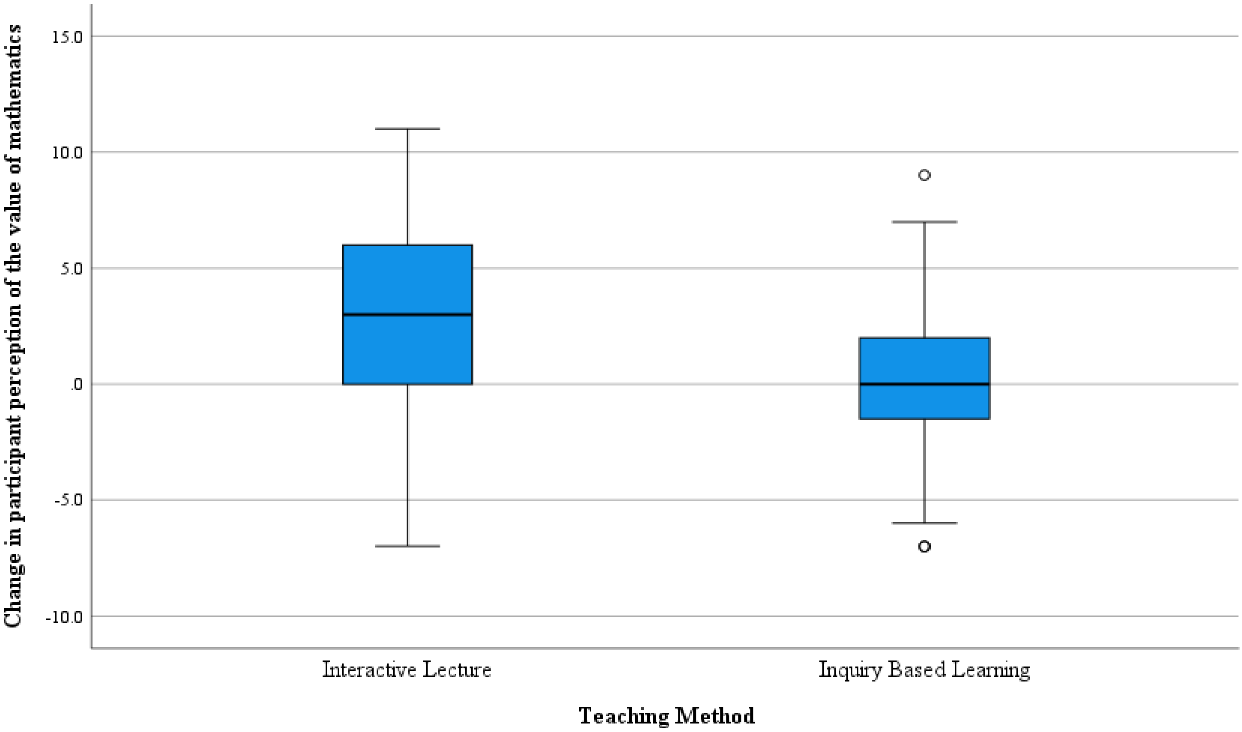
To test for normality, the Shapiro-Wilk statistic was considered, and both cases were considered non-significant ($W = 0.940$ and sig = 0.968 > 0.05), indicating normality of the data was not violated. An independent sample *t*-test was conducted, and Levine's test for equality of variances was non-significant ($F = 3.395$ and sig = 0.068 > 0.05). Therefore, equal variances are assumed. The *t*-test value is statistically significant with $p = 0.015$, with the Interactive Lecture group ($M = 81.640$, SD = 10.470) with a higher mean score than the Inquiry-Based Learning group. This *t*-test reported scores 5.447 points higher scores for the Interactive Lecture group, 95% CI [1.064, 9.712], than for the Inquiry-Based Learning cohort ($M = 76.252$, SD = 13.135), $t(119) = 2.467$, $p = 0.015$, two-tailed, $d = 0.450$. Since $p = 0.015 < 0.05$ and Cohen's $d = 0.450$ represents a medium effect size, there is a difference between the mean overall course grade of the participants enrolled in courses taught by the two different instructional methods.

4.5.2 Students' attitudes towards mathematics from ATMI

4.5.2.1 Students' perceptions on the value of mathematics

An independent samples *t*-test was used to compare the change in scores reported on the ATMI by the interactive lecture participants (*n* = 39) and the inquiry-based learning participants (*n* = 51). It should be noted that this sample is smaller than the total number of participants, as not all of the research participants filled out both the pre- and post-survey; this was done during class time, but since the research participants were unknown to the researcher instructor, there was no way during the term to determine if the participants had filled out each survey. Neither Shapiro-Wilk statistic was significate, indicating the assumption of normality was not violated. Levene's test was also non-significant, thus equal variances can be assumed. The *t*-test was statistically significant, with the interactive lecture group (*M* = 2.87, *SD* = 4.44) reporting change in their reported scores of the value of mathematics higher, 95%CI [0.94, 4.32], than the inquiry-based learning group (*M* = 0.24, *SD* = 3.61), *t*(88) = 3.106, *p* = 0.003, two-tailed, *d* = 0.66. A box plot comparing the scores for these two instructional groups is shown in Figure 4-2.

Figure 4-2
Comparison of the change in student perception of the value of mathematics, as reported on the ATMI



To further investigate these differences, ten of the 40 questions in the ATMI asked students to rate their perception of the value of mathematics. Paired *t*-tests were done for each of these questions, comparing the pre-test average for participants who were enrolled in the interactive lecture course, with the post-test average for the same participants. Table 4-5 compares the pre- and post-average of the students enrolled in the interactive lecture course. Of the ten questions pertaining to the value of mathematics, statistically significant differences were found in nine questions at the $\alpha = 0.05$ significance level (and eight would be statistically significant at the $\alpha = 0.01$ significance level). The *p*-value for the questions that were significantly significant at the $\alpha = 0.05$ level are highlighted in light green, while the *p*-value for the questions that were significantly significant at the $\alpha = 0.01$ level are highlighted in dark green. These results provide strong evidence of significant change, supporting the *t*-test scores reported above.

Table 4-5
ATMI survey results for the Value of mathematics, for students in Interactive Lecture course

Question	**Pre avg.**	**Post avg.**	**Diff. in avg.**	***t*-test *p*-value**
1. Mathematics is a very worthwhile and necessary subject.	3.73	4.23	.50	0.0039
2. I want to develop my mathematical skills.	3.77	4.23	.46	0.0093
4. Mathematics helps develop the mind and teaches a person to think.	3.86	4.28	.42	<0.0001
5. Mathematics is important in everyday life.	3.52	4.11	.59	0.0004
6. Mathematics is one of the most important subjects for people to study.	3.16	3.81	.65	0.0013
7. High school math courses would be very helpful no matter what I decide to study.	3.20	3.74	.54	0.0064
8. I can think of many ways that I use math outside of school.	3.89	4.11	.22	0.1245
35. I think studying advanced mathematics is useful.	3.07	3.62	.55	0.0099
36. I believe studying math helps me with problem solving in other areas.	3.48	4.00	.52	0.0013
39. A strong math background could help me in my professional life.	3.59	3.96	.37	0.0386

However, when the same 10 questions for student perceptions of the value of mathematics are analyzed, different results occur. Table 4-6 shows that none of the questions produced statistically significant results (at any standard significance level) for students enrolled in an IBL course.

Table 4-6
ATMI survey results for the Value of mathematics, for students in an IBL course

Question	**Pre avg.**	**Post avg.**	**Diff. in avg.**	***t*-test *p*-value**
1. Mathematics is a very worthwhile and necessary subject.	3.89	4.13	.24	0.0976
2. I want to develop my mathematical skills.	3.98	4.14	.16	0.2201
1. Mathematics helps develop the mind and teaches a person to think.	4.18	4.18	.00	0.9999
5. Mathematics is important in everyday life.	3.77	3.86	.09	0.5251
6. Mathematics is one of the most important subjects for people to study.	3.55	3.77	.22	0.2106
7. High school math courses would be very helpful no matter what I decide to study.	3.46	3.46	.00	0.9999
8. I can think of many ways that I use math outside of school.	3.71	3.89	.18	0.2382
35. I think studying advanced mathematics is useful.	3.23	3.30	.07	0.1552
36. I believe studying math helps me with problem solving in other areas.	3.70	3.75	.05	0.7301
39. A strong math background could help me in my professional life.	3.48	3.70	.22	0.2279

4.5.2.2 Students' perceptions on their enjoyment of mathematics

An independent sample *t*-test was used to compare these changes reported in student's ratings of their enjoyment of mathematics on the Attitudes Towards Mathematics Inventory. The data was normal ($W = 0.969$ and 0.980) and equal variances were assumed ($F = 1.470$ and sig = 0.229 > 0.05), but the *t*-test was found to be not significant $t(88)=1.115$, p = 0.268, two-tailed. Thus, there was not enough evidence to suggest that there is a difference between the mean enjoyment scores of the two cohorts.

Ten of the questions in the ATMI asked students to rate their own enjoyment of mathematics. A paired *t*-test was again conducted, to determine if there were any significant changes within each cohort. (In the table below, reverse coded questions are marked with a *; those items have been decoded before analysis.) Table 4-7 shows significant change (at $\alpha = 0.5$ in light green, or $\alpha = 0.01$ in dark green), showing statistically significant improvement in the perception of students' enjoyment of mathematics in those students enrolled in an interactive lecture course. These results provide strong evidence of change in these participants.

Table 4-7
ATMI survey results for the Enjoyment of mathematics, for students in an Interactive Lecture course

Question	Pre avg.	Post avg.	Diff. in avg.	*t*-test *p*-value
3. I get a great deal of satisfaction out of solving a mathematics problem.	3.39	4.04	.65	0.0016
24. I have usually enjoyed studying mathematics in school.	2.84	3.28	.44	0.0666
25.* Mathematics is dull and boring.	3.05	3.55	.50	0.0059
26. I like to solve new problems in mathematics.	2.95	3.53	.58	0.0036
27. I would prefer to do an assignment in math than to write an essay.	3.34	3.98	.64	0.0159
29. I really like mathematics.	2.66	3.19	.53	0.0239
30. I am happier in a math class than in any other class.	2.16	2.62	.46	0.0289
31. Mathematics is a very interesting subject.	3.16	3.64	.48	0.0100
37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.	3.00	3.49	.49	0.0120
38. I am comfortable answering questions in math class.	3.02	3.51	.49	0.0138

However, Table 4-8 shows corresponding results for students enrolled in an IBL section of the course. Only one question of the 10 rating the enjoyment of the enjoyment in mathematics showed significant change over the semester for the students in the IBL course. It should be noted that averages for all ten questions about student enjoyment of mathematics which were asked before the start of the course are higher for the IBL course students than they are for the interactive lecture students. Higher pre-course starting averages gives those cohorts less room to move, but the six of the post-test averages for the interactive lecture groups were higher than those of the IBL students. One post-test question average was higher for the IBL students, and three of the post-test question averages were within 0.01 of one another, so considered equal.

Table 4-8
ATMI survey results for the Enjoyment of mathematics, for students in an IBL course

Question	Pre avg.	Post avg.	Diff. in avg.	*t*-test *p*-value
3. I get a great deal of satisfaction out of solving a mathematics problem.	3.82	3.91	.09	0.6172
24. I have usually enjoyed studying mathematics in school.	3.13	3.14	.01	0.9652
25.* Mathematics is dull and boring.	3.30	3.54	.24	0.0075
26. I like to solve new problems in mathematics.	3.41	3.55	.14	0.4422
27. I would prefer to do an assignment in math than to write an essay.	3.59	3.70	.11	0.6515
29. I really like mathematics.	3.02	3.25	.23	0.2860
30. I am happier in a math class than in any other class.	2.55	2.61	.06	0.7683
31. Mathematics is a very interesting subject.	3.34	3.43	.09	0.6264
37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.	3.41	3.45	.04	0.8243
38. I am comfortable answering questions in math class.	3.48	3.43	-.05	0.7770

4.5.2.3 Students' perceptions of their self-confidence in mathematics

An independent sample *t*-test was used to compare the change in student's ratings of their self-confidence in mathematics on the Attitudes Towards Mathematics Inventory. The Shapiro-Wilk statistic was considered, and both cases were considered significant ($W = 0.965$ and 0.963), again indicating the *t*-test assumption of normality was not violated. Levine's test for equality of variances was also not significant ($F = 0.753$ and sig = 0.388 > 0.05); therefore, equal variances are assumed. The *t*-test value is not statistically significant, with the Interactive Lecture group ($M = 2.539$, SD = 8.519) reporting a similar mean as the Inquiry-Based Learning cohort ($M = 2.549$, SD = 7.220), $t(88) = -0.006$, $p = 0.995$, two-tailed, $d = -0.0013$. Further analysis was conducted on the ATMI questions within the cohorts of students, to determine if any significant changes were detected within the cohorts.

Fifteen of the 40 questions on the AMTI asked participants questions rating their own self-confidence in mathematics. A paired *t*-test was used compare the pre- and post-courses participant responses for this subscale on the ATMI, as shown in Table 4-9. In the participants enrolled in the interactive lecture sections of the course, a significant change (at $\alpha = 0.05$) in eleven items on the paired *t*-test comparing pre- and post-test responses, as shown in Table 4-9.

Table 4-9

ATMI survey results for Confidence in mathematics, for students in Interactive Lecture course

Question	**Pre avg.**	**Post avg.**	**Diff. in avg.**	***t*-test *p*-value**
9. *Mathematics is one of my most dreaded subjects.	3.20	2.85	-.35	0.2293
10. *My mind goes blank and I am unable to think clearly when working with mathematics.	3.18	2.55	-.63	0.0141
11. *Studying mathematics makes me feel nervous.	3.16	2.51	-.65	0.0047
12. *Mathematics makes me feel uncomfortable.	3.02	1.94	-1.08	<0.0001
13. *I am always under a terrible strain in a math class.	2.89	2.23	-.66	0.0034
14. *When I hear the word mathematics, I have a feeling of dislike.	3.09	2.34	-.75	0.0040
15. *It makes me nervous to even think about having to do a mathematics problem.	2.84	2.17	-.67	0.0035
16. Mathematics does not scare me at all.	2.84	3.06	.22	0.3662
17. I have a lot of self-confidence when it comes to mathematics.	2.66	2.98	.32	0.1821
18. I am able to solve problems without too much difficulty.	2.93	3.19	.26	0.2092
19. I expect to do fairly well in any math class I take.	3.07	3.55	.48	0.0105
20. *I am always confused in mathematics class.	2.95	2.19	-.76	0.0001
21. *I feel a sense of insecurity when attempting mathematics.	3.20	2.62	-.58	0.0098
22. I learn mathematics easily.	3.70	3.21	-.49	0.0219
40. I believe I am good at solving math problems.	3.07	3.47	.40	0.0378

Student responses to their confidence in mathematics decreased in their responses to ten of the questions; nine of these changes were statistically significant (highlighted in green). Questions 11, 12, 15, and 16 showed statistically significant decreases in areas relating to comfortability in mathematics, and questions 10, 14, 19, 20, 22, and 40 showed statistically significant decrease in students' perception of doing well in mathematical class. It might be argued that a lower significance level (lower than $\alpha = 0.05$) should be used here, since multiple data questions are being analyzed. This Bonferroni adjustment, with a lower significance level ($\alpha = 0.01$), would yield seven of the items to be considered statistically significant and indicative of significant change. (The questions that would show statistical significance at the lower significance level are highlighted in dark green.)

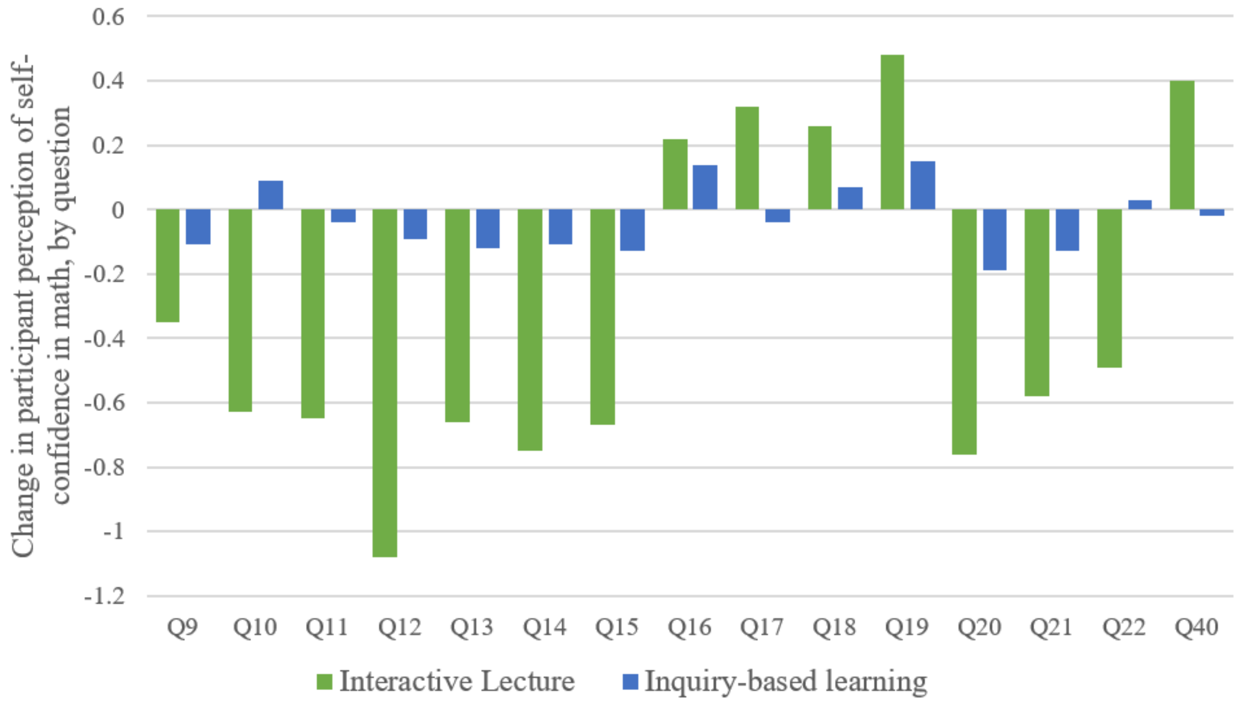
Table 4-10
ATMI survey results for Confidence in mathematics, for students in an IBL course

Question	**Pre avg.**	**Post avg.**	**Diff. in avg.**	***t*-test *p*-value**
9. *Mathematics is one of my most dreaded subjects.	3.07	2.96	-.11	.6553
10. *My mind goes blank and I am unable to think clearly when working with mathematics.	2.64	2.73	.09	.6403
11. *Studying mathematics makes me feel nervous.	2.86	2.82	-.04	.8376
12. *Mathematics makes me feel uncomfortable.	2.61	2.52	-.09	.6690
13. *I am always under a terrible strain in a math class.	2.64	2.52	-.12	.5392
14. *When I hear the word mathematics, I have a feeling of dislike.	2.84	2.73	-.11	.6219
15. *It makes me nervous to even think about having to do a mathematics problem.	2.64	2.61	-.03	.8796
16. Mathematics does not scare me at all.	3.04	3.18	.14	.4947
17. I have a lot of self-confidence when it comes to mathematics.	2.93	2.89	-.04	.8328
18. I am able to solve problems without too much difficulty.	3.14	3.21	.07	.6989
19. I expect to do fairly well in any math class I take.	3.48	3.63	.15	.4104
20. *I am always confused in mathematics class.	2.71	2.52	-.19	.2612
21. *I feel a sense of insecurity when attempting mathematics.	2.86	2.73	-.13	.4886
22. I learn mathematics easily.	3.11	3.14	.03	.8678
40. I believe I am good at solving math problems.	3.48	3.46	-.02	.9098

For these questions which pertain to the confidence of a student in mathematics, no questions showed statistical significance (at any reasonable alpha value) for students enrolled in the inquiry-based learning course, as shown in Table 4-10. Thus, the slight changes in these questions are not due to significant change in the self-confidence of the students.

Figure 4-3 compares the average differences for each of the 15 questions relating to self-confidence on the ATMI. The interactive lecture average differences are in green, while the inquiry-based learning average differences are in blue. The average change in participant perception of their self-confidence is *negative* for ten of the questions; this will be further discussed in Chapter 5.

Figure 4-3
Comparison of the change in student perception of their self-confidence in mathematics, as reported on the ATMI



4.5.2.4 Students' perceptions of their motivation in mathematics

An independent sample *t*-test was used to compare the change in these participants' ratings of their motivation in mathematics on the Attitudes Towards Mathematics Inventory. The data was normal ($W = 0.983$ and 0.979) and equal variances were assumed ($F = 0.228$ and Sig = 0.634), but the *t*-test was found to be not significant $t(88)=0.618$, $p = 0.538$, two-tailed. Thus, there was not enough evidence to suggest that there is a difference between the mean motivation scores of the two cohorts.

Additional analysis was performed on the five questions from the ATMI which pertained to motivation in mathematics. Table 4-11 shows the results of this item analysis.

Table 4-11
ATMI survey results for Motivation in mathematics, for students in an Interactive Lecture course

Question	**Pre avg.**	**Post avg.**	**Diff. in avg.**	***t*-test *p*-value**
23. I am confident that I could learn advanced mathematics.	2.84	3.60	.76	.0022
28.* I would like to avoid using mathematics in college.	2.98	2.45	-.53	.0133
32. I am willing to take more than the required amount of mathematics.	2.52	2.83	.31	.2021
33. I plan to take as much mathematics as I can during my education.	2.32	2.66	.34	.1419
34. The challenge of math appeals to me.	2.75	3.21	.46	.0372

Of these 5 questions pertaining to motivation in mathematics, three showed statistical significance at the $\alpha = 0.05$ level (and it should be noted that only one question, 23, showed statistical significance at the $\alpha = 0.01$ level).

The results for the Motivation item analysis for students enrolled in a IBL section of the course are shown below, in Table 4-12. None of the questions showed statistical significant changes at any significance level for students who were enrolled in an IBL course.

Table 4-12
ATMI survey results for Motivation in mathematics, for students in an IBL course

Question	**Pre avg.**	**Post avg.**	**Diff. in avg.**	***t*-test *p*-value**
23. I am confident that I could learn advanced mathematics.	3.05	3.38	.33	.1185
28.* I would like to avoid using mathematics in college.	3.13	2.82	-.31	.1513
32. I am willing to take more than the required amount of mathematics.	2.52	2.80	.28	.1744
33. I plan to take as much mathematics as I can during my education.	2.18	2.50	.32	.1192
34. The challenge of math appeals to me.	3.04	3.16	.12	.5618

4.5.3 Effect of teaching method by gender

A 2x2 factorial between group analysis (ANOVA) was used to investigate the effects the teaching method and gender had on the overall course grade for all participants in the study. A Shapiro-Wilk test was performed to ensure the assumptions of normality and homogeneity of variance were not violated. The interaction of teaching method and gender was not found to be significant, ($F = 0.261$, Sig = 0.610); there was no evidence to show that there is a significant interaction between teaching method and gender on participant course grades.

4.5.4 Correlation between grades and change in self-confidence

The relationship between a student grade in the course and mathematical self-confidence was considered, to see if higher grades produced a greater change in self-confidence. To assess the size and direction of this relationship between the overall course grade and the change in self-confidence of participants in the course, a bivariate Pearson's coefficient was considered. However, the Shapiro-Wilk test revealed that the normality assumption was violated ($W = 0.973$, *Sig* = 0.057 and $W = 0.928$, $p < 0.001$), so a Spearman's Rho test was performed instead. Each participant provided independent data and both scales are ordinal, so the assumptions were met. However, Spearman's Rho did not indicate the presence of a strong correlation between student overall grades and the change in self-confidence, $rs = 0.191$, $p = 0.071$, two-tailed, $N = 90$.

4.5.5 Relationship between gender and teaching method and modality

To estimate the proportion of variance of the overall course grades in the course that can be accounted for by the student's gender, teaching method, and modality of the course, a standard multiple regression analysis (MRA) was performed. The assumptions for an MRA were evaluated before completing the analysis. First, stem-and-leaf plots and boxplots indicated that each variable considered was roughly normally distributed, and free from any univariate outliers. Inspection of the normal probability plot of standardized residuals, as well as the scatterplot of standardized residuals against the standardized predicted values showed that the assumptions of normality, linearity and homoscedasticity of the residuals were not violated. Next, outliers were considered, but because the Mahalanobis distance, 6.331, did not exceed the critical $\chi^2=16.266$ for $df=3$, $\alpha=0.001$ for any cases in the data set, there were no multivariate outliers.

The MRA indicated that gender and modality each accounted for a non-significant portion of the variability in overall grades in the course (gender: $t(117)=0.530$, $p = 0.597$, modality: $t(117)=0.670$, $p=0.504$), while the teaching method was significant $t(117)=-2.405$, $p = 0.018$. Thus, students learning with IBL or interactive lecture showed significance, while the differences in learning for males versus females, and online versus face-to-face, was non-significant. The variability of the three variables accounted for 5.6% of the variability in the overall grades in the course ($R^2=0.056$, adjusted $R^2=0.032$, $F(3,117)=2.315$, $p=0.079$). The unstandardized (B) and standardized (β) regression coefficients, and squared semi-partial correlations (sr^2) for each predictor in the regression model are reported in Table 4-13.

Table 4-13
Unstandardized (B) and standardized (β) regression coefficients, and squared semi-partial correlations (sr2) for each predictor variable in a multiple regression model predicting overall course grade

Variable	*B* [95% CI]	β	sr^2
Gender	1.241 [-3.393, 5.875]	0.049	0.002
Teaching Method	-5.287 [-9.641, -0.933]	-0.216*	0.047*
Modality	1.613 [-3.155, 6.382]	0.061	0.003

4.6 Research Question Two

The original goal of this research was to determine the effects of inquiry-based learning in a general education mathematics class. Typically, the course is only offered in a face-to-face environment, during the school semesters. (The general education mathematics course is not offered in summer terms.) However, the COVID-19 pandemic allowed the opportunity to include further research questions due to mandated online teaching for an academic year, so one of the research questions remains to be answered: What impact does inquiry-based learning have on student’s cognitive and affective learning in a face-to-face GE mathematics course at a liberal arts university, as compared to learning by interactive lecture? In this section, only research participants who were enrolled in the general education mathematics course during face-to-face semesters, taught in a classroom on campus, are considered.

4.6.1 **Assessments in face-to-face instruction**

Quantitative data was compared for all participants who were enrolled in face-to-face sections of the course. Results from tests comparing these scores are presented in this section.

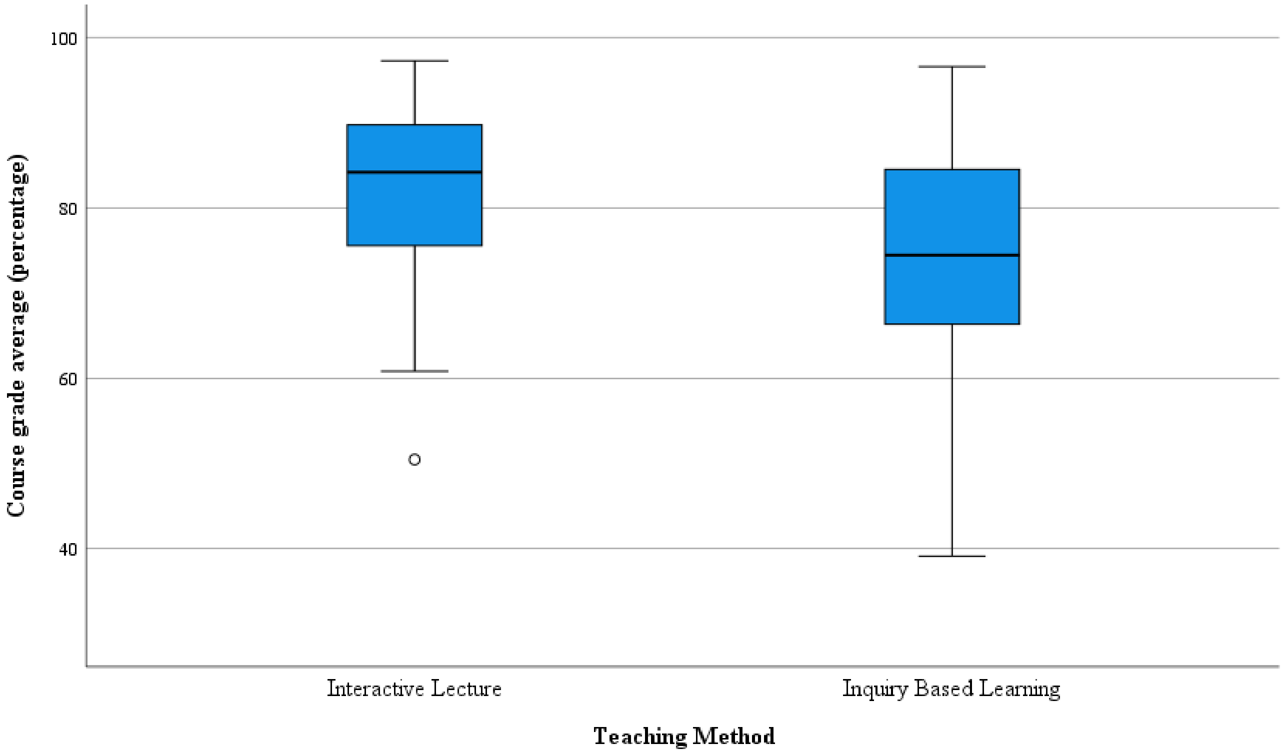
4.6.1.1 Final exam scores and Overall course grades

An independent samples *t*-test was used to compare the Final Exam scores for 83 participants who were in the face-to-face sections of the course. This includes 38 students who were taught in an interactive lecture section of the course, and 45 who were taught in an inquiry-based learning section. The interactive lecture group scored higher (*M* = 78.84, *SD* = 16.42) than the inquiry-based learning group (*M* = 66.99, *SD* = 18.53). The Shapiro-Wilk statistic for this test was significant (interactive lecture *W* = 0.922, Sig = 0.011; inquiry-based learning *W* = 0.935, Sig = 0.014), indicating the assumption of normality was violated, thus, a Mann-Whitney *U* test was performed as well. The Mann-Whitney *U* test indicated that the Final Exam scores of the interactive lecture group (*Mean Rank* = 50.79, *n* = 38) were significantly higher than those of the inquiry-based learning group (*Mean Rank* = 34.58, *n* = 45), *U* = 521, *z* = -3.055, *p* = 0.002, two-tailed. This effect can be described as "medium" effect (*r* = -0.335).

The impact of inquiry-based learning on the overall grade averages of the participants who were enrolled in a face-to-face section of the course was tested next, using an independent samples *t*-test. Overall course grades were also found to be statistically significantly higher for the interactive lecture group (*M* = 81.41, *SD* = 11.23) than for the inquiry-based learning group (*M* = 75.36, *SD* = 12.64). The independent samples *t*-test was used to compare the overall course averages (including all quizzes, exams and the final exam) for the 83 participants (IBL *n* = 45, interactive lecture *n* = 38). Neither Shapiro-Wilk statistic was significant (interactive lecture *W* = 0.926, Sig = 0.92; inquiry-based learning *W* = 0.972, Sig = 0.200), indicating the assumption of normality was not violated. Levene's test was also non-significant (*F* = 1.063, Sig = 0.306), thus equal variances are assumed. The *t*-test was statistically significant, with the interactive lecture group (*M* = 81.41, *SD* = 11.23) scoring 6.05% points higher, 95% CI [.79, 11.32] than the inquiry-based learning group (*M* = 75.36, *SD* = 12.64), *t*(81) = 2.288, *p* = 0.025, two-tailed, *d* = 0.50. A comparison of the overall course grade average is shown in Figure 4-4.

Figure 4-4

Comparison of course grade average for students in face-to-face sections



4.6.2 **Participants' attitude towards mathematics (from ATMI) in face-to-face courses**

Next, the ATMI scores for the participants enrolled in face-to-face sections of the course were compared. Tests were run on each of the four sub-scales of the ATMI: self-confidence, value, enjoyment, and motivation.

4.6.2.1 Participants' perception of their self-confidence in mathematics

The change in the self-confidence of the participants in the face-to-face sections of the course was compared using an ANOVA, comparing the change of participants in the inquiry-based learning course to the participants in the interactive lecture course. Inspection of the skewness, kurtosis, Shapiro-Wilk statistics, W = 0.951, Sig = 0.260 and W = 0.961, Sig = 0.283, and Levene's statistic, $F(1, 56)$ = 0.344, Sig = 0.560, indicated that the assumption of normality was supported and the assumption of homogeneity of variance was not violated. However, the ANOVA revealed there is no statistically significant difference between the perception of self-confidence in mathematics between the two groups who were taught face-to-face, $F(1, 56)$ =

0.048, $p = 0.827$. This differs from Section 4.5.2.3, which compared the combined face-to-face and online participants in the research study; the test for all research participants revealed statistical significance in the interactive lectures students whose results showed decreased confidence in mathematics between the pre- and post-survey.

4.6.2.2 Participants' perception of the value of mathematics

To compare the change in the perception of the value of mathematics, as reported by the research participants on the ATMI, an ANOVA was also used. The ANOVA compared the change in scores for the 10 questions pertaining to the value of mathematics of the participants who were enrolled in a face-to-face inquiry-based learning course to those who were enrolled in a face-to-face interactive lecture course. Inspection of the skewness, kurtosis, Shapiro-Wilk statistics, $W = 0.95$, Sig = 0.517 and $W = 0.951$, Sig = 0.143, and Levene's statistic, $F(1, 56) = 2.300$, Sig = 0.135, indicated that the assumption of normality was supported and the assumption of homogeneity of variance was not violated. The ANOVA was statistically significant, indicating there is significant evidence that the change of the participants' perception of the value of mathematics was not equal for the interactive lecture and inquiry-based learning groups, $F(1, 56) = 10.790$, $p = 0.002$, $\eta^2 = 0.193$. Further investigation of this impact using planned contrasts revealed that participants taught with interactive lecture ($M = 3.280$, $SD = 4.354$) was associated with a large increase in perception of the value of mathematics as compared to those taught with inquiry-based learning ($M = -0.151$, $SD = 3.598$), $t(56) = 3.285$, $p = 0.002$, with Cohen's measure of effect size $f = 0.490$ and Cohen's $d = 0.878$, a large effect size.

4.6.2.3 Participants' perception of the enjoyment of and motivation in mathematics

No statistically significant difference was found using an ANOVA to compare the participants' change in enjoyment of mathematics over the course in the face-to-face classes, $F(1, 56) = 2.581$, $p = 0.114$, nor with the participants' change in motivation towards mathematics of the course, $F(1, 56) = 0.972$, $p = 0.328$.

4.7 Research Question Three

The COVID-19 pandemic forced two terms of courses to be conducted fully online: Fall 2020 and Spring 2021. Many of the university's students were off campus the whole academic year, and the core mathematics classes were taught synchronously online. It should be noted that the university did allow some students who had courses or activities which were necessary to have face-to-face components to be conducted on campus in person. These included musical groups, natural science labs, and sporting groups. However, mathematics courses, including this general education mathematics class, were conducted fully online for both semesters. This presented the opportunity to include an additional research question: How does an inquiry-based learning GE mathematics course differ from an interactive lecture course in a fully online classroom?

The two terms (Fall 2020 and Spring 2021) that data was collected from fully online courses had the lowest research participation rates from students. Students from three sections of the course were given the opportunity to participate in Fall 2020, and students from two sections were given the opportunity in Spring 2021. Overall, there were 126 students enrolled in those five sections of the course, and only 38 agreed to be research participants. In all, only 30.2% of the students enrolled in the course participated in the research. (In one section of the course with 26 students, only two participated in the research study.) This was a low participation rate during these two terms, as compared to the face-to-face sections where 83 out of 157 (or 52.9%) of the students enrolled chose to participate. The low participation during the online sections of the course are most likely due to the uncertainty of the students, as the world faced a global pandemic, and they were enrolled in college courses online, and distanced from those around them.

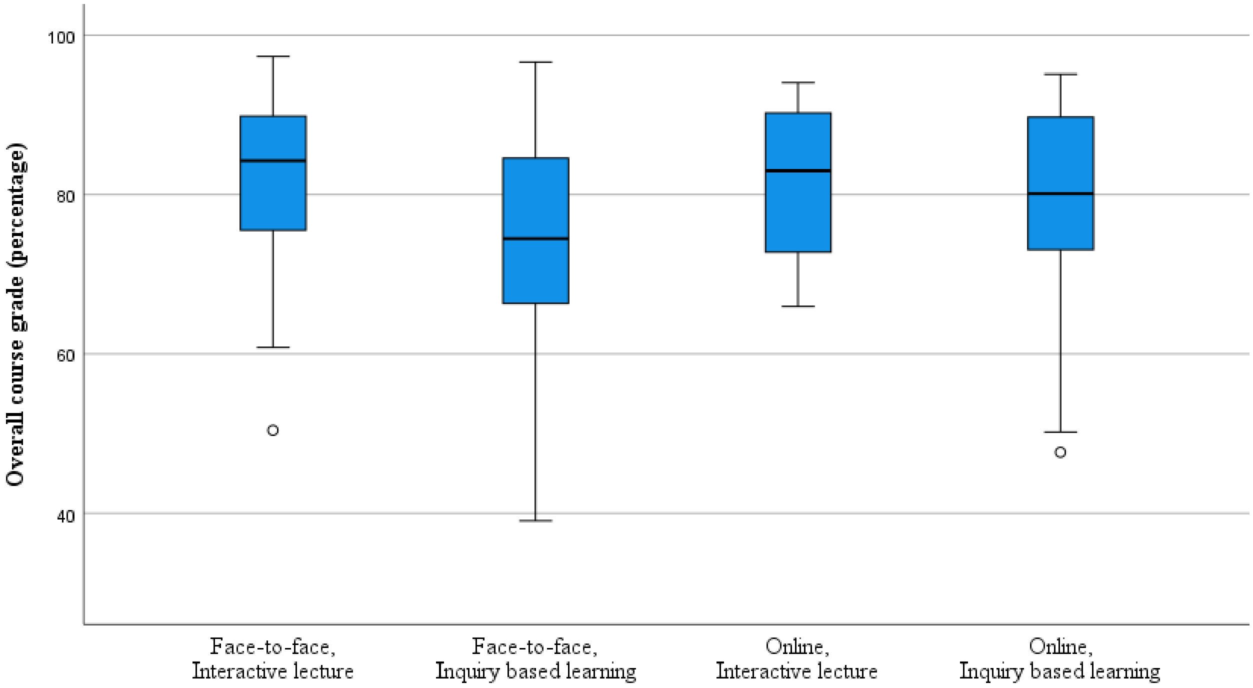
4.7.1 **Comparing the teaching methods and modalities**

Before considering the fully online courses, a one-way between groups analysis of variance was performed in SPSS to investigate the impact that the teaching method and modality (online vs face-to-face, interactive lecture vs inquiry-based learning) had on the overall course grade of the participants. Figure 4-5 shows box-and-whisker plots for the overall course grades for these four groups (by teaching method and modality); these plots did not reveal any notable differences. Inspection of the skewness, kurtosis and Shapiro-Wilk statistics supported the assumption of normality, and equal variances were assumed, $F(3, 117) = 0.359$. The ANOVA also showed the differences were non-significant, and could be due to chance alone $F(3, 117) =$

2.293, $p = 0.082$. Therefore, further testing was performed to compare the online cohorts and the face-to-face cohorts individually.

Figure 4-5

Comparison of overall course grade by teaching method and modality



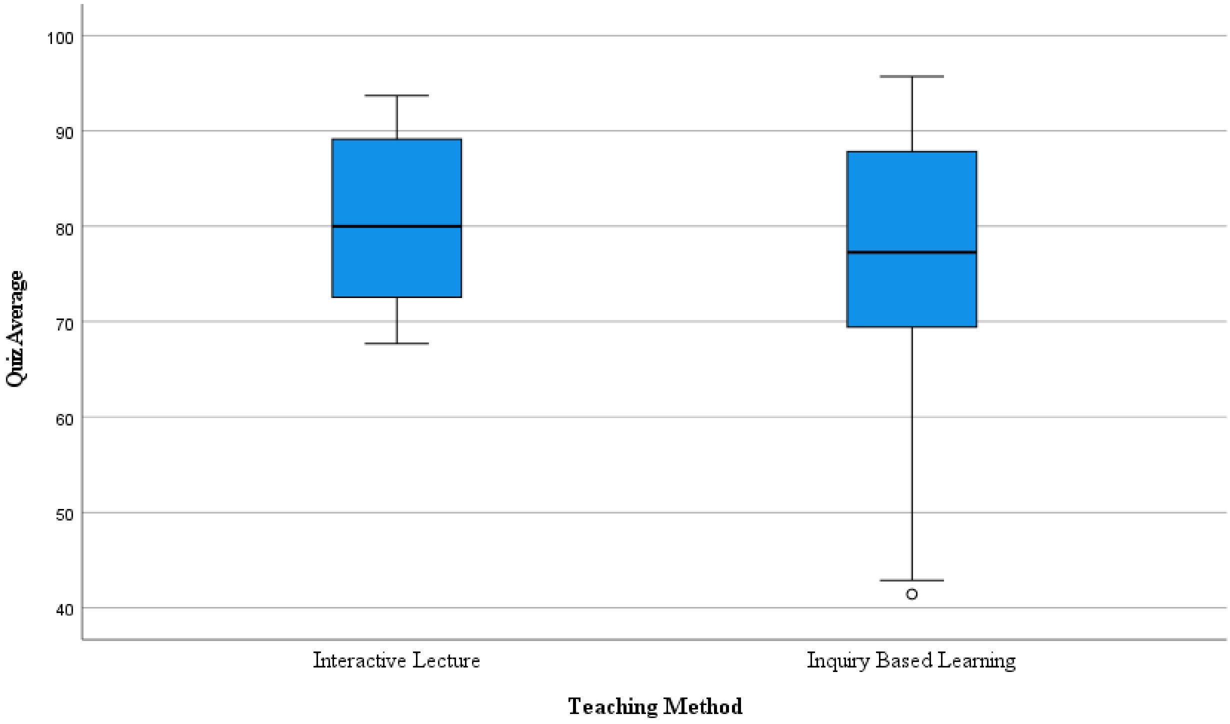
4.7.2 Assessments in online instruction

Three different components of assessments for the participants who took the course fully online were considered to determine if any differences exist between the participants who took the course taught by inquiry-based learning and those taught by interactive lecture. First, the quiz scores from the course were compared, then the final exam scores were compared, and finally, the overall course assessment averages were compared.

4.7.2.1 *Quiz scores*

Throughout the term of enrollment, students in the fully online sections of the course took five unit quizzes. Scores from these unit quizzes were totaled, and the average on those quizzes were compared using an independent samples *t*-test. The independent samples *t*-test was used to compare the average of the five quiz scores for each participant in the inquiry-based learning online sections (*n* = 20) to the participants in the interactive lecture online sections (*n* = 18). Neither Shapiro-Wilk statistic was significant, indicating that the assumption of normality was not violated. Levene's test for equality of variances was also not significant, so equal variances were assumed. The *t*-test was not statistically significant, with the Inquiry-based learning group (*M* = 76.0, *SD* = 16.01) reporting similar scores as the Interactive lecture group (*M* = 81.5, *SD* = 8.75), *t*(36) = 1.305, *p* = 0.200, two-tailed. As seen in by these statistics and in Figure 4-7, the Interactive lecture students had slightly higher quiz averages, but the inquiry-based learning students had a greater standard deviation and spread, so these noted differences could be attributed to random chance alone.

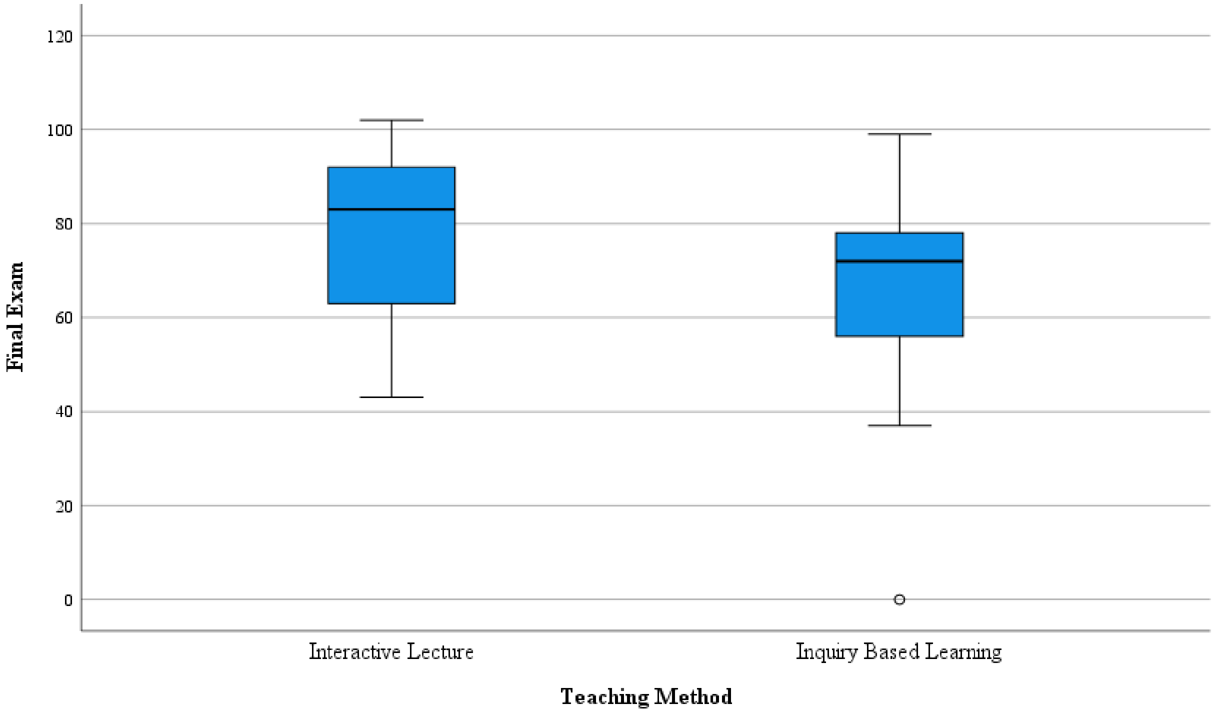
Figure 4-6
Box plot comparing Quiz Averages for Online students



4.7.2.2 Final exam scores

An independent samples *t*-test was run to compare the final exam scores for the participants in the inquiry-based learning online sections (*n* = 20) to the participants in the interactive lecture online sections (*n* =18). Neither the Shapiro-Wilk statistic nor Levene's test was significant, indicating the assumption of normality was not violated and equal variances were assumed. The *t*-test was also not statistically significant, with the Inquiry-based learning group (*M* = 51.0, *SD* = 6.92) reporting similar scores as the Interactive lecture group (*M* = 50.3, *SD* = 6.73), *t*(36) = -0.303, *p* =0.764, two-tailed. The Final Exam Scores for Online students are compared in a box plot in Figure 4-7.

Figure 4-7
Box plot comparing Final Exam scores for Online Students



4.7.2.3 Overall course assessment average

An independent samples *t*-test was also used to investigate the impact that inquiry-based learning had on the overall grade average of participants in online sections of the course. When comparing the overall grade average of the participants, the data were non-normal (Interactive lecture *W* = 0.938, Inquiry-based learning *W* = 0.889). Therefore, a Mann-Whitney *U* test was run. The Mann-Whitney *U* test indicated that the overall course grade average of the inquiry-based learning students (*Mean Rank* = 18.83, *n* = 20) was not statistically significantly different from the interactive lecture students (*Mean Rank* = 20.25, *n* = 18), *U* = 166.5, *z* = -0.395, *p* = 0.696 (two-tailed, exact).

4.7.3 Students' attitude towards mathematics (from ATMI) in online instruction

An independent samples *t*-test was run to compare the change in students' perception of their self-confidence in participants enrolled in the inquiry-based learning sections of the course (*n* = 20) to change in participants enrolled in the interactive lecture sections of the course (*n* = 18). Neither Shapiro-Wilk statistic was significant, indicating that the assumption of normality was not violated. Levene's test was also non-significant, thus equal variances are assumed for these groups. The *t* test was not statistically significant, with the inquiry-based learning section (*M* = 0.43, *SD* = 6.02) reporting changes in self-confidence similar to the interactive lecture group (*M* = 1.33, *SD* = 5.30), *t*(30) = -0.451, *p* = 0.655, two-tailed.

Independent samples *t*-tests were also run to compare the change in students' perception of the value of mathematics, their enjoyment of mathematics, and motivation towards mathematics. In running each of these tests, the Shapiro-Wilk statistics were significant, indicating the assumption of normality was not violated; Levene's test was non-significant in each case, thus equal variances were assumed each time. Each *t*-test was found to be statistically insignificant; the *t*-statistic and *p*-values are given in Table 4-14.

Table 4-14
Results from Independent Samples t-test for ATMI results of online students

	Inquiry-based learning (M, SD)	**Interactive lecture (M, SD)**	***t*-statistic**	***p*-values**
Self-confidence	(0.43, 6.02)	(1.33, 5.30)	*t*(30) = -0.451	*p* =0.655, two tailed
Value	(0.94, 3.62)	(2.14, 4.67)	*t*(30) = -0.819	*p* =0.420, two tailed
Enjoyment	(1.50, 4.88)	(1.14, 5.05)	*t*(30) = -0.202	*p* =0.841, two tailed
Motivation	(-.28, 2.65)	(-.57, 2.85)	*t*(30) = -0.301	*p* =0.766, two tailed

4.8 Qualitative Data Analysis

4.8.1 Instructor reflections

The notes and reflections of the instructor/researcher were analyzed first. These notes were taken throughout the courses studied, during and at the end of each course. The instructor/researcher wrote free-form notes, based on how each class period went, comments heard from students before class and during passing periods, and overall observations and reflections on class periods. These were written weekly throughout the terms, then were reviewed and coded using inductive content analysis. Several themes emerged as the notes were reviewed following the data collection. These themes were analyzed within the context of the study and within the conceptual framework for the study.

4.8.1.1 *Theme 1: Difficulties with independent small group work in IBL courses*

There were immediate challenges noted by the educator-researcher each term when implementing an IBL classroom. First, students were expected to work in small groups, working through the instructor-designed activities to find patterns and make conclusions each class period. Designing note packets, which had enough scaffolding that students could work independently, proved to be challenging. Students in the general education mathematics classes come from a variety of mathematics backgrounds, so providing notes which were accessible yet challenging to all students were difficult to design. These course notes were generated based on both social constructivist (active learning, group work) and cognitive constructivist (deep thinking, rich mathematical ideas) ideas. They were designed as such to allow students to work in small groups, making conclusions based on previous knowledge, to learn new information, theorems and make conclusions. In particular, course notes for topics which required prerequisite knowledge (algebra, geometry, or probability) were difficult to make challenging yet accessible to all students enrolled, and course notes for conceptually difficult topics (Cantor's proofs about infinity and power sets) were difficult to make simple enough yet still accessible to all students.

Additionally, in the inquiry-based learning sections, managing group work was a constant, challenging task. In all five terms, motivating students to talk to their group mates in the IBL courses was difficult. Expectations for "good group work" were shared with students at the start of each semester, and included each student focused and working distraction-free on the task at hand, communicating with the other members of the group, and sharing any ideas or patterns with the others. Some students preferred to try to work alone, some groups worked silently, and some would not share their ideas or check in with their classmates. This took constant reminding from the educator-researcher, and explaining to the students and research participants many times what “good group work” looked like.

During the face-to-face sections, where students and the researcher-educator were physically in the classroom together, the educator could scan the classroom and listen for the groups to share their ideas. In the online sections, this was even more challenging, as the students were physically distant in their Zoom breakout rooms. Several times at the end of a class period, students would casually tell me, “no one wanted to talk” and “it was just silent” in the Zoom breakout rooms. While this is anecdotal evidence, comments like this were heard throughout the online terms. Providing well-scaffolded notes which were accessible to all students, and implementing productive and communicative groups proved to be very challenging aspects of inquiry-based learning to implement each term.

One of the biggest obstacles to “good group work” was for each group of students to being their discussion or sharing ideas each class period. Because the students often felt like they were not the strongest mathematics student, groups would often sit in silence. Over the terms, one observation made by the instructor-researcher was that students would start generating and sharing ideas together if they had a “group leader” each class period. The “group leader” did not need to know all the answers, but was assigned the task of asking questions, encouraging and fostering conversation within the group. To assign a random or different group leaders each time, the educator would come up with a small “ice breaker” question to assign a group leader; these “ice breaker” questions included “Who has the most siblings?” “Who has the largest shoe size?” “Who woke up earliest this morning?” These questions would encourage students to start a conversation in their group to determine who the “group leader” was that day. Once the “group leader” was established, that leader began asking questions in the group, and the groups had an easier time communicating and working together. Data was not specifically collected for how well groups worked together after a leader was appointed, but anecdotally it was consistently noted.

4.8.1.2 Theme 2: Negative connotations of IBL

Throughout the different semesters, that while many students had a positive experience in the inquiry-based classroom overall, that many students had difficulty, especially at the start of the term, with the different roles in the classroom. This is a common issue in IBL classrooms, as students transition from more passive to more active learning. Many students were used to "applying a formula" or "waiting for the teacher's answer," so were easily frustrated at the start of the term, when they noted the role of the educator in the classroom was more of "guide on the side" instead of the "sage on the stage" (White-Clark et al., 2008, p.40). On various occasions, a student would ask when he or she would be shown how to answer an exercise, and were often frustrated when the researcher-educator would not just "give them the answer." By the end of each term, though, students were more comfortable sharing ideas and working in groups.

In particular, students with learning challenges tended to get more frustrated by the IBL course than in the lecture course. These learning challenges included dyslexia, auditory processing disorder, and dyscalculia; the instructor was made aware of student learning challenges through the university's Disability Access Services (DAS) center. (The Director of the DAS assesses and monitors students with learning challenges, and provides information to instructors about support for each student.) Students with learning challenges often asked to see an example or solutions worked out to fully understand each concept, as they had learned from past mathematics classes to take careful notes of each topic. Generating new ideas, patterns or generalizations was often very difficult for this set of students. Students with learning challenges tended to be less frustrated in the interactive lecture courses.

4.8.1.3 Theme 3: Negative preconception of a college mathematics course

At the start of each term, there were always a handful of students (in each course) who had very negative experiences in a mathematics classroom in the past, and who believed they simply "couldn't do math." These students would come in with an attitude towards mathematics, and often wished they didn't have to enroll in a college-level math course. However, our university requires the course for all first-year students, regardless of their major. In the educator reflections, it was noted that each term students would start with a very negative preconception of mathematics, and this often prevented them from trying their hardest at the start of the term.

4.8.1.4 Theme 4: Positive experiences with the content of the mathematics course

In both the interactive lecture and inquiry-based learning sections of the course, there were students who near the end of the term would discuss how much they liked the content of the course. While they were used to typical algebra-based mathematics courses, the content of this mathematics course included topics which were relevant in their everyday lives. Many students would talk about that at the end of the term, and would return to the research-educator several terms later, still thinking about the mathematical topics. Students in particular would like that they could see the application and value of the mathematical ideas in their everyday lives. Several students, in both the IBL and interactive lecture course, shared that they went home over a weekend or a break, and shared the mathematical ideas with their parents or families.

4.8.1.5 Theme 5: Positive experiences with inquiry-based learning

While students initially had a very negative outlook on IBL (as described in 4.8.1.2), by the end of the term, many admitted to having a positive experience with IBL. They would often describe the group work as challenging, yet enjoyable. Several students each term would share their experiences of a moment when their group "got it," and would talk and explain their answers to one another. There were moments of *productive struggle* throughout class; productive struggle is defined by the National Council of Teachers of Mathematics as students delving "more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions (2014, p. 48). Productive struggle in a classroom should challenge students within their educational targets (Townsend et al., 2018); productive struggle in this mathematics class allowed students from different mathematical backgrounds to use both social and cognitive constructivist ideals, working together to complete challenging tasks within the zone of proximal development for each student. As described in Section 2.5.1, the instructor-designed course notes were scaffolded so that students of all mathematical abilities could work together to form patterns and make conclusions. The moments where groups of students worked together and understood new material seemed to boost students' self-confidence in mathematics. In the educator reflections, it was noted many times when students would share how much they enjoyed working in small groups and "discovering" new ideas.

4.8.2 Semi-structured interviews

The semi-structured interviews contained a mix of students who were in an interactive lecture course or inquiry-based learning course, and who were in a face-to-face section or fully online section of the course. These were originally planned to be focus groups, with 3-5 students in each group, but because they were conducted on Zoom and so many participants opted not to participate, most students were interviewed individually. Because of the timing of these interviews (several weeks after each course had ended and final grades had been submitted), many research participants did not respond to repeated requests for interview. As a result, 13 research participants shared their reflections on the course with the interviewer. Of those 13 students, 7 participated in an interactive lecture section of the course (3 face-to-face and 4 online), and 6 participated in an inquiry-based learning section of the course (4 face-to-face and 2 online). These interviews reinforced themes from the personal educator reflections, specifically Theme 1 and Theme 5. Additional themes were also generated by the comments and reflections of the participants.

4.8.2.1 *Theme 1: Difficulties with independent small group work*

As noted in the instructor reflections, the students who participated in IBL courses often remarked about the difficulty of working in independent small groups. This was true for students enrolled in both modalities of the course, but was most prominent for students who were in an online version of the course.

> **Student 5 (IBL, online):** My only thing I didn’t like was sometimes I’d be in a breakout room for like 10-15 minute with people who just didn’t want to talk to me, so just to kind of hang out there.
>
> **Student 6 (IBL, online):** I liked how we went in breakout rooms, but if she left us too long in there,…then we would just not talk.

Reflecting back on the course, this was particularly hard to manage in the online sections of the course. In the face-to-face sections, the educator-researcher regularly roved the room, and would frequently check in on groups who weren’t communicating. However, in the online sections, the educator would check in to each Zoom breakout room, but it was difficult to know which group was struggling at any point in time. Difficulties with Zoom breakout rooms was a common issue among all students. Students who were in the interactive lecture sections also faced this same challenge. The students would have short periods of time (2-3 minutes) to discuss answers or ask questions about an exercise.

> **Student E (Interactive lecture, online):** The only hindrance I had was I hated Zoom breakout rooms,…no one would ever answer me or talk in the Zoom rooms and it's just like, hey! We're supposed to talk about a problem, and no one's talking. We didn't do them that often either.

During these interviews, all the comments from participants about the difficulties working in small groups were verbalized by students who had been enrolled in the online sections of the course. These students were enrolled in the course during the 2020-2021 academic year, when all classes were taught online, and were unique issues to the online sections. No participants enrolled in face-to-face sessions of the course mentioned difficulties or frustrations working in non-communicative small groups.

4.8.2.2 Theme 2: Negative connotations of inquiry-based learning

Of the six students who participated in the interviews, none reported a negative connotation of inquiry-based learning. While the instructor heard comments from students (Section 4.8.1.2) in the course, this theme was not present in the small group interviews.

4.8.2.3 Theme 3: Negative preconceptions of a college mathematics course

The instructor noted student dissatisfaction with a general education mathematics class for all students (Section 4.8.1.3), but those comments were predominately made in the first half of the semester. As the interviews were conducted after the term was finalized and final grades had been recorded, participants did not make comments about the requirement of the college mathematics course.

4.8.2.4 *Theme 4: Positive experiences with the content of the mathematics course*

In the semi-structured interviews, students from both the inquiry-based learning sections and the interactive lecture sections made comments about the content of the course. Because the course was designed by the faculty to include topics which were new to most undergraduate freshmen, several comments were made about the content of the course (Fibonacci numbers, geometry) and problem solving.

> **Student 3 (IBL, face-to-face):** You got like a lot of different types of math all in one instead of like in high school was kind of catered towards like a year of geometry or a year of statistics but it was cool how you got different types of math all in one semester.
>
> **Student C (Interactive lecture, face-to-face): [**We] reiterated understanding the meaning behind every single concept that we learned, from like the Fibonacci numbers to the geometries.
>
> **Student F (Interactive lecture, online):** I was so used to the "plug and chug" math and I was really good at that. And I would say I was probably better at the plug and chug than like the problem solving math, so ..at the beginning of the semester, it was kind of hard for me to make the equations of figure out the answers to some of the problems. [The course] game me more of a mathematical way to go about like some problems and patterns.

Students from both the inquiry-based learning and interactive lecture courses made similar comments relating to this theme; no notable differences were made by the participants in the interviews.

4.8.2.5 Theme 5: Positive experiences with inquiry-based learning

Students in both the inquiry-based learning and interactive lecture courses were asked question 5 (Table 3-5), *How did lessons presented in class help or hinder your understanding of the topics presented?* None of the students in the interactive lecture course commented on any particular aspect of class that was most helpful, but of the six students who participated in the interviews, five reported having a positive experience with inquiry-based learning, particularly the group work and explaining solutions to their classmates. As described in Section 2.3.2, students worked on in-class tasks which were based on social constructivism. The quotes below are from those participants who viewed the group work and interactions with their peers as having a positive influence on their learning.

> **Student 1 (IBL, face-to-face):** It [small groups] really helped with leadership in the sense, where someone was like, okay, someone's gonna have to figure this out. IT helped with leadership and learning how to teach each other, because we all, we were all kind of in this together, and at times, it was definitely tricky and probably frustrating…But it was definitely a cool experience and you saw a lot of people step up and try to learn in different ways.
>
> **Student 2 (IBL, face-to-face):** At the beginning, not everybody knew each other so it's like a regular classroom. But I feel like over the semester,…her making us do group

> work with different people and putting us in different groups kind of forced us to talk to other people. But a lot of times in math classes, people are scared to ask questions, and I feel like in the core math class a lot of people who normally wouldn’t ask questions feel free to ask questions without feeling embarrassed or dumb.
>
> **Student 3 (IBL, face-to-face):** The interactiveness of the class helped me stay engaged and learn more about the topics, with the group work and what [Student 4] was saying with the activities that we do. During class, because a lot of times I feel like college classes are easy just to sit and be lectured at, and it is hard to focus all the time with that because your mind starts wandering, but by being hands-on and always with group activities and active, in class activities, just always keeps you engaged and focused on the topic.
>
> **Student 4 (IBL, face-to-face):** I actually really enjoyed the group and being able to actually discuss with peers and get multiple ideas and just everyone’s thoughts.
>
> **Student 6 (IBL, online):** She has us explain to the class how to do a certain problem, even if it was kind of nerve wracking.

Overall, these quotes seem to show that the participants valued the small group interaction and ability to discuss the mathematics topics with their small groups. Even though sharing their ideas in the small groups made the students nervous sometimes, they found value and enjoyment in the interaction with their peers, and found it to be an effective learning environment.

4.8.2.6 Theme 6: Student responses related to the ATMI sub-scale: Self-Confidence

Throughout the interviews, students often verbalized a change in confidence in mathematics, particularly about verbalizing their thoughts or solutions to exercises presented in class. Students made these comments to answer the questions of their overall experience in the mathematics class, and when asked if they felt confident sharing their ideas or explanations about mathematics.

> **Student 2 (IBL, face-to-face**): I feel like in the core math class a lot of people who normally wouldn’t ask questions feel free to ask questions without feeling embarrassed or dumb.
>
> **Student 3 (IBL, face-to-face):** I feel more confident…[Professor] makes me feel more confident and sharing with other people or helping other people with those concepts because I’ve confidently learned. …When you do share with others, it makes you more confident in the ideas and topics that she taught us.

Students in the interactive lecture course commented on their gain in confidence in mathematics as well.

> **Student A (Interactive lecture, face-to-face):** I think [core mathematics] did give me more confidence in myself as a student, to be able to go, "oh, there is math I can do." … I learned I would be totally confident sharing those ideas with anyone. … I don't think I was at all confident in it before.

There were students in the interactive lecture sections of the course who expressed hesitancy in an improvement in confidence in mathematics. In the classroom, they did not have as much discussion time as the inquiry-based learning sections.

> **Student C (Interactive lecture, face-to-face):** I don't feel too comfortable because I'm not sure if I still understand, like the background behind it, yes, I do have a better understanding, but definitely understand, like the concepts that we learned I do most of the time, but I would be hesitant a little bit just because it was something new for me.

These comments support the research by McLeod (2019), which explains that students must be engaged in an active learning process, in which they construct new knowledge both through social constructivism as well as cognitive constructivism. Active learning, whether through interactive lecture or inquiry-based learning, is related to the self-confidence of students in understanding and explaining mathematics.

4.8.2.7 Theme 7: Student responses related to the ATMI sub-scale: Value

Through the interviews, students also verbalized a change in their perception of the value of mathematics. This sometimes related to the content of mathematics studied, but also related to the classroom environment, applying mathematics to real life situations, and their view of mathematics in their future careers or studies.

4.9 Chapter summary

This chapter reported the findings from a mixed-methods study of the impact of inquiry-based learning in a first-year general education mathematics class which examined course learning outcomes as well as students' perceptions of their self-confidence in mathematics, the value of mathematics, their enjoyment of mathematics, and their motivation towards mathematics. Quantitative data was collected from the research participants throughout their semester of study by gathering course assessment scores, including quizzes, exams and a comprehensive final examination. Students' perceptions towards mathematics was collected through the ATMI, which was administered at the start and end of each course.

Qualitative data was gathered though instructor reflections as well as semi-structured research participant interviews. These interviews were analyzed and several themes emerged from the qualitative data which helped explain the quantitative results. First, to examine differences in students' understanding of key concepts in interactive lecture and IBL, assessments from the course were analyzed. Initial analysis of the quantitative data via an independent samples *t*-test showed statistically significant results ($p = 0.015$), with scores for the inquiry-based learning cohort 5.447 points higher than for the interactive lecture cohort.

Additionally, student attitudes towards mathematics were collected using the ATMI and tested using an independent samples *t*-test. The interactive group reported a higher change toward the value of mathematics ($p = 0.003$). ATMI questions were further analyzed to determine where there were statistically significant changes in attitudes from pre- to post-test. There were statistically significant changes in the attitude towards mathematics for the students enrolled in the interactive lecture course, as described in Section 4.5.2.

The quantitative data was further analyzed, comparing the two teaching methods within face-to-face instruction and within online instruction. For students who were instructed in a face-to-face classroom, an ANOVA was performed and found that students in the interactive lecture had a medium increase in overall course grades as compared to those who were in the inquiry-based learning classroom ($p = 0.023$, effect size $f = 0.263$, Cohen's $d = 0.508$). The students in the interactive lecture course also scored 6.05% points higher on the final exam ($p = 0.025$). There were no statistically significant differences on the ATMI sub-scales of self-confidence, motivation in mathematics, or enjoyment of mathematics for the students taught with the two different teaching methods in the face-to-face sections of the course. However, an ANOVA revealed that there was a slight increase in the perception of the value of mathematics in students who were in the interactive lecture course ($p = 0.002$, $f = 0.490$, Cohen's $d = 0.878$). There were no statistically significant differences found in student overall grades for students enrolled in online sections of the course.

Qualitative data was gathered through instructor reflections and semi-structured interviews. Several themes emerged from the qualitative data which further explained the quantitative results. Specifically, while there were no significant differences in self-confidence on the ATMI, students in the inquiry-based learning sections reported positive experiences with IBL and their confidence in explaining mathematics to others, with difficulties working in small groups at times.

Chapter 5 will discuss these findings, and how these results answer each of the research question for this study. The limitations of this study will also be discussed, the chapter will conclude with the implications from this study for other instructors of a general education mathematics course and for future researchers in mathematics education.

5 Summary, Conclusions, and Recommendations

5.1 Introduction

Undergraduate general education mathematics courses are perceived by students as difficult, irrelevant, and often a course students dread (Fielding & Makar, 2008). It is not uncommon for educators to struggle as well, trying to present content in a meaningful and understandable way to students, especially those who are not pursuing a STEM degree (Brady, 2014; Clinkenbeard, 2015). A pedagogical model, inquiry-based learning, may provide a solution for both students and educators. Inquiry-based learning combines cognitive and social constructivist ideals, blending them in an environment that allows for students to learn mathematical concepts by working with peers and making conclusions in a small group setting (Laursen & Rasmussen, 2019).

Almost 20,000 studies focusing on inquiry-based learning in mathematics classrooms have been located on Google Scholar in the past decade, but few have focused on a GE mathematics class. In many ways, IBL seems ideal to implement in a GE mathematics classroom (Caswell, 2017; Hotchkiss & Fleron, 2014; Zion & Mendelovici, 2012), as an IBL course can help increase critical thinking skills, motivation, and engagement (Caswell, 2017), engaging student in active constructivism of mathematical concepts (Zion & Mendelovici, 2012). Furthermore, few research studies have considered the impact of the remote online learning during the COVID-19 global pandemic in an inquiry-based learning classroom, particularly for first-year university students which makes this study of importance to the field (Moliner et al., 2022; Znidarsic et al, 2022).

As IBL impacts how a student interacts with the material, peers, and the instructor, studying the impact of an IBL classroom must examine not only students' achievement of key concepts, but also their perceptions of their classroom environment and their attitudes about mathematics and themselves as a mathematics student. This chapter will include a summary of the thesis and research objectives for this study. A summation of the specific findings and conclusions from the data analysis in Section 5.2. Section 5.3 will present the limitations and potential biases in the study, and Section 5.4 identifies the implications of this study, for other general education mathematics classrooms and further research studies. Finally, concluding remarks and a summary of the findings are in Section 5.5.

5.2 Summary of key findings

The results from Chapter 4 will be further explained in the following three sections, to answer each of the research questions.

5.2.1 Research Question One summary

How does an IBL GE mathematics course for first year students differ from a traditional interactive lecture in terms of student understanding of key topics? Attitudes towards mathematics? Mathematical self-concept?

The quantitative data show that there are differences in student understanding of key concepts, attitudes towards mathematics and mathematical self-concept between the inquiry-based learning and interactive lecture classrooms. The overall grades in the course, computed as a simple average of points earned on assessments (quizzes, exams, final exam) differed between the two teaching methods. Somewhat surprisingly, a *t*-test revealed the interactive lecture group scored higher (M = 81.640, SD = 10.470) than the inquiry-based learning group (M = 76.252, SD = 13.135) with statistical significance (p = 0.015, two-tailed).

Participants in this study were from a variety of majors, and had very different mathematical backgrounds, and approximately half of the students were enrolled in the mathematics class (and their first year of college) during a worldwide pandemic. Inquiry-based learning was unfamiliar to most students enrolled in the course, and many had completed only basic high school graduation requirements and often struggled in mathematics. The differences noted here, with the interactive lecture students scoring statistically significantly higher than the inquiry-based learning students, could be because students with weaker mathematics skills often prefer direct instruction, with a teacher presenting a step-by-step process and giving examples before working a similar exercise on their own (Cooper et al., 2017). For non-STEM students and students who had weaker mathematics skills upon enrolling in the course, the interactive lecture may have felt more familiar, which may explain the higher mean scores for the interactive lecture participants taught face-to-face.

Additionally, during two semesters of this study, students were isolated at home, frequently with other responsibilities, often without a quiet place to do schoolwork, and many had increased pandemic-related stress (Bonsangue & Clinkenbeard, 2021). Students who returned to the classroom for the Fall 2021 semester were fully masked and socially distanced while learning in the classroom. Both of these present barriers in learning – whether at home learning through Zoom, or masked in the classroom. Learning via direct instruction may have been more familiar and comfortable, as many students believe they “cannot learn mathematics effectively without teacher guidance” (Mukaka et al., 2021, p. 6). In the inquiry-based learning classes, communicating with peers to construct new knowledge either on Zoom or masked presents additional challenges. The participants in this study may have found the interactive lecture more familiar and less stressful during and after the global pandemic, which may explain the interactive lecture group having a mean assessment score higher than the inquiry-based learning group.

Quantitative results from the Attitudes Toward Mathematics Inventory (ATMI) survey showed statistical significance ($p = 0.003$, $d = 0.661$) in comparing the change in students’ perception of the value of mathematics from the start of the course to the completion of the course. This was further supported though a paired t-test comparing the pre- and post-survey results by question, which revealed that there was statistical significance in the improvement of the participants’ perception of the value of mathematics on nine of the ten questions on the ATMI by students in the interactive lecture course. No questions revealed statistically significant improvement/change for participants in the IBL course. However, it should be noted that while there was no statistically significant improvement/change for participants in the IBL course, the IBL pre-course average scores were higher than the pre-course average scores for the interactive lecture group on 27 of the 40 questions on the ATMI. Higher pre-course average scores for the IBL course left less room for improvement in the four sub-scales, which may explain the more consistent improvement in scores for the interactive lecture participants.

The course taught in this research includes applications of mathematics to real-world scenarios, including using non-Euclidean geometry to determine the great-circle route of aircraft, using the Fibonacci numbers and the golden ratio to show efficiency in plant growth, and applying data to real world scenarios such as election theory, disease spread, and medical testing. While all students had the same exercises to develop and work, the application and value of mathematics may have been more stressed in the teacher-directed interactive lecture course, resulting in a gain of participants’ perception of the value of mathematics. Throughout each course, the instructor-researcher made every effort to ensure that all students covered the same content, the students might have perceived a higher importance of the value and application in the instructor-led interactive lecture sections of the course, as opposed to the student- and group-led inquiry-based learning sections.

There were no statistically significant differences noted on the independent samples *t*-tests for the ATMI survey sub-scales of enjoyment or motivation in mathematics, and there was no statistically significant difference noted in the change in the self-confidence of students between the two teaching methods. Research from Kogan & Laursen, and Laursen shows a boost in mathematical self-confidence in STEM students enrolled in IBL courses (Kogan & Laursen, 2014; Laursen, 2014); however, the participants in this study were from a variety of mathematical backgrounds, and less than ten percent pursued a STEM major.

The results from the ATMI were also analyzed using a paired *t*-test comparing the pre- and post-survey averages by question for the sub-scales of enjoyment, self-confidence and motivation in mathematics. The test revealed that there were statistical significant changes in individual question in the participants' perception of these sub-scales. There were statistically significant positive changes in nine of the ten questions regarding enjoyment of mathematics for the interactive lecture students, while the only one question for the inquiry-based learning students showed statistical significance. There were also statistically significant positive changes in three of the five questions regarding the motivation of participants in the interactive lecture course, while no questions showed statistical significance for the inquiry-based learning participants.

Most notably, there was statistical significance in eleven of the 15 questions pertaining to self-confidence in mathematics for the interactive lecture participants, while no questions showed statistical significance for the inquiry-based students' self-confidence. Of the eleven questions which showed statistically significant changes, in nine of the questions pertaining to self-confidence, the average decreased over the term. In other words, the participants' average self-confidence scores for nine of the questions was *higher* at the start of the term, and decreased over the term in the interactive lecture course. The course taught in this research includes topics covering the influential "great ideas" of mathematics, culminating each term in the study of infinity and Georg Cantor's proof of multiple sizes of infinity. While this idea fits the course objectives and goals and allows students to think deeply about mathematics, learning about a difficult concept such as the different sizes of infinity in the last few weeks of the course may have forced some students to question their own abilities and understanding of mathematics, resulting in lower scores for self-confidence in mathematics. In the teacher-led interactive lecture course, students often vocalized their questions about "what they know" throughout the unit on infinity, and answering questions on the ATMI shortly afterwards may have led to the decreased scores in self-confidence in mathematics.

There have not been studies analyzing the impact of the learning environment in an inquiry-based general education mathematics classroom, but there have been studies examining the impact of the learning environment in undergraduate STEM classes. Research has shown that IBL classrooms show improved learning, persistence and student retention in mathematics (Laursen, 2013; Laursen, 2014; Kogan & Laursen, 2014), a recent study has found that even when students are engaged in active learning, they aren't aware that they are learning as much (Flaherty, 2019a). Additional research on the impacts of active learning found that "students can be misled by the inherent disfluency associated with the sustained cognitive effort required for active learning, which can in turn have a negative impact on their actual learning" (Deslauriers et al., 2019). Because engaging in active learning is more difficult than passive learning, students often feel they have learned more from a lecture (Deslauriers et al., 2019; Flaherty, 2019b); students may have felt like they learned less, so less change in self-confidence in mathematics was noted by the participants.

While there was no quantitative difference in the groups' overall scores in the course or in the improvement of their motivation towards mathematics, enjoyment of mathematics, or self-confidence in mathematics, the qualitative data shows slightly different results. Students who enjoyed the inquiry-based learning environment also were sometimes frustrated in groups, if they could not solve an exercise or when their group worked quietly without sharing their results. As stated in Section 2.2, social constructivist (group) work can result in a classroom environment where "some students may participate eagerly while others sit out the session waiting for answers to develop" (Noddings, 1990, p. 17). The qualitative data from the semi-structured interviews show that the students in the inquiry-based learning courses discuss their comfort level sharing or explaining mathematics more than the interactive lecture courses.

Students in both groups worked the same in-class and homework exercises; however, the researcher regularly instructed the interactive lecture group, who reported a higher change in their perception of the value in mathematics as described in Section 4.5.2. This change in participant perception of the value of mathematics may be the result of direct instruction and examples of everyday applications presented by the instructor course; understanding the usefulness and application of mathematics in everyday life has been showed to improve student perception on the value of mathematics (Masingila, 2015; Young-Loveridge et al., 2006).

5.2.2 Research Question Two summary

What effect does inquiry-based learning have on students' achievement in a face-to-face general education mathematics course at a liberal arts university, as compared to learning by interactive lecture?

The quantitative data show that the scores on the comprehensive Final Exam were higher for the interactive lecture group ($M = 78.84$, $SD = 16.42$) than they were for the inquiry-based learning group ($M = 66.99$, $SD = 18.53$). The Mann-Whitney U test indicated statistical significance of this result ($z = -3.055$, $p = 0.002$), with medium-sized effect ($r = -0.335$). Overall course grades were also found to be statistically significantly higher for the interactive lecture group ($M = 81.41$, $SD = 11.23$) than for the inquiry-based learning group ($M = 75.36$, $SD = 12.64$). The independent samples t-test showed significance with $t(81)=2.288$, $p = 0.025 < 0.05$, two-tailed, with Cohen's $d = 0.50$. Thus, the mean difference between the two groups is equal to half the standard deviation, a medium effect size (Allen et al., 2014; Cohen, 1988).

Quantitative data from the ATMI showed that there were no statistically significant differences between the attitude towards mathematics in terms of student self-confidence, enjoyment of mathematics or motivation towards mathematics, but the interactive lecture group showed a higher positive change in their perception of the value of mathematics, $t(56) = 3.285$, $p = 0.002$ with Cohen's $d = 0.878$, a larger effect size (Allen et al., 2014; Cohen, 1988).

These results seem to show that students who were in the interactive lecture course earned slightly higher grades – both on the Final Exam and overall, in the course. These scores, however, only report the demonstration of key concepts on the assessments within the course. It does not take into account a student's previous knowledge in mathematics, which was not accounted for in this research. While the topics covered in the course were typically new to all students in the classroom, students with stronger quantitative skills when enrolling in the course were not considered. Note – one way to triangulate the data and explain some of these differences could have been through collecting additional data – including SAT or ACT scores. However, students at the university used in this research were not all required to take any standardized exam for application or enrollment at the university, so no consistent scores could be collected.

There were no statistically significant changes in quantitative data for any of the four sub-categories of attitudes towards mathematics, as measured by the Attitudes Toward Mathematics Inventory. This was an unexpected result. Throughout the instructor's reflections of the courses, several students each term in the inquiry-based learning sections of the course would share their positive experiences of their group work. This seemed to show the value of mathematics and problem-solving, and an increased self-confidence in themselves as mathematicians as they "got it" for the first time. Students in the course often worked through productive struggle (Townsend et al., 2018) in their zone of proximal development, bridging the gap between their knowledge and what they could achieve with peer collaboration (Vygotsky, 1978, p. 86). These experiences shared by the students were unique in the inquiry-based learning sections of the course, so increased perception of the value of mathematics and self-confidence in mathematics was expected. However, the quantitative data showed no statistically significant differences. In the post-course participant interviews, students from the IBL course discussed the struggles they had throughout the course, and the satisfaction of the moment where they first understood a new topic.

5.2.3 **Research Question Three summary**

How do the student cognitive and affective results in an IBL GE mathematics course differ from an interactive lecture course, in a fully online classroom?

Remote learning can provide opportunities for students and educators to remain connected while working from their homes and without limitations to access to a physical classroom (Ray, 2020). However, the data collected for the fully online classrooms of this study were collected during the COVID-19 global pandemic, where social distancing was state and nationally mandated. In these pandemic circumstances, where all students were learning remotely and isolated from others, mathematics was anticipated to be an additional learning challenge for many students (Mukuka et al., 2021; Znidarsic et al., 2022).

Vygotsky's ideals of social constructivism were built upon ideals in traditional face-to-face settings; transforming these and promoting the interpersonal interactions require collaborative forums in an online space (Sedaghatjou et al., 2023). For this study, collaborative synchronous forums were used for both teaching methods. Inquiry-based learning regularly used Zoom breakout rooms and live Zoom chat, while the interactive lecture used synchronous Zoom video calls with constant feedback from the educator both in the chat box and audibly; both provided experiences for "sharing and interpersonal engagement" (Sedaghatjou et al., 2023, p. 369) to allow students opportunities for social and cognitive constructivism. There was no advantage to one teaching method over the other in this online environment; any difference in scores for the affective and cognitive domains for the IBL and interactive lecture students is not statistically significant. This is consistent research conducted in Zambia during the COVID-19 global pandemic, which found that "most students hold a belief that they cannot learn mathematics effectively without teacher guidance in a face-to-face environment" (Mukuka et al., 2021, p. 6), In this research as well, in the fully remote online environment with collaborative synchronous forms, no statistically significant differences were found between the inquiry-based learning and the interactive lecture groups.

The quantitative data show that there were no significant differences in any of the scores of participants in this study; the quiz scores, final exam scores, and overall grades for the inquiry-based learning online participants were compared to those from the interactive lecture online participants, and no significant differences were found. Additionally, no statistically significant differences were found in the changes of the four ATMI sub-categories. Thus, in this study, there were no statistically significant differences between the two teaching methods for the participants who were taught in the fully online course during the Fall 2020 and Spring 2021 semesters; one possible explanation for this may be that the courses were switched to fully online courses during the COVID-19 pandemic, which may have been an unexpected confounding factor in the research study.

5.3 Limitations to the study

Efforts were made to ensure that the data collected for this research study is free from bias or error, but inherent biases or limitations may still exist because the data was collected from 18 to 20-year-old human subjects; all research participants were in this age range. This research was completed over five semesters, both face-to-face and fully online, and used a quasi-experimental, explanatory mixed methods design. The sample was a convenience sample (the research subjects were the researcher's students during those five terms), and with a small sample (121 research participants) and four different classroom environments (traditional lecture and inquiry-based learning, online and face-to-face), it is difficult to detect and explain difference in the data, and to generalize those results.

When students volunteered to participate in the study, it was made very clear to the students that their participation in the research study had absolutely no impact on their course grade. The names of the research participants were kept confidentially by a department colleague until the completion of each term, and the researcher only learned the names of the participants several weeks after the semester had ended and final grades had been posted. Other mathematics faculty members administered the Attitudes Toward Mathematics Inventory and conducted the semi-structured participant interviews. This was done so that research participants could confidentiality speak truthfully and candidly about their experience. Neither the ATMI nor the interviews can be free of human bias, but every effort was made by the researcher and the interviewer to minimize their effects and any response bias (Creswell & Creswell, 2018). Response bias would occur if "responses do not accurately reflect the views of the sample and the population" (Creswell, 2015, p. 394); while all efforts were made to minimize response bias, there is no way to determine if any response of a participant differed from those in the general population of the course or at the university.

The semi-structured interviews were conducted several weeks after the course ended and final grades had been posted, to give the research participants time to reflect on the course and its impact, and to ensure that participants knew their responses had no impact on their course grades. This was intended to have a positive impact on the quantity and quality of the qualitative data. However, this meant that many of the research participants did not participate in the interviews, either because they simply ignored the email requests to interview, or were no longer enrolled in the university (some students transferred and others dropped out of the university). This greatly reduced the number of qualitative data collected. However, those who participated in the interviews and those who did not participate may have shared particular characteristics and opinions on the course, but the similarities between interview participants and non-participants cannot be fully determined or addressed.

With such a small sample size at a single university, conclusions must be made very cautiously, as a small sample can increase risk (in the participants' responses, the interpretation of those results, and generalizations) and bias (Cohen et al., 2018; Creswell, 2015). The highest proportion of students participated the first term of research (Fall 2019, with 43 research participants of the 56 enrolled), and the lowest proportion of student participated during the first semester of online teaching during the COVID-19 pandemic (Fall 2020, with 19 research participants of the 75 enrolled students). The researcher offered many of the recommendations suggested by Saleh and Bista (2017) to increase response rate, including targeting a population that may hold interest in the research and results, explanation of how data is collected and handled, and assurance of anonymity and confidentiality of their responses, but overall, only 121 participants of the 283 enrolled students participated. This represents a 43% response rate for the research, which is considered sufficient for generalizations and reducing bias or errors (Fosnacht et al., 2017).

Making broader conclusions will decrease the validity of the results found here. However, what happened in the classroom (both face-to-face and online) over these five terms is reported here, and the impact that was noted for the students in the classes. The group of students who were learning together in the general education math class studied here was unique, as all majors (STEM and non-STEM) were learning the same mathematics content in the general education classroom. Additionally, this data was collected before, during, and after the COVID-19 global pandemic, so offers a unique perspective on student achievement and attitudes towards mathematics during that time. Two semesters (62 participants) were enrolled in the two semesters before COVID-19 and learned in traditional face-to-face classrooms, two semesters (38 participants) were enrolled in the two semesters during the height of the COVID-19 global pandemic and learned through Zoom in a fully online synchronous classroom, and one semester

(21 participants) were enrolled “post” COVID-19, and were taught in a face-to-face classroom with face coverings and social distancing between students and the instructor.

There are many different variations of an inquiry-based classroom, including the design of the cognitive constructivist activities which take place in class. In this study, teacher-designed scaffolded course notes were designed as a guide for the social constructivist groups in the classroom. However, a larger scale study involving students at different universities, with different instructors and a different mix of student in the classroom may not have the same results as this study. Additionally, the university at which this data was collected is predominately Caucasian and Hispanic, so a larger scale study which better represents the general population (including African-American, Asian, and others) could help generalize results further.

5.4 Implications of this study

5.4.1 Implications for educators using Inquiry-Based Learning for a general education mathematics class

This section will contextualize some of the results and reflections on the data analysis, and existing research on inquiry-based mathematics classroom.

5.4.1.1 *Design and Development*

When an instructor makes the conscious choice to use inquiry-based learning in a mathematics classroom, he or she should have clear goals and student outcomes in mind. One of the goals when implementing inquiry-based leaning in this general education mathematics classroom was to allow students to discuss mathematics content with their peers (social constructivist) to learn rich mathematical ideas (cognitive constructivist). While the GE college-level mathematics class is the last formal mathematics class many students take, other useful skills are developed through mathematics. In that light, inquiry-based learning in a mathematics classroom has been shown to develop persistence and understanding of core concepts (Laursen, 2013). Additionally, IBL courses have been shown to improve STEM students’ confidence and ability, as well as the enjoyment of, mathematics courses (Mayfield & Dunham, 2015). To implement an IBL classroom for a general education mathematic classroom, the course must be carefully designed and developed.

There is no set structure for an IBL course, and in fact, there are many different levels of inquiry within the IBL title. There should be balance between the social and cognitive constructivist ideas implemented in the classroom, giving students time to work collaboratively, but also giving the students higher-level thinking tasks. These in-class exercises should be supplemented with independent practice exercises for the students. A major focus in this research was on the in-class activities and course notes; more time should have been spent to further develop the supplemental homework exercises.

Designing and developing the course notes for an IBL course is time-consuming. To implement such a course, the instructor must have time to develop scaffolded exercises and examples, so that students can work to make conclusions and generalizations about rich mathematical ideas on their own. While there are some existing course notes for an IBL courses (particularly developed with the Journal of Inquiry-Based Learning in Mathematics www.jiblm.org website), no course notes for a general education mathematics course such as this (covering "big ideas" in mathematics such as Voting Theory, Fibonacci Numbers and the Golden Ratio, Infinity and Cantor's Theorem) existed when this research began. These course notes were developed with colleagues over several terms to parallel the content being taught in the more traditional interactive lecture classrooms.

5.4.1.2 Implementation

When implementing an inquiry-based classroom, an instructor must be prepared for the work it takes to help students become effective active learners. In an IBL course, students must "come to every class prepared, are accountable to other students, and actually must apply what they are learning on a day-to-day basis" (Love et al., 2015, p. 756). Some students will not be as prepared daily as they should, and the instructor must act as a cheerleader, encouraging students to work together on challenging tasks. As an instructor, some days would have been easier to revert to the "direct instruction" model, instead of having students work slowly in their groups. Because groups are self-paced and often work at a slower rate than with direct instruction, there is often less content which can be covered in an IBL course (Yoshinobu & Jones, 2012). However, the sacrifice of content is counterbalanced because IBL-taught students typically have higher cognitive outcomes, higher academic achievement (Hassi et al., 2011), increased peer collaboration, and “affective gains in confidence, persistence, and positive attitude about mathematics” (Laursen et al., 2014, p. 409).

5.4.1.3 Instruction

Teaching mathematics using inquiry-based learning, and in particular to those students who have not participated in inquiry learning and who may have previous preconceived notions of mathematics or their own mathematical ability, proved to be challenging. Many college students have mathematics anxiety and a low mathematics self-concept (Brady, 2014; Clinkenbeard, 2015; Li et al., 2021) and expected to be taught with direct instruction during class. Participation in an inquiry-based learning course forces students to be active during classtime, share their ideas and generalizations, and be willing to make mistakes in a small group. The participants in this research did not know beforehand by which method they would be taught with, and signed up for the mathematics course with no knowledge of the teaching method which would be used throughout the course.

Inquiry-based learning was much easier to implement in a face-to-face setting for this population of students. Even in a face-to-face setting, many students needed prodding and encouragement to share their mathematical ideas, usually because they feared being incorrect. Being physically present in the room, the educator-researcher could listen in for students, and encourage those who weren’t sharing ideas or participating to engage with their group. Silence in an inquiry-based classroom is a lost opportunity for students to share ideas and discuss observations with one another.

During the first term of research, the educator-researcher noted that groups would often sit and work independently, without sharing ideas; no one wanted to “break the ice” and be the first to talk. To circumvent this, in future terms, each class period would begin with an “ice breaker question” for the group, to assign a *group leader*. The group leader did not need to make presentations or make all the generalizations for the group, but was responsible for asking questions, encouraging the group to talk to one another, and keeping the group on task. This was found to be effective, as groups would immediately being talking with one another about and then would turn towards the mathematical topic and directed course notes. When class periods started with these “ice breaker” questions, there seemed to be much less silence at the start of and throughout the class period. A list of sample “ice breaker” questions used for inquiry-based learning classes throughout this research is given in Table 5-1. Once groups started their exercises during each class period, the educator-researcher was available to answer questions, and would listen in to groups checking for struggling groups.

Table 5-1.
Sample "ice breaker" questions for inquiry-based learning groups

Who is the tallest student in the group?
Who has the most siblings?
Who has the most pets?
Who traveled the furthest over the weekend?
Who has the longest hair?
Whose hometown is the furthest from our campus?
Who woke up earliest this morning?
Who woke up the closest to class time this morning?
Who has visited the most U.S. states in their lifetime?

Working with groups in the online format during COVID-19 was even more challenging. The researcher-instructor provided Zoom breakout rooms, with 4-5 students in each breakout room. As discussed in Section 3-3 and Table 3-1, students would collaborate in the Zoom breakout rooms using a Google Jamboard (a shared online document that each student could edit, so that the other group members could see, as shown in Table 3-2). The educator-researcher had each group's Jamboard on her laptop, and would monitor the Jamboards to watch for any struggling groups. However, this proved to be harder than in the face-to-face environment; keeping groups on tasks and engaged, while remote, was extremely difficult. Many students preferred to stay "muted" and idea sharing was much harder online, even with a peer group leader during each class period. Research has shown that students preferred to stay "muted" and not share their ideas in an online class, out of fear of being wrong or judged by their peers, and the permanency of their thoughts in the recorded online format (Blackley et al., 2021).

5.4.2 Recommendations for implementing inquiry-based learning

To best implement inquiry-based learning, particularly in a general education mathematics course which services a variety of majors and has a majority of non-STEM majors, the following recommendations are made through the research results of this study. The most effective inquiry-based learning environment for a general education mathematics course is in a face-to-face classroom, not an online classroom. This research was designed to be conducted face-to-face, but the COVID-19 pandemic forced two of the semesters to be taught online. Like many other educators, moving the course to online during the pandemic was extremely challenging (Sedaghatjou et al., 2023).

First, inquiry-based learning requires engagement of students on a regular basis. This is easiest to do in a fully face-to-face classroom. While this research included two terms of fully online courses, the online environment was less than ideal for IBL for general education mathematics. The educator and research participants all noted that there was regularly silence in the Zoom breakout rooms; silence on Zoom during group work time is lost opportunity in an inquiry-based classroom. Inquiry-based learning requires every opportunity for students to share ideas, explain observations to their peers, or make new conjectures, and those are most easily achieved in the face-to-face classroom. For most undergraduate students, a face-to-face classroom feels “normal” and safe; learning remotely online while being “stuck” at home did not feel normal to students, and it impacted how they felt in class, and how the researcher felt during class as well.

Second, students need an environment where they feel safe to share their ideas, developed through social constructivism. The students enrolled in the general education mathematics class often had a “first idea” which was not completely correct, but needed to share those ideas and talk with their peers to fine-tune their ideas and correct their initial observations. An environment where students feel “safe”, knowing they can share their ideas, make mistakes, and learn from those mistakes is imperative. As an educator, students needed to be reminded regularly that they could make mistakes, and that they could ask *questions* without labeling them as “dumb questions.” In a face-to-face class, students could freely share ideas in a community of learners, while online often felt like they were being watched or recorded.

Finally, in a face-to-face classroom, students can regularly see the educator, which seems “normal”. Even though the instructor is not directing the class and is acting as a “guide on the side” (White-Clark et al., 2008) in an inquiry-based classroom, students are acting as cognitive constructivists and often need guidance while developing ideas. In the face-to-face classroom, students have someone that they can look to if they are completely stuck, do not understand a major concept, or have a question. This allows students to assist students as they struggle with mathematics in their zone of productive struggle (Townsend et al., 2018). In the online classroom, the students often felt alone, and did not always have an educator present to ask questions or verify their ideas. However, the results from this study showed that the interactive lecture students scored statistically significantly higher than the inquiry-based students in these face-to-face classrooms (Section 4.6.1.1 and Section 4.6.1.2); such differences were not present in comparison of the online courses.

For all of these reasons, based on this research, implementing inquiry-based learning in a general education mathematics class is recommended in a face-to-face environment, until new technology enables greater interaction with students in an online environment, or students become more willing to interact and share ideas in an online environment. Providing students with task scaffolding, peer support, a teacher motivated to help them succeed and a positive classroom environment each will support students as they develop their affective and cognitive domains through mathematics (Townsend et al., 2018). The students in a general education classroom, who often need more support and encouragement, benefit from the physical space and presence together in a classroom.

5.4.3 Implications for further research

This study built on previous research, which had considered implications of inquiry-based learning in upper-division mathematics and science classes. This study adds to that research, as it considers the impact in a general education classroom with a more intellectually diverse group of students. To the researcher's knowledge, this is the first study investigating the impact of IBL in a general education mathematics classroom at the collegiate level. This research study is unique as it gives a view of a GE mathematics class, and the impact on students before, during, and after the COVID-19 pandemic. Very few research studies have investigated the learning environment at the collegiate level. While there has been research done at primary and secondary levels, and in upper-division college classes, none have been done in a general education classroom.

This research was conducted at one university, in one instructor's classroom, so students in both the control (interactive lecture) and experimental (inquiry-based learning) groups were exposed to the same curriculum, in the same order and at the same pace, and were given the same assessments at the end of each unit. Thus, the differences noted were social (i.e., were they working through new content with their peers in active learning, or were they learning from an instructor presenting the material) and physical (i.e., working closely with a small group, either in the face-to-face setting or online, or facing and watching an instructor). By examining student understanding of core content based on in-class assessments, student attitudes towards mathematics and their perceptions of the learning environments, this study considers inquiry-based learning in both the cognitive and affective domains. Based on this research, these are recommendations for future research.

These tight controls allowed for differences to be noted from the 121 research participants. However, if similar research could be completed in multiple classrooms with more research participants, a greater generalization of findings could be found. Further research is recommended in similar general education classrooms, which encompass a wide breadth of majors and deep mathematical content, to determine if similarities are found. If a larger study were conducted, there could be greater generalizations made; a recommendation would be to conduct a greater survey (more classrooms, more instructors, more universities), similarly investigating the impact of the classroom environment, in both the social and cognitive domains, and the impact the learning environment has on learning of key concepts, student attitudes towards mathematics and their self-confidence in mathematics. Furthermore, future research would allow for more data in more typical face-to-face environment, without a global pandemic.

5.5 Final summation

Mathematics educators teaching general education mathematics classroom are uniquely positioned to impact student views of mathematics. This thesis examined the impact of inquiry-based learning in a general education mathematics classroom, both in a traditional classroom setting (pre-COVID) and in an online setting (during COVID). Instruction via inquiry-based learning changes the relationship between teacher and student in the classroom, so both cognitive and affective domains of learning were considered. Student learning outcomes and performance on in-class assessments were considered, as well as students’ perception of their attitudes towards mathematics and their perspective of their learning and learning environment. Based on the quantitative and qualitative data collected in this study, the impact of inquiry-based learning on a general education mathematics class is minimal. These learning domains were assessed based on students' perspective of their learning environment, their attitudes towards mathematics, and their understanding of key concepts.

Overall, this study provided surprising results. When comparing the inquiry-based learning participants to the interactive lecture students in Section 4.5, the notable differences were found in improved scores in the participants ATMI scores for the value of mathematics. Additionally, there was a statistically significant decrease in responses about self-confidence in mathematics for all research participants, as noted in Section 4.5.2.3. The overall course grades were statistically significantly higher for the research participants in the interactive lecture group as well.

In the face-to-face courses, the interactive lecture students scored statistically significantly higher on the final exam as well as the overall course grade, as compared to the inquiry-based learning course. However, in those face-to-face sections, there were no notable differences in the ATMI scores. In the online courses, no statistically significant results were found in any of the quantitative data – understanding of key concepts as shown on the course quizzes, exams, and scores on the ATMI in the four sub-scales. This was somewhat surprising, but the data for the online courses was collected in the height of the worldwide COVID-19 pandemic, and students may have been surviving through classes, instead of thriving, as they would in a less stressful, more normal time.

These results are different than the results found in the previous research studies (Kogan & Laursen, 2014; Laursen et al., 2011). However, the timing of this research study is unique, given the opportunity to collect data just before, and during the COVID-19 pandemic. From Fall 2020 through Fall 2021, during the COVID-19 pandemic, students returned to classrooms, either online through an online communications platform such as Zoom or in a classroom with masks and social distancing in place. Data was collected for this study each of those three terms; in this time, students who learned mathematics online reported poor communication from instructors and difficulty adapting to online learning (Baticulon et al., 2021). Mathematics students during the COVID-19 pandemic also reported that they found learning face-to-face more enjoyable than learning online, and that poor academic performance during the time may be attributed to an unfamiliar learning environment (Bringula, 2021). The participants in this study were learning during that same time period and may have found the environment in the synchronous interactive lecture more familiar.

In this study, the educator found that even with inquiry-based course notes, there was a great barrier of familiarity with open-ended questions and communication within small groups in the classroom. Working in small groups, discussion and creating new ideas, was unfamiliar to many general education students. Having a designated group leader for each class period helped students more clearly lead discussions and work effectively in their small groups. Instructors who wish to implement inquiry-based learning in a general education mathematics classroom should allow time for students to become familiar in the learning environment. Educators must also carefully craft lessons and course notes which lead to inquiry in the classroom (Fitzgerald et al., 2019).

In conclusion, while many researchers have found that inquiry-based learning increases student self-confidence and achievement in mathematics courses for STEM majors (Kogan & Laursen, 2014; Laursen et al., 2011), in this study, participants who were a majority of non-STEM majors, were enrolled in a general education mathematics course, The participants in courses which were taught by the more traditional interactive lecture had higher scores on course assessments than those taught by inquiry-based learning, and those in the interactive lecture courses had larger gains on the Attitude Towards Mathematics Inventory (ATMI) (Appendix A) in their perception of the value of mathematics, enjoyment, confidence, and motivation in mathematics than the participants enrolled in inquiry-based learning. This may have been because in an uncertain time, the learning style of inquiry-based learning was more familiar for them. However, becoming more accustomed to an inquiry-based learning environment may result in increased self-confidence and achievement in non-STEM students, as it does with STEM students (Kogan & Laursen, 2014; Laursen et al., 2011).

There is no single method by which to best teach mathematics, but as Hungarian-American mathematician Paul Halmos wrote, “The only way to learn mathematics is to do mathematics.*”* Educators should engage students in *doing* mathematics to *learn* mathematics. This can be achieved by providing students with inquiry assignments, allow students to work in small works communicating their ideas, clearly defining roles in the classroom, and allowing students the opportunity to become familiar with the process of *doing* mathematics in the classroom. General education students becoming more accustomed to and familiar with an inquiry-based environment will give these students the opportunity to *learn mathematics by doing mathematics*.

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APPENDIX A: Attitudes Towards Mathematics Inventory

NOTE: The Attitudes towards Mathematics Inventory was developed by Martha Tapia and George Marsh (2004), and is discussed in Section 3.5.2. The ATMI was used in my research study with the permission of Martha Tapia.

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Darken the circle that most closely corresponds to how the statements best describes your feelings. Use the following response scale to respond to each item.

PLEASE USE THESE RESPONSE CODES:
A – Strongly Disagree
B – Disagree
C – Neutral
D – Agree
E – Strongly Agree

1. Mathematics is a very worthwhile and necessary subject.
2. I want to develop my mathematical skills.
3. I get a great deal of satisfaction out of solving a mathematics problem.
4. Mathematics helps develop the mind and teaches a person to think.
5. Mathematics is important in everyday life.
6. Mathematics is one of the most important subjects for people to study.
7. High school math courses would be very helpful no matter what I decide to study.
8. I can think of many ways that I use math outside of school.
9. Mathematics is one of my most dreaded subjects.
10. My mind goes blank and I am unable to think clearly when working with mathematics.
11. Studying mathematics makes me feel nervous.
12. Mathematics makes me feel uncomfortable.
13. I am always under a terrible strain in a math class.
14. When I hear the word mathematics, I have a feeling of dislike.
15. It makes me nervous to even think about having to do a mathematics problem.
16. Mathematics does not scare me at all.
17. I have a lot of self-confidence when it comes to mathematics
18. I am able to solve mathematics problems without too much difficulty.
19. I expect to do fairly well in any math class I take.
20. I am always confused in my mathematics class.
21. I feel a sense of insecurity when attempting mathematics.
22. I learn mathematics easily.
23. I am confident that I could learn advanced mathematics.
24. I have usually enjoyed studying mathematics in school.
25. Mathematics is dull and boring.
26. I like to solve new problems in mathematics.
27. I would prefer to do an assignment in math than to write an essay.
28. I would like to avoid using mathematics in college.
29. I really like mathematics.
30. I am happier in a math class than in any other class.
31. Mathematics is a very interesting subject.
32. I am willing to take more than the required amount of mathematics.
33. I plan to take as much mathematics as I can during my education.
34. The challenge of math appeals to me.
35. I think studying advanced mathematics is useful.
36. I believe studying math helps me with problem solving in other areas.
37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.
38. I am comfortable answering questions in math class.
39. A strong math background could help me in my professional life.
40. I believe I am good at solving math problems.

(Tapia & Marsh, 2004)

Scoring Key for the **Attitude Towards Mathematics Inventory**

Scoring
Each question was scored by students on a scale of 1-5.

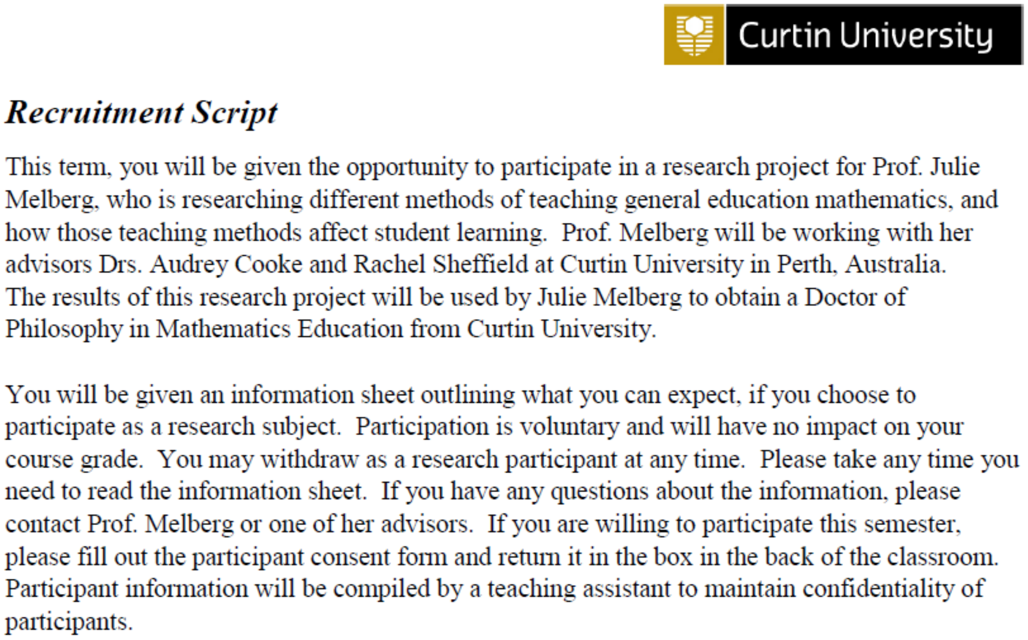
Subscales
Self-confidence : Items 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 & 40
Value: Items 1, 2, 4, 5, 6, 7, 8, 35, 36 & 39
Enjoyment: Items 3, 24, 25, 26, 27, 29, 30, 31, 37 & 38
Motivation: Items 23, 28, 32, 33 & 34

Reverse Coding
The following items are reverse items.

9. Mathematics is one of my most dreaded subjects.
10. My mind goes blank and I am unable to think clearly when working with mathematics.
11. Studying mathematics makes me feel nervous.
12. Mathematics makes me feel uncomfortable.
13. I am always under a terrible strain in a math class.
14. When I hear the word mathematics, I have a feeling of dislike.
15. It makes me nervous to even think about having to do a mathematics problem.
20. I am always confused in my mathematics class.
21. I feel a sense of insecurity when attempting mathematics.
25. Mathematics is dull and boring.
28. I would like to avoid using mathematics in college.

For analysis: Score = 6 - item12 to determine the correct value for analysis.

APPENDIX B: Recruitment Script



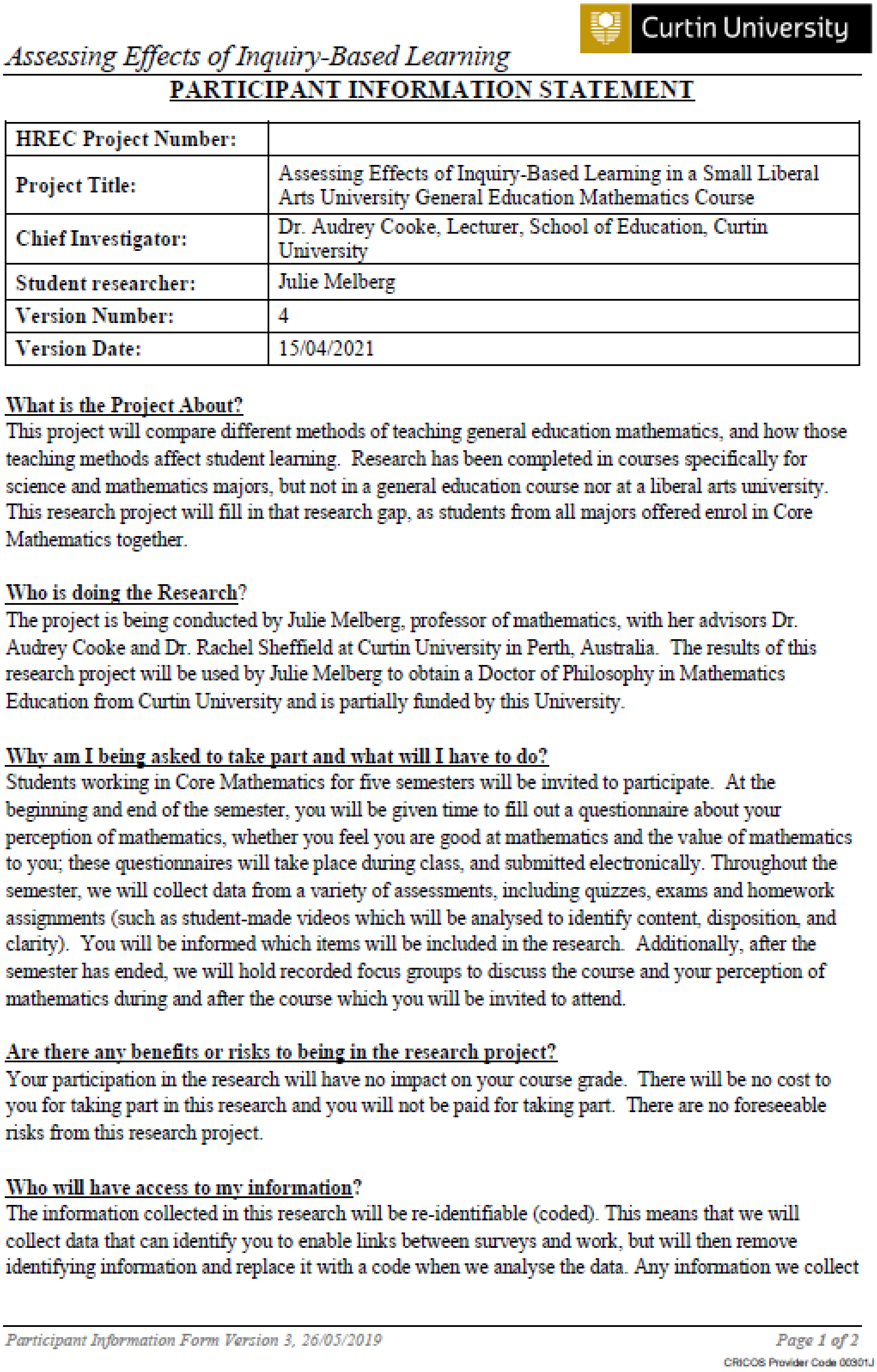
Curtin University

Recruitment Script

This term, you will be given the opportunity to participate in a research project for Prof. Julie Melberg, who is researching different methods of teaching general education mathematics, and how those teaching methods affect student learning. Prof. Melberg will be working with her advisors Drs. Audrey Cooke and Rachel Sheffield at Curtin University in Perth, Australia. The results of this research project will be used by Julie Melberg to obtain a Doctor of Philosophy in Mathematics Education from Curtin University.

You will be given an information sheet outlining what you can expect, if you choose to participate as a research subject. Participation is voluntary and will have no impact on your course grade. You may withdraw as a research participant at any time. Please take any time you need to read the information sheet. If you have any questions about the information, please contact Prof. Melberg or one of her advisors. If you are willing to participate this semester, please fill out the participant consent form and return it in the box in the back of the classroom. Participant information will be compiled by a teaching assistant to maintain confidentiality of participants.

APPENDIX C: Participant Information Sheet



Curtin University

Assessing Effects of Inquiry-Based Learning

PARTICIPANT INFORMATION STATEMENT

HREC Project Number:	
Project Title:	Assessing Effects of Inquiry-Based Learning in a Small Liberal Arts University General Education Mathematics Course
Chief Investigator:	Dr. Audrey Cooke, Lecturer, School of Education, Curtin University
Student researcher:	Julie Melberg
Version Number:	4
Version Date:	15/04/2021

What is the Project About?

This project will compare different methods of teaching general education mathematics, and how those teaching methods affect student learning. Research has been completed in courses specifically for science and mathematics majors, but not in a general education course nor at a liberal arts university. This research project will fill in that research gap, as students from all majors offered enrol in Core Mathematics together.

Who is doing the Research?

The project is being conducted by Julie Melberg, professor of mathematics, with her advisors Dr. Audrey Cooke and Dr. Rachel Sheffield at Curtin University in Perth, Australia. The results of this research project will be used by Julie Melberg to obtain a Doctor of Philosophy in Mathematics Education from Curtin University and is partially funded by this University.

Why am I being asked to take part and what will I have to do?

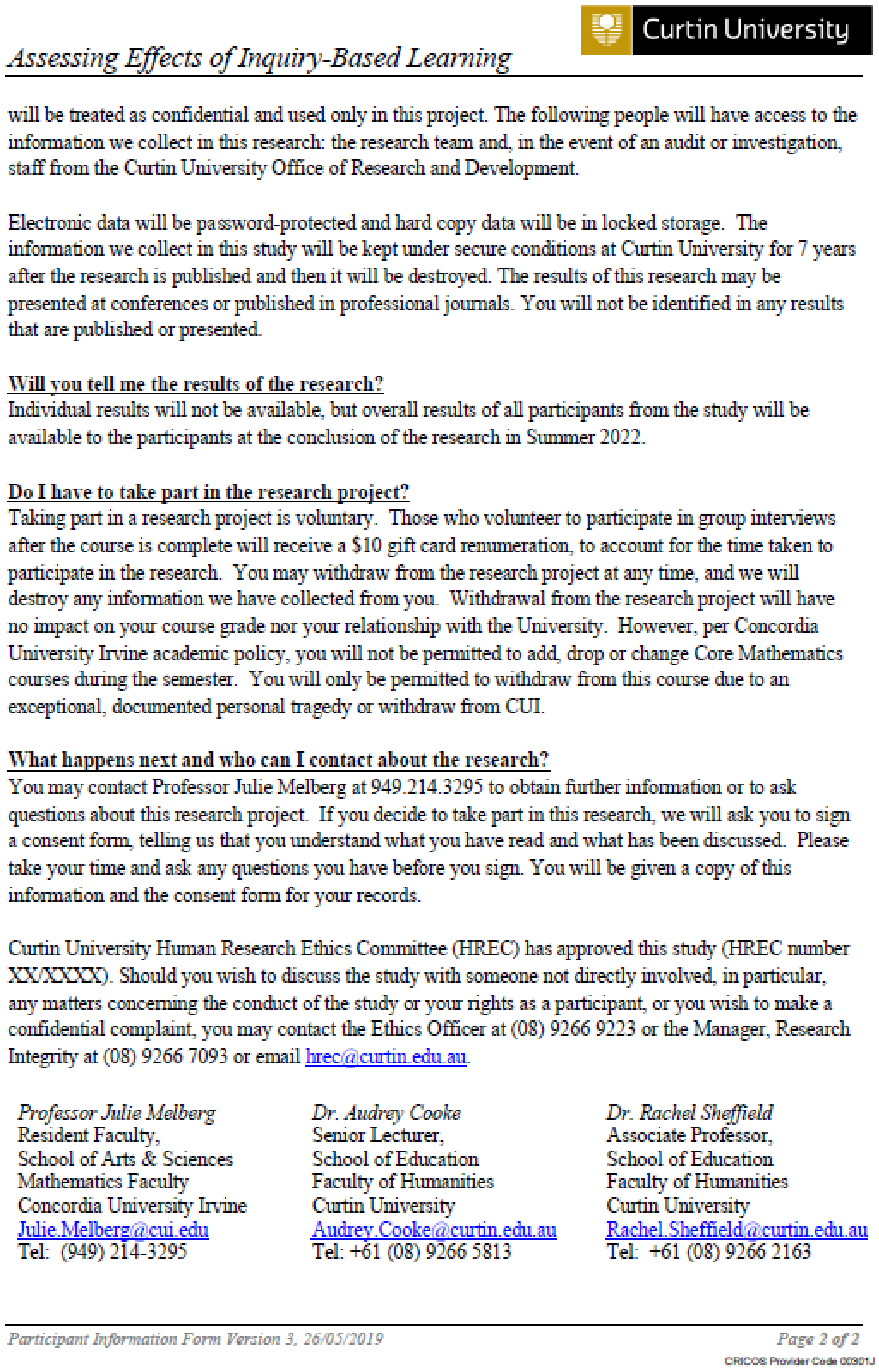
Students working in Core Mathematics for five semesters will be invited to participate. At the beginning and end of the semester, you will be given time to fill out a questionnaire about your perception of mathematics, whether you feel you are good at mathematics and the value of mathematics to you; these questionnaires will take place during class, and submitted electronically. Throughout the semester, we will collect data from a variety of assessments, including quizzes, exams and homework assignments (such as student-made videos which will be analysed to identify content, disposition, and clarity). You will be informed which items will be included in the research. Additionally, after the semester has ended, we will hold recorded focus groups to discuss the course and your perception of mathematics during and after the course which you will be invited to attend.

Are there any benefits or risks to being in the research project?

Your participation in the research will have no impact on your course grade. There will be no cost to you for taking part in this research and you will not be paid for taking part. There are no foreseeable risks from this research project.

Who will have access to my information?

The information collected in this research will be re-identifiable (coded). This means that we will collect data that can identify you to enable links between surveys and work, but will then remove identifying information and replace it with a code when we analyse the data. Any information we collect



will be treated as confidential and used only in this project. The following people will have access to the information we collect in this research: the research team and, in the event of an audit or investigation, staff from the Curtin University Office of Research and Development.

Electronic data will be password-protected and hard copy data will be in locked storage. The information we collect in this study will be kept under secure conditions at Curtin University for 7 years after the research is published and then it will be destroyed. The results of this research may be presented at conferences or published in professional journals. You will not be identified in any results that are published or presented.

Will you tell me the results of the research?
Individual results will not be available, but overall results of all participants from the study will be available to the participants at the conclusion of the research in Summer 2022.

Do I have to take part in the research project?
Taking part in a research project is voluntary. Those who volunteer to participate in group interviews after the course is complete will receive a $10 gift card renumeration, to account for the time taken to participate in the research. You may withdraw from the research project at any time, and we will destroy any information we have collected from you. Withdrawal from the research project will have no impact on your course grade nor your relationship with the University. However, per Concordia University Irvine academic policy, you will not be permitted to add, drop or change Core Mathematics courses during the semester. You will only be permitted to withdraw from this course due to an exceptional, documented personal tragedy or withdraw from CUI.

What happens next and who can I contact about the research?
You may contact Professor Julie Melberg at 949.214.3295 to obtain further information or to ask questions about this research project. If you decide to take part in this research, we will ask you to sign a consent form, telling us that you understand what you have read and what has been discussed. Please take your time and ask any questions you have before you sign. You will be given a copy of this information and the consent form for your records.

Curtin University Human Research Ethics Committee (HREC) has approved this study (HREC number XX/XXXX). Should you wish to discuss the study with someone not directly involved, in particular, any matters concerning the conduct of the study or your rights as a participant, or you wish to make a confidential complaint, you may contact the Ethics Officer at (08) 9266 9223 or the Manager, Research Integrity at (08) 9266 7093 or email hrec@curtin.edu.au.

Professor Julie Melberg
Resident Faculty,
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Dr. Audrey Cooke
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Dr. Rachel Sheffield
Associate Professor,
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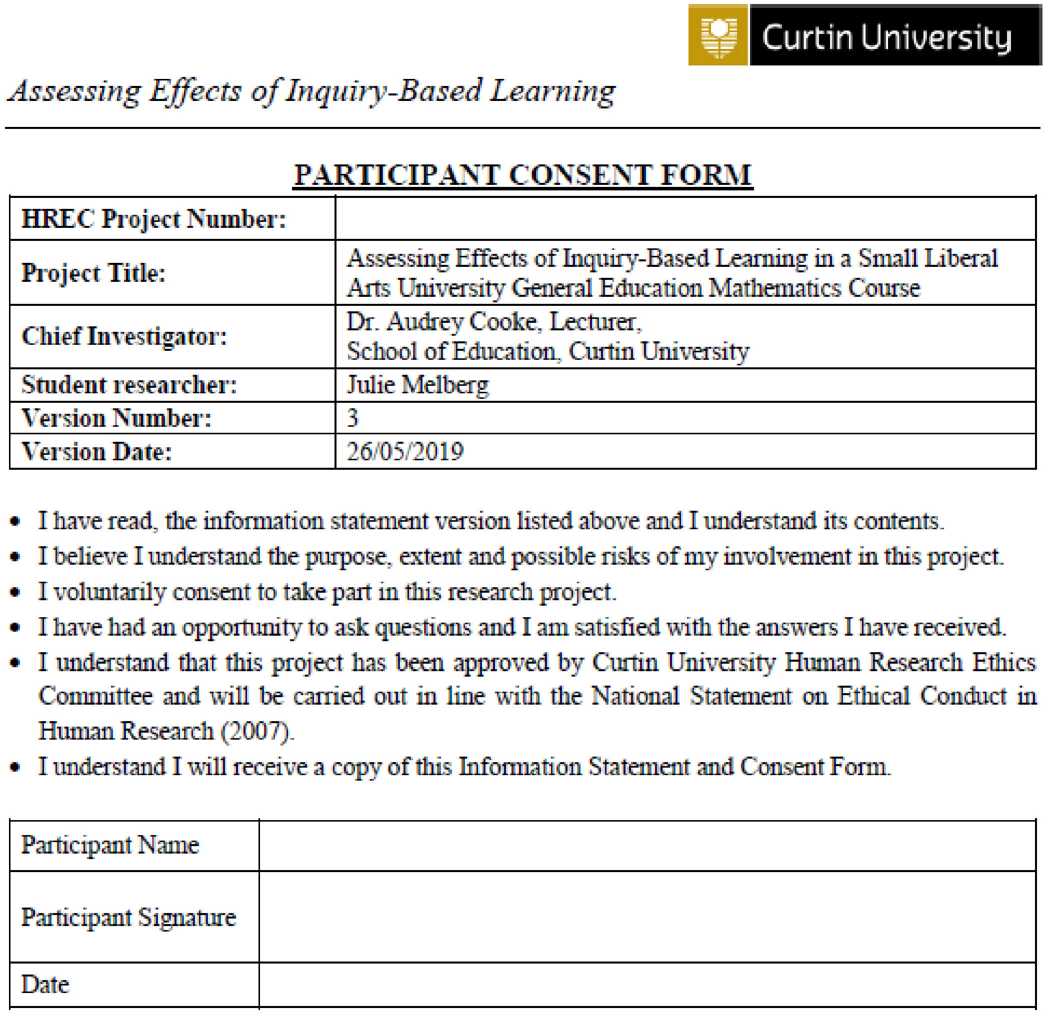
APPENDIX D: Participant Consent Form



Curtin University

Assessing Effects of Inquiry-Based Learning

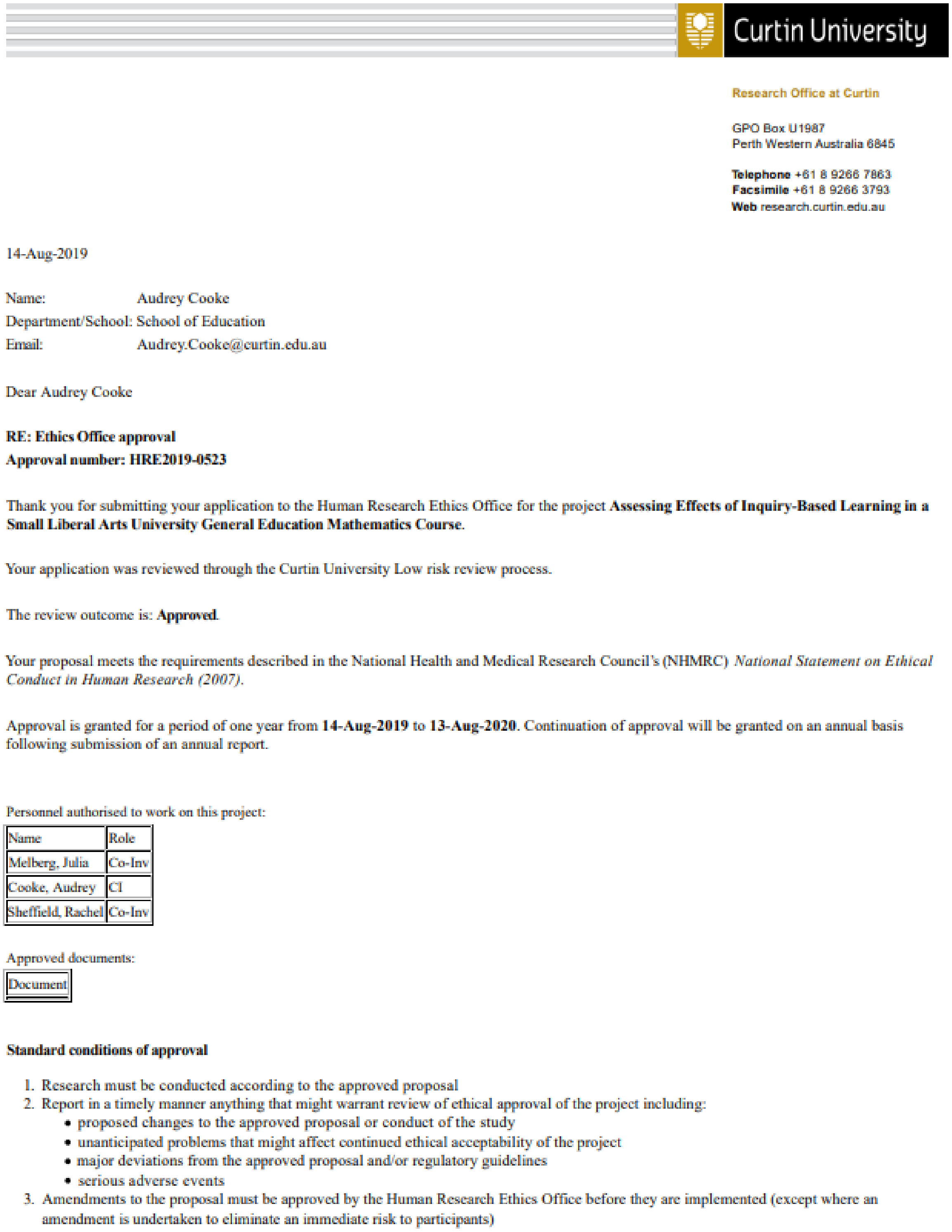
PARTICIPANT CONSENT FORM

HREC Project Number:	
Project Title:	Assessing Effects of Inquiry-Based Learning in a Small Liberal Arts University General Education Mathematics Course
Chief Investigator:	Dr. Audrey Cooke, Lecturer, School of Education, Curtin University
Student researcher:	Julie Melberg
Version Number:	3
Version Date:	26/05/2019

- I have read, the information statement version listed above and I understand its contents.
- I believe I understand the purpose, extent and possible risks of my involvement in this project.
- I voluntarily consent to take part in this research project.
- I have had an opportunity to ask questions and I am satisfied with the answers I have received.
- I understand that this project has been approved by Curtin University Human Research Ethics Committee and will be carried out in line with the National Statement on Ethical Conduct in Human Research (2007).
- I understand I will receive a copy of this Information Statement and Consent Form.

Participant Name	
Participant Signature	
Date	

APPENDIX E: Curtin University Human Research Ethics Approval



Curtin University

Research Office at Curtin

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Perth Western Australia 6845

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Web research.curtin.edu.au

14-Aug-2019

Name: Audrey Cooke
Department/School: School of Education
Email: Audrey.Cooke@curtin.edu.au

Dear Audrey Cooke

RE: Ethics Office approval
Approval number: HRE2019-0523

Thank you for submitting your application to the Human Research Ethics Office for the project **Assessing Effects of Inquiry-Based Learning in a Small Liberal Arts University General Education Mathematics Course**.

Your application was reviewed through the Curtin University Low risk review process.

The review outcome is: **Approved**.

Your proposal meets the requirements described in the National Health and Medical Research Council's (NHMRC) *National Statement on Ethical Conduct in Human Research (2007).*

Approval is granted for a period of one year from **14-Aug-2019** to **13-Aug-2020**. Continuation of approval will be granted on an annual basis following submission of an annual report.

Personnel authorised to work on this project:

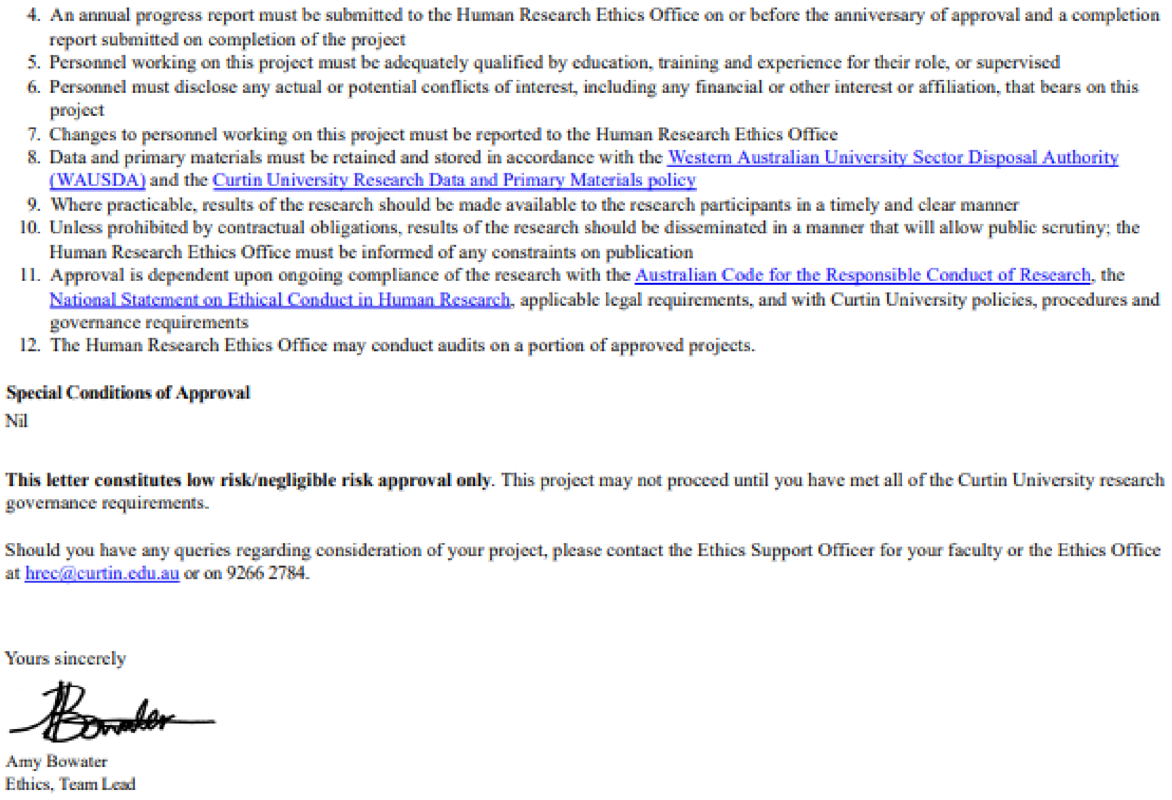
Name	Role
Melberg, Julia	Co-Inv
Cooke, Audrey	CI
Sheffield, Rachel	Co-Inv

Approved documents:

Document

Standard conditions of approval

1. Research must be conducted according to the approved proposal
2. Report in a timely manner anything that might warrant review of ethical approval of the project including:
    - proposed changes to the approved proposal or conduct of the study
    - unanticipated problems that might affect continued ethical acceptability of the project
    - major deviations from the approved proposal and/or regulatory guidelines
    - serious adverse events
3. Amendments to the proposal must be approved by the Human Research Ethics Office before they are implemented (except where an amendment is undertaken to eliminate an immediate risk to participants)



4. An annual progress report must be submitted to the Human Research Ethics Office on or before the anniversary of approval and a completion report submitted on completion of the project
5. Personnel working on this project must be adequately qualified by education, training and experience for their role, or supervised
6. Personnel must disclose any actual or potential conflicts of interest, including any financial or other interest or affiliation, that bears on this project
7. Changes to personnel working on this project must be reported to the Human Research Ethics Office
8. Data and primary materials must be retained and stored in accordance with the Western Australian University Sector Disposal Authority (WAUSDA) and the Curtin University Research Data and Primary Materials policy
9. Where practicable, results of the research should be made available to the research participants in a timely and clear manner
10. Unless prohibited by contractual obligations, results of the research should be disseminated in a manner that will allow public scrutiny; the Human Research Ethics Office must be informed of any constraints on publication
11. Approval is dependent upon ongoing compliance of the research with the Australian Code for the Responsible Conduct of Research, the National Statement on Ethical Conduct in Human Research, applicable legal requirements, and with Curtin University policies, procedures and governance requirements
12. The Human Research Ethics Office may conduct audits on a portion of approved projects.

Special Conditions of Approval

Nil

This letter constitutes low risk/negligible risk approval only. This project may not proceed until you have met all of the Curtin University research governance requirements.

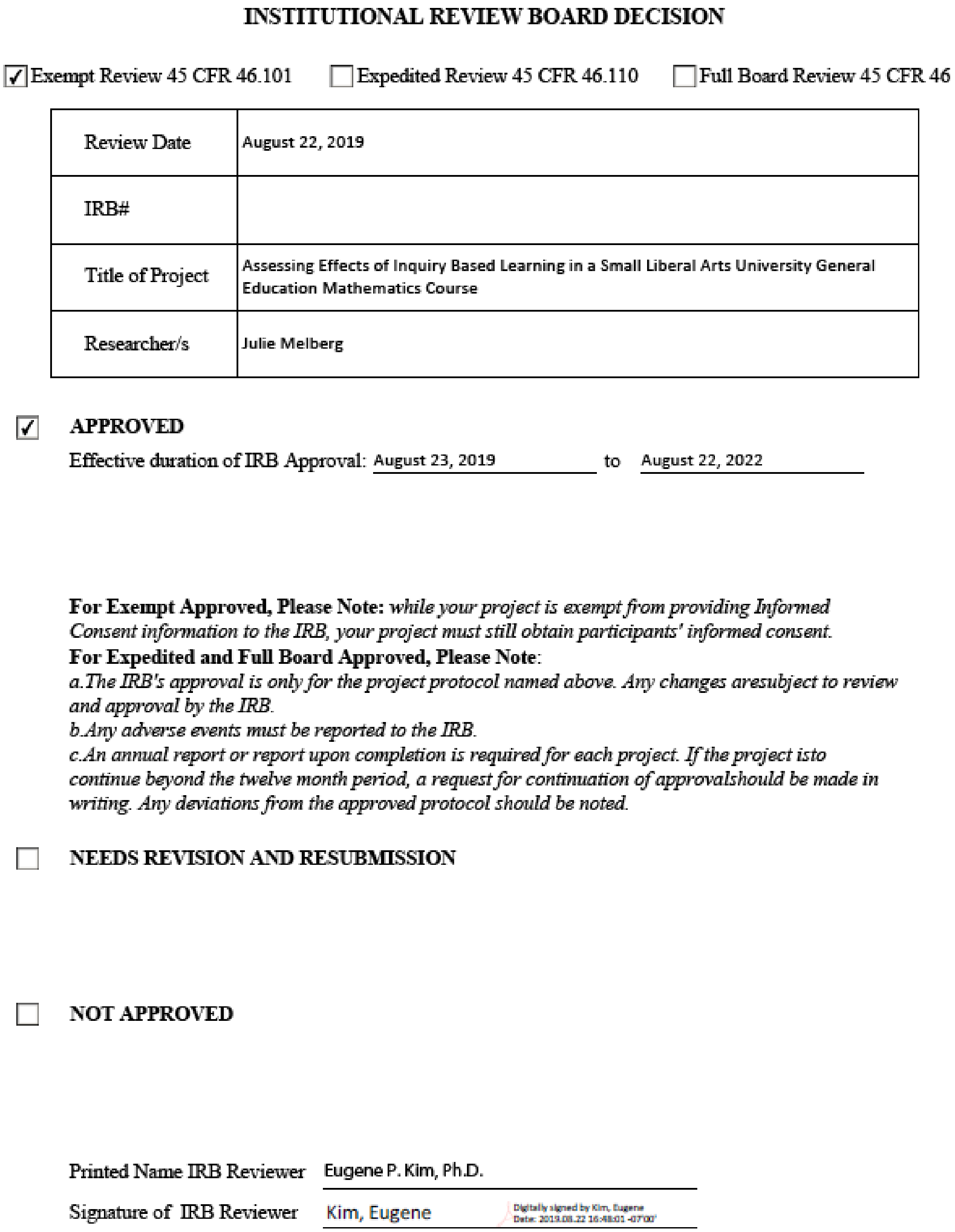
Should you have any queries regarding consideration of your project, please contact the Ethics Support Officer for your faculty or the Ethics Office at hrec@curtin.edu.au or on 9266 2784.

Yours sincerely

ABowater

Amy Bowater
Ethics, Team Lead

APPENDIX F: University Institutional Review Board Approval



INSTITUTIONAL REVIEW BOARD DECISION

☑ Exempt Review 45 CFR 46.101 ☐ Expedited Review 45 CFR 46.110 ☐ Full Board Review 45 CFR 46

Review Date	August 22, 2019
IRB#	
Title of Project	Assessing Effects of Inquiry Based Learning in a Small Liberal Arts University General Education Mathematics Course
Researcher/s	Julie Melberg

☑ **APPROVED**

Effective duration of IRB Approval: August 23, 2019 to August 22, 2022

For Exempt Approved, Please Note: *while your project is exempt from providing Informed Consent information to the IRB, your project must still obtain participants' informed consent.*

For Expedited and Full Board Approved, Please Note:

a.The IRB's approval is only for the project protocol named above. Any changes aresubject to review and approval by the IRB.

b.Any adverse events must be reported to the IRB.

c.An annual report or report upon completion is required for each project. If the project isto continue beyond the twelve month period, a request for continuation of approvalshould be made in writing. Any deviations from the approved protocol should be noted.

☐ **NEEDS REVISION AND RESUBMISSION**

☐ **NOT APPROVED**

Printed Name IRB Reviewer Eugene P. Kim, Ph.D.

Signature of IRB Reviewer Kim, Eugene Digitally signed by Kim, Eugene Date: 2019.08.22 16:48:01 -07'00'

APPENDIX G: Sample "The Nature of Mathematics" Course Notes

Fibonacci Numbers

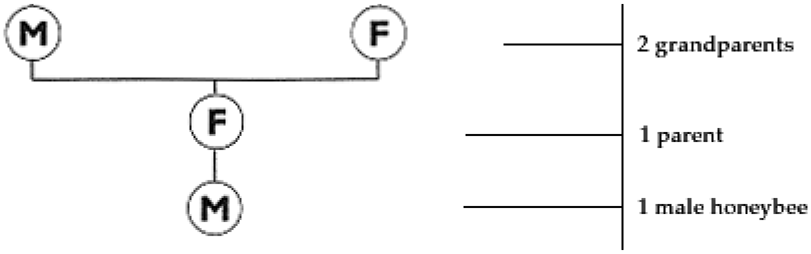
Example 1: Honeybee populations

A male honeybee is called a drone, which is the product of an unfertilized egg.
A female honeybee is called a worker bee, which is the product of a fertilized egg.
Hence, a drone has only a mother, but a worker bee has both a mother and a father.

Consider the family tree of a male honeybee. It would have only one parent (a mother). It would have two grandparents (since the mother, a female, would have both parents.)

The original bee, parent and grandparent generations are drawn in the family tree below.

(a) Draw four more generations.



(i) How many great-grandparents does the bee have?

(ii) How many great-great-grandparents does the bee have?

(iii) How many great-great-great-grandparents does the bee have?

(iv) How many great-great-great-great-grandparents does the bee have?

(b) Do you notice a pattern? Could you predict the number of bees in the next generation? How?

(c) This is called the FIBONACCI sequence. List the first 10 terms of the sequence.

(d) How do you determine a number in the Fibonacci sequence? (You can write your answer in words or as a formula!)

Example 2: Notation

The notation used to denote a Fibonacci number is F_n. For example, F_1 represents the first Fibonacci number, and F_6 represents the sixth Fibonacci number.

(a) Determine each of the following:

(i) F_{12} =

(ii) $F_3 + F_5$ =

(iii) $F_7 - F_5$ =

(iv) $(F_1)^2 + (F_3)^2$ =

(v) F_{15} =

(vi) $F_4 + F_7$ =

(vii) $F_1 + F_2 + F_7$ =

(viii) $(F_1) + (F_2)^2 + (F_3)^3$ =

(b) Consider the formula $F_n + F_{n+2}$. What is the value of the formula when ...

$n = 2$? $n = 5$? $n = 7$?

(c) Consider the formula $(F_n)^2 + (F_{n+2})^2$. What is the value of the formula when ...

$n = 1$? $n = 2$? $n = 4$?

==============================CHECKPOINT==========================

Example 3: Patterns

One of the reasons for mathematicians' fascination with Fibonacci numbers is the many patterns and settings in which they arise. In fact, there is an entire journal, *Fibonacci Quarterly*, devoted to the Fibonacci numbers and other similar numbers. Let's investigate these identities.

(a) Write down and evaluate the sum of the first three Fibonacci numbers; i.e., 1+1+2 =?

1+1+2 = 4

(b) Write down and then evaluate the sum of the first four Fibonacci numbers.

1+1+2+3 = ______

(c) Continuing the pattern, write down and evaluate the sum of the first five Fibonacci numbers.

(d) Write down and evaluate the sum of the first six Fibonacci numbers.

(e) Write down and evaluate the sum of the first seven Fibonacci numbers.

(f) What do you notice about your answers to (a) through (e)? (Are they related to the Fibonacci numbers?)

(g) Consider the sum of the first 8 Fibonacci numbers. Does your observation from part (f) still give the correct sum?

(h) Make a conjecture (educated guess). That is, write the formula for the sum of the first *n* Fibonacci numbers. $1 + 1 + 2 + 3 + 5 + \ldots + F_n =$

APPENDIX H: Sample Questions from "The Nature of Mathematics" Final Exam

The following is a sample of questions on a university-wide final exam. Students in the IBL and the interactive lecture course for this study, as well as other students enrolled in the course with other professors, were given a similar questions on the university-wide departmental final exam.

1. Consider the game of Dodgeball as shown below. Which player is the winner and why?

Player 1:						*Player 2:*					
X	X	O	O	O	X	O	O	X	O	O	X
O	X	X	X	X	O						
O	O	O	O	X	X						
O	O	X	X	O	X						
O	O	X	O	X	X						

(A) Player 1, because his row matches Player 2's row.
(B) Player 2, because his row matches one of Player 1's rows.
(C) Player 1, because his row is different than Player 2's row.
(D) Player 2, because his row is different than each of Player 1's rows.
(E) None of the above

3. You have brought two unmarked buckets to a stream. The buckets hold 5 liters and 3 liters of water, respectively. How can you obtain exactly 4 liters of water to take home?

11. Suppose you have a Golden Rectangle cut out of a piece of paper. Now suppose you fold it in half along its base and then in half along its width. You have just created a new, smaller rectangle. Is that rectangle a Golden Rectangle? Justify your answer.

12. Which of the following is not a place that Fibonacci numbers are found in nature?
(A) In the depth of the roots of a California sago palm tree.
(B) In the number of spirals of the bottom of a pinecone
(C) The family tree of the male honeybee; that is, the number of parents, grandparents, great-grandparents, etc. form a Fibonacci sequence
(D) In the number of spirals in the center of a sunflower
(E) None of the above—that is, they are all examples of occurrences of the Fibonacci numbers in nature

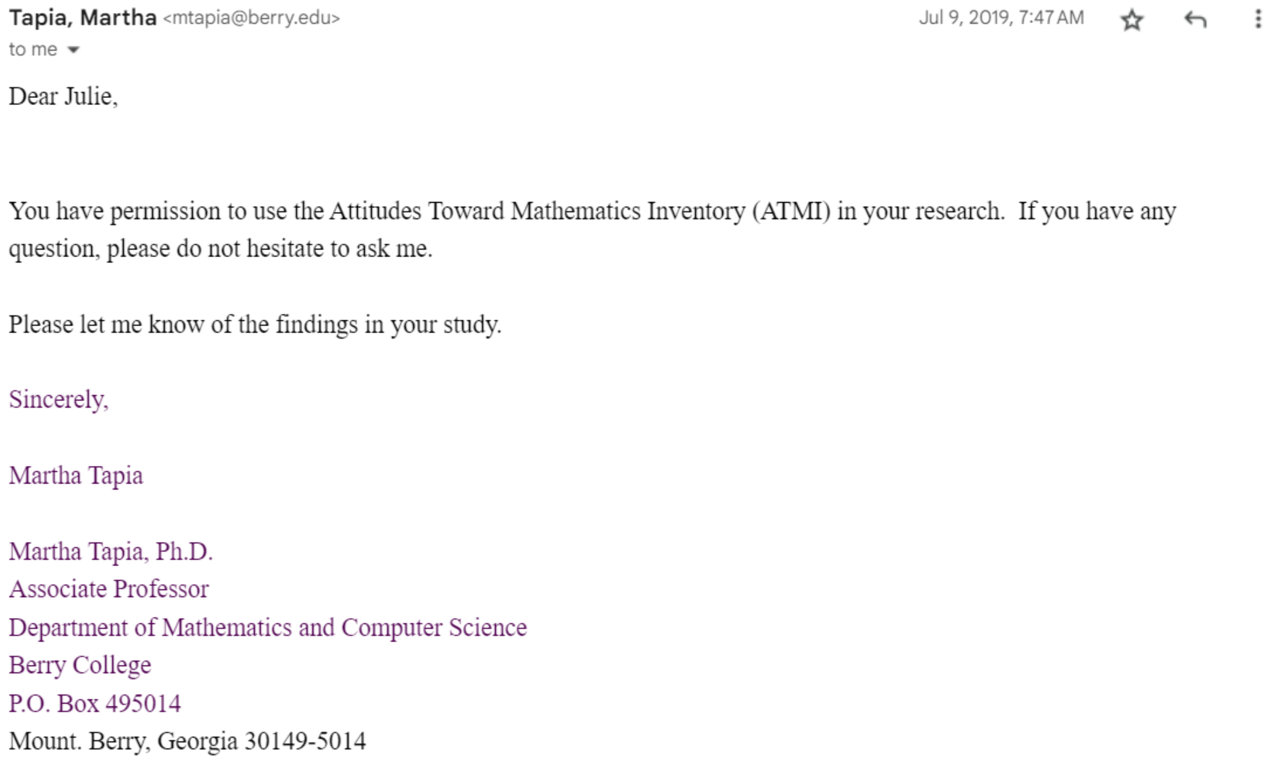
Determine whether each of the following is true for (A) all geometries, (E) Euclidean geometry only, (S) Spherical geometry only or (H) hyperbolic geometry only.
17. Parallel "lines" are equidistant from one another.
18. The sum of the angles of a triangle can be 205 degrees.
19. If equals are added to equals, then the sums are equal.
23. Vertical angles are equal..
28. This geometry guides airline routes around the world.

30. Why are the Common Notions separated from the other axioms in the *Elements*?
(A) The Common Notions are common to all the sciences.
(B) The Common Notions are less important than the other axioms.
(C) The Common Notions are proven in the *Elements*.
(D) All of the above.
(E) None of the above.

33. Give a careful calculation for the Fermi estimate: How many Rubik's Cube toys fill up our math classroom? Begin with the following assumptions:
* Assume our math classroom is 20ft x 20ft x 10ft.
* Assume that the room is completely empty
* Assume each Rubik's Cube toy is 3in x 3in x 3 in.

39. For the 7:30 am classes in this building, every student who has a class and wants a parking space is able to find one. Does that mean there is a one-to-one correspondence between students with 7:30 am classes and parking spaces in the parking lot? Explain your answer.

APPENDIX I: Permission to Use ATMI in Research



Tapia, Martha <mtapia@berry.edu>
to me

Jul 9, 2019, 7:47 AM

Dear Julie,

You have permission to use the Attitudes Toward Mathematics Inventory (ATMI) in your research. If you have any question, please do not hesitate to ask me.

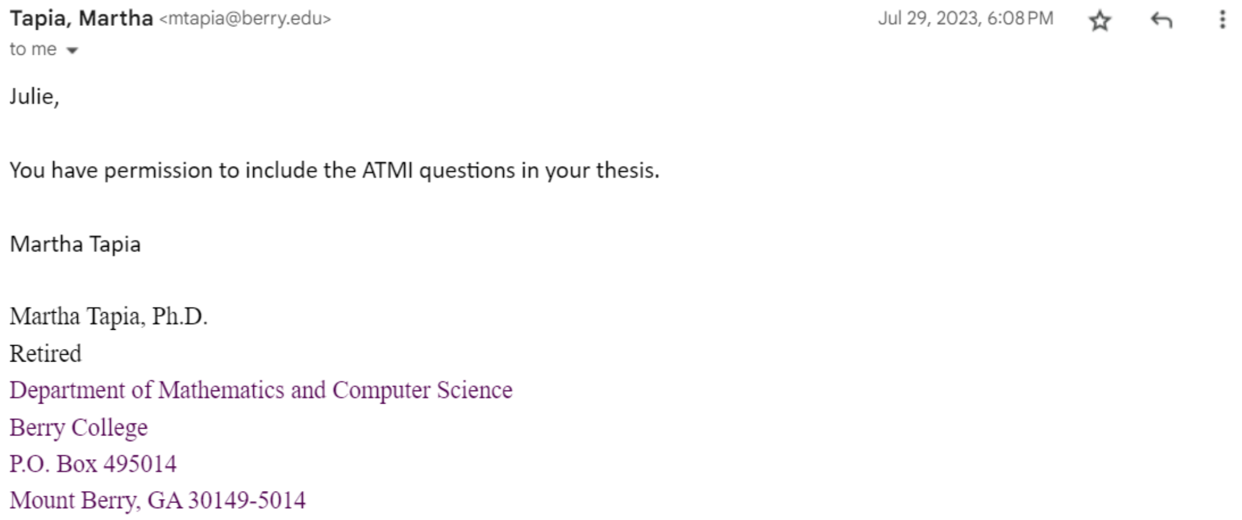
Please let me know of the findings in your study.

Sincerely,

Martha Tapia

Martha Tapia, Ph.D.
Associate Professor
Department of Mathematics and Computer Science
Berry College
P.O. Box 495014
Mount. Berry, Georgia 30149-5014

APPENDIX J: Permission to include ATMI Questions in Thesis



Tapia, Martha <mtapia@berry.edu>
to me

Jul 29, 2023, 6:08 PM

Julie,

You have permission to include the ATMI questions in your thesis.

Martha Tapia

Martha Tapia, Ph.D.
Retired
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